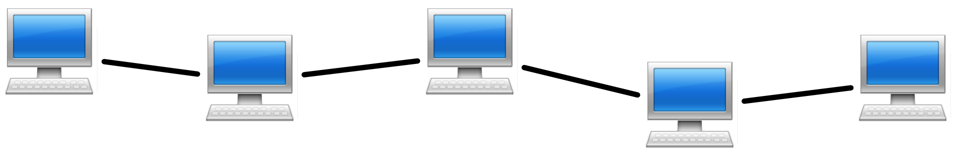
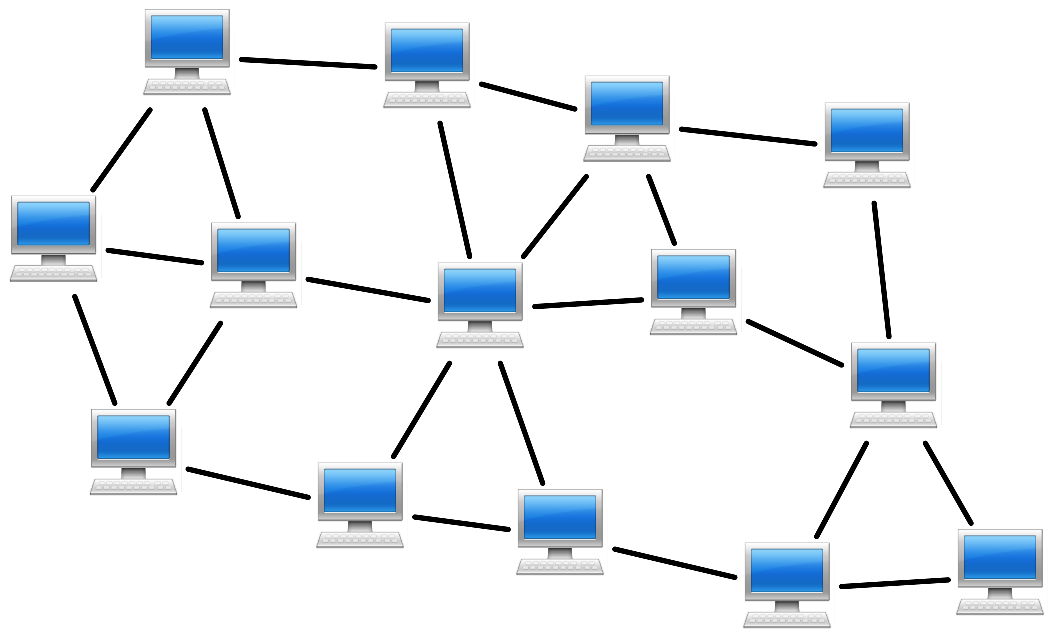
- **Weeks 1–2: informal introduction**
  - network = path 
- **Week 3: graph theory**
- **Weeks 4–7: models of computing**
  - what can be computed (efficiently)?
- **Weeks 8–11: lower bounds**
  - what cannot be computed (efficiently)?
- **Week 12: recap**

# Week 12

– Conclusions

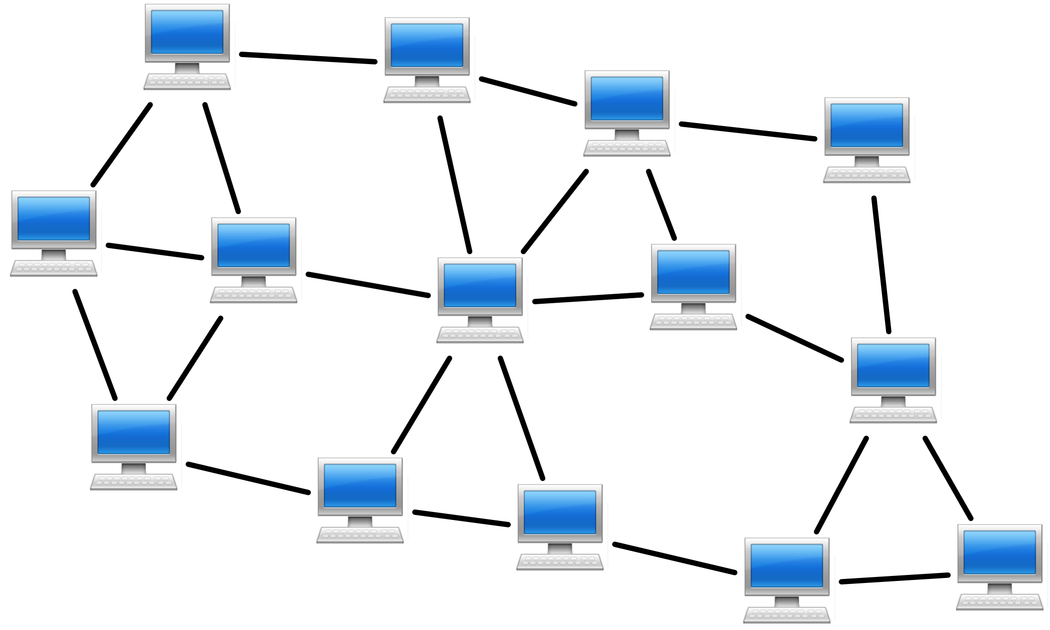
Recap:
# Distributed algorithms

**Algorithms for computer networks**



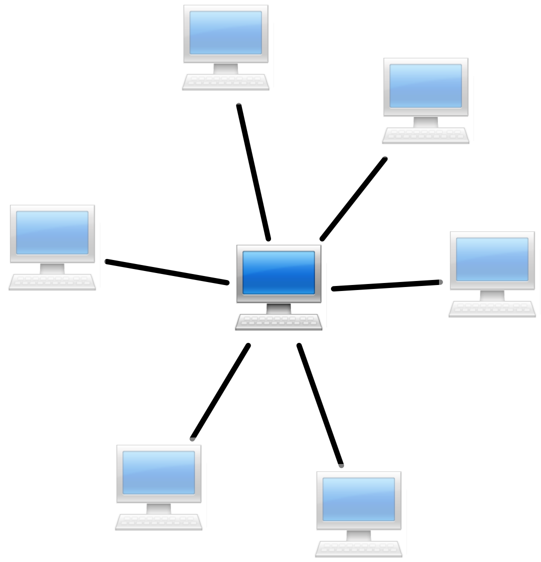
Recap:
# Distributed algorithms

**Identical computers in an unknown network, all running the same algorithm**



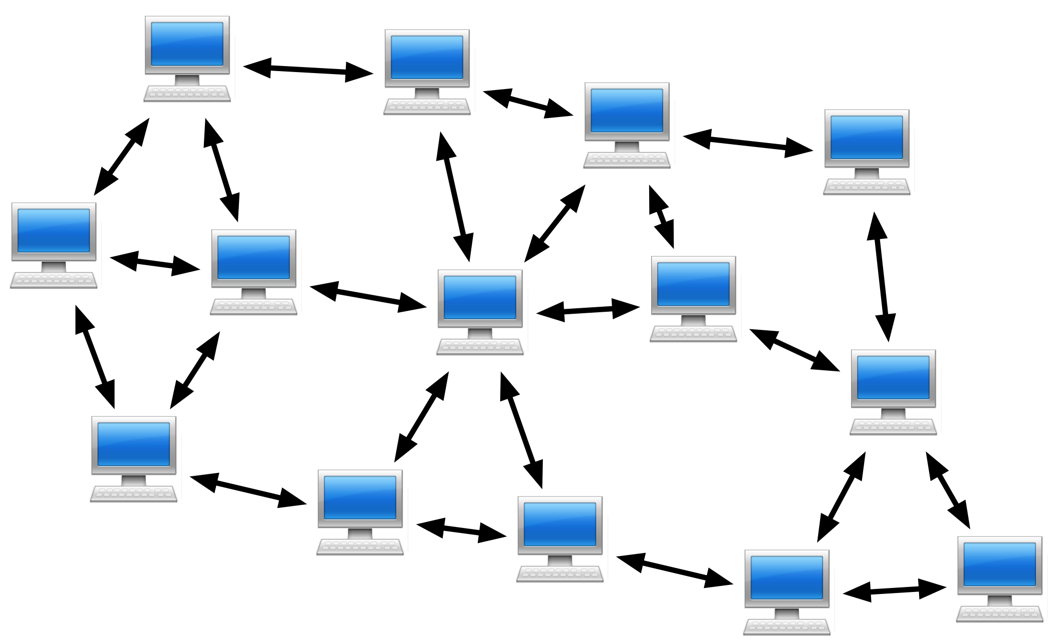
Recap:
# Distributed algorithms

**Initially each computer only aware of its immediate neighbourhood**



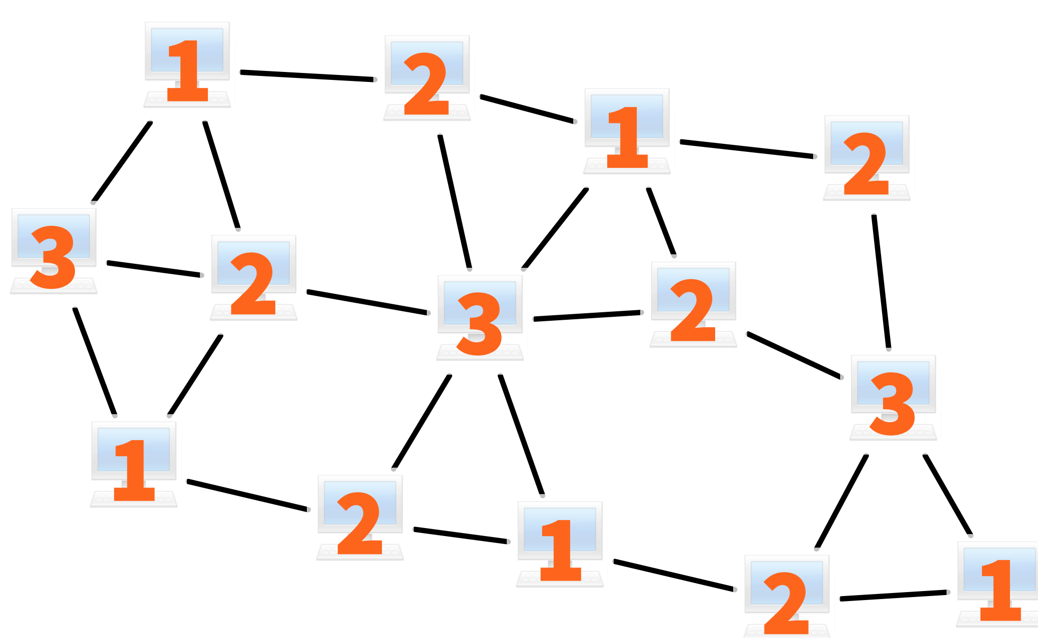
# Recap:
# **Distributed algorithms**

**Nodes can exchange messages with their neighbours to learn more…**



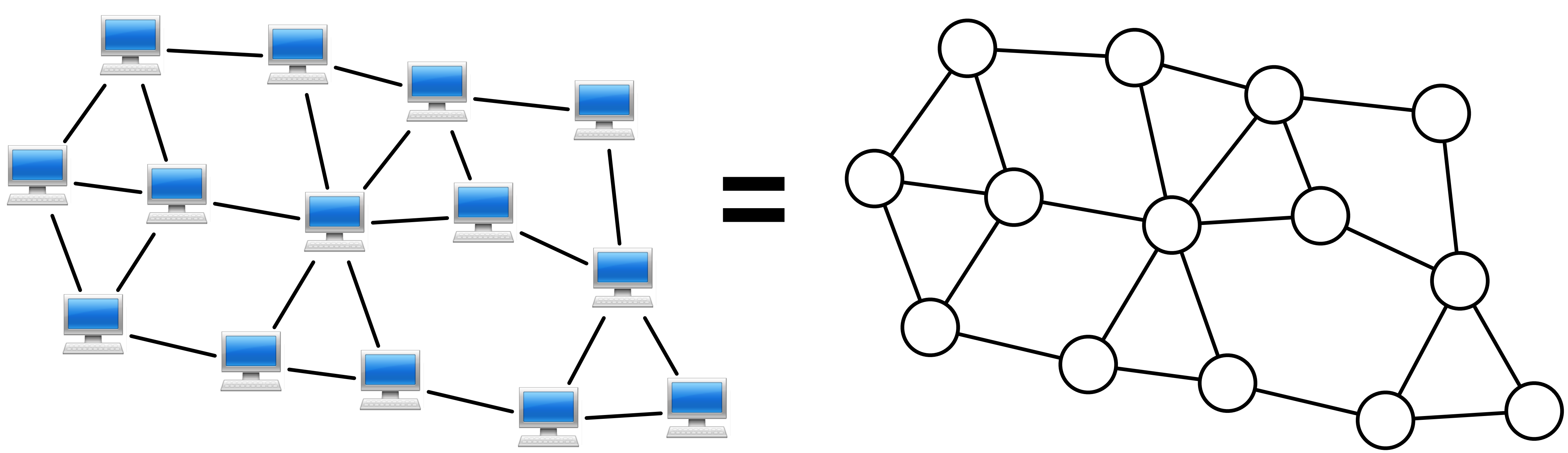
Recap:
# Distributed algorithms

**Finally, each computer has to stop and produce its own local output**



# Recap:
# **Distributed algorithms**

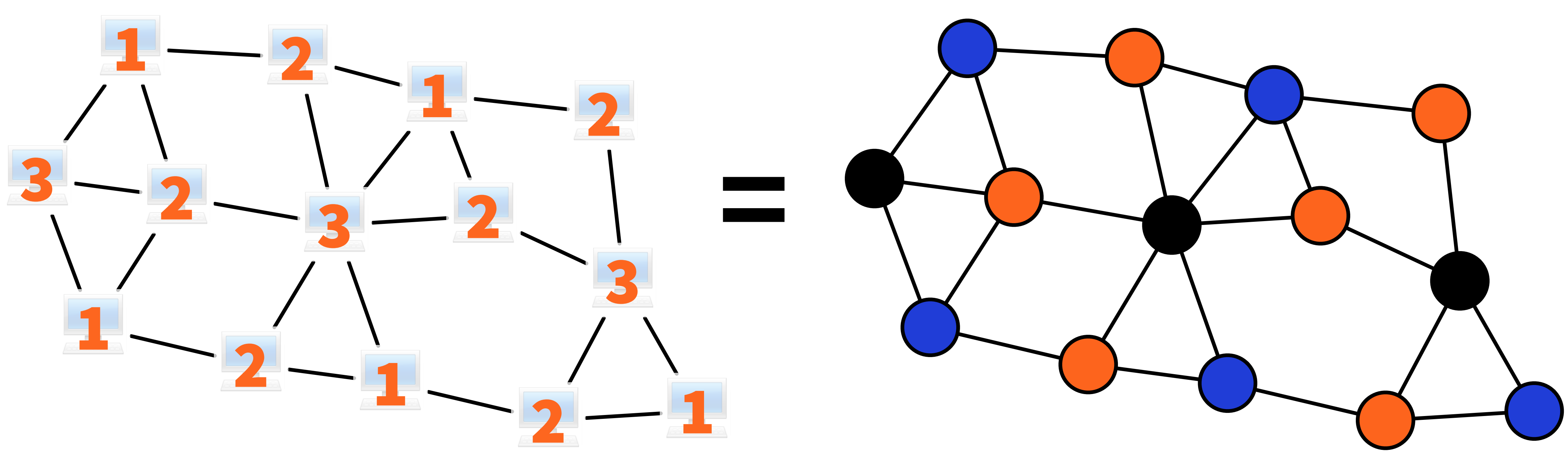
**Focus on graph problems:**
**network topology = input graph**



# Recap:
# **Distributed algorithms**

**Focus on graph problems:**

**local outputs = solution** (here: graph colouring)



# Recap:
# **Distributed algorithms**

**Typical research question:**

*“How fast can we solve graph problem X?”*

**Time = number of communication rounds**

# What have we learned?

- **Dealing with *unknown systems***
- **Dealing with *partial information***
- **Dealing with *parallelism***
- **Applications beyond distributed computing:** fault tolerance, online, streaming, multicore…

# Learning objectives

- Models
- Algorithms
- Lower bounds
- Graph theory

# Objective 1:
# **Models of computing**

- **Precisely what is a *"distributed algorithm"***
- **In each of these models:**
  - PN, LOCAL, CONGEST
  - deterministic, randomised

# Objective 2: **Algorithms**

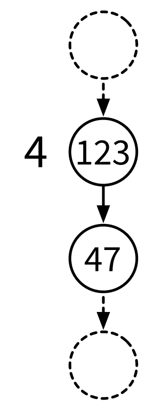
- **Colouring paths:** LOCAL, $O(\log^* n)$
- **Colouring graphs:** LOCAL, $O(\log n)$ w.h.p.
- **Gather everything:** LOCAL, $O(\mathrm{diam}(G))$
- **Bipartite maximal matching:** PN, $O(\Delta)$
- **All-pairs shortest paths:** CONGEST, $O(n)$

Week 1

# Algorithm P3CBit:
# **Fast colour reduction**

$\boldsymbol{c_0 = 123 = 01111{\color{orange}0}11_2}$ (my colour)
$\boldsymbol{c_1 = \phantom{1}47 = 00101{\color{orange}1}11_2}$ (successor's colour)
$\boldsymbol{i} = {\color{orange}\mathbf{2}}$ (bits numbered 0, 1, 2, … from right)
$\boldsymbol{b} = {\color{orange}\mathbf{0}}$ (in my colour bit number *i* was 0)

$\boldsymbol{c = 2 \cdot 2 + 0 = 4}$ (my new colour)




*$k = 8$, reducing from $2^8 = 256$ to $2 \cdot 8 = 16$ colours*

Week 7

Algorithm P3CBit:
**Fas**

$c_0$ = 12
$c_1$ = 4
$i$ = 2 (
$b$ = 0 (

$c$ = 2·2

*k*

# Algorithm idea 3

- **Colour palette: {1, 2, …, Δ + 1}**
- **Active with probability 1/2**
- **If *active*, pick a random *free* colour**
  - not used by any neighbour that has stopped
- **Try again if conflicts…**

Week 5

Algorithm P3CBit:
**Fas**

$c_0$ = 12
$c_1$ = 4
$i$ = 2 (
$b$ = 0 (

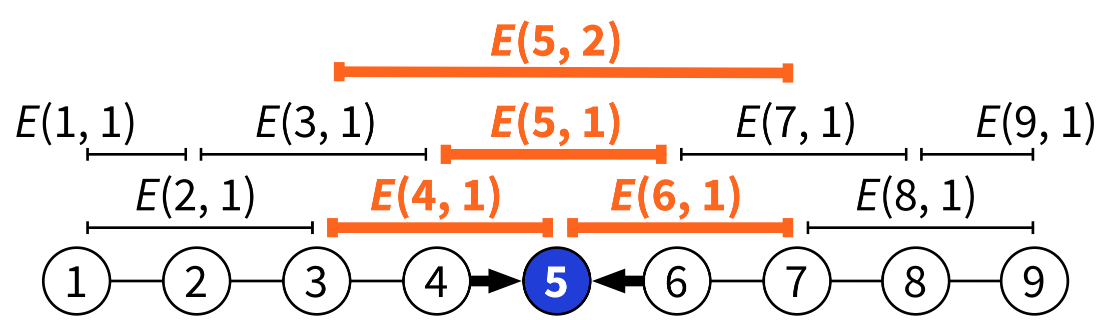
$c$ = 2·2

**Alg**

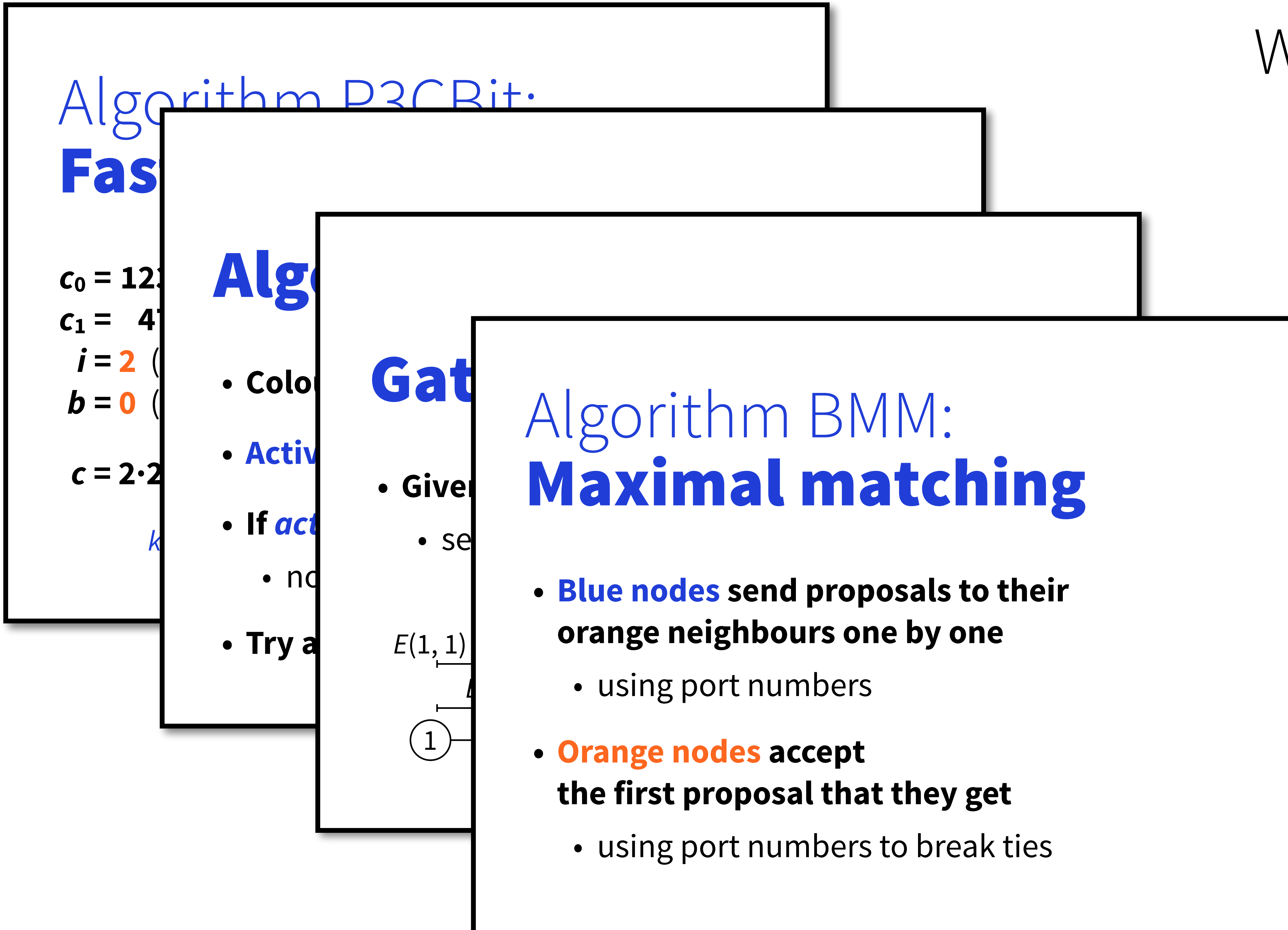
- **Colo**
- **Activ**
- **If *act***
  - no
- **Try a**

# Gathering everything

- **Given $E(v, r)$, we can learn $E(v, r + 1)$ in 1 round**
  - send $E(v, r)$ to all neighbours, take union

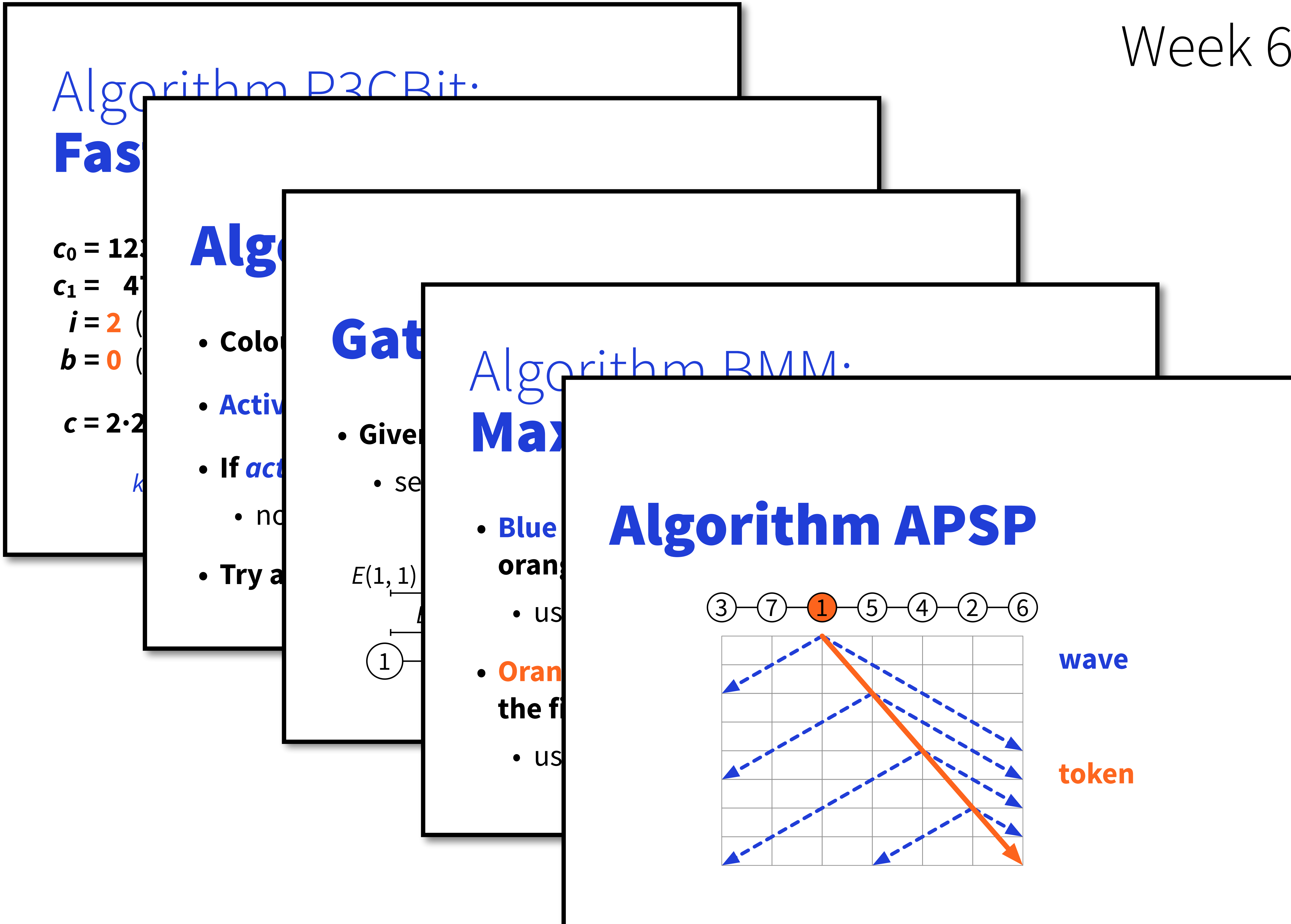


# Week 4



Algorithm P3CBit:
Fas
$c_0$ = 12
$c_1$ = 4
$i$ = 2
$b$ = 0
$c$ = 2·2
Alg
Colo
Activ
If act
nc
Try a
Gat
Giver
se
$E(1, 1)$
1
Algorithm BMM:
Maximal matching
Blue nodes send proposals to their orange neighbours one by one
using port numbers
Orange nodes accept the first proposal that they get
using port numbers to break ties

# Week 6


Algorithm P3CBit:
Algorithm BMM:
Algorithm APSP
3 7 1 5 4 2 6
wave
token

# Objective 2:
# **Algorithms**

- **Reductions!**
- **Graph colouring is a very useful subroutine**

# Objective 3:
# **Lower bounds**

- **Covering maps:**
  what cannot be solved at all in PN model
- **Local neighbourhoods:**
  what cannot be solved fast in any model
- **Ramsey's theorem:**
  what cannot be solved in $O(1)$ time

# Objective 4: **Graph theory**

- **Basic definitions**
- **Connections between graph problems**
  - e.g. maximal matching → small vertex covers
- **Ramsey's theorem**
  - at least for $c = 2$, $k = 2$

# What else is studied in distributed computing?

- Fault-tolerance
- Asynchrony
- Shared memory
- Physical models
- Robot navigation
- Nondeterminism
- Complexity measures
- High-performance computing
- Practical aspects of networking …

# What next?

- **ICS-E4020 Programming Parallel Computers**
  - 5th period, 5 credits, intensive course
  - programming modern parallel computers: multicore, GPU, memory hierarchies …
  - hands-on programming exercises
  - main goal: make it as fast as you can!

# What next?

- **Just ask if you want to do more!**
  - master’s thesis topics?
  - summer internships?
  - doctoral studies?

# Practicalities

- **2nd mid-term exam: 11 December**
  - remember to register on time!
- **Course feedback: deadline 12 December**
  - this is a new course,<br>feedback very important!
  - 1 extra point in grading

# What to expect in the exam?

- See the learning objectives!
- Do not think that you can safely forget what we learned during the 1st period!
- Expect both algorithm design and lower bound proofs

# Examples of old exam problems

- **Prove: no deterministic PN-algorithm that finds a *minimum* vertex cover in cycle graphs, given a *minimal* vertex cover**

# Examples of old exam problems

- Prove: no deterministic PN-algorithm that finds a 6-colouring in cycle graphs given a maximal independent set

# Examples of old exam problems

- Counting problem: all nodes output $|V|$
- Prove: no deterministic PN-algorithm for cycle graphs
- Prove: no $o(n)$-time deterministic LOCAL-algorithm for cycle graphs

# Examples of old exam problems

- Prove: no deterministic PN-algorithm for maximal matching in arbitrary graphs

# Examples of old exam problems

- Prove: no deterministic $o(n)$-time PN-algorithm for weak 2-colouring in paths of length $\geq 3$

# Examples of old exam problems

- Give an elementary proof that
  any graph with 6 nodes contains
  a clique with 3 nodes or
  an independent set with 3 nodes

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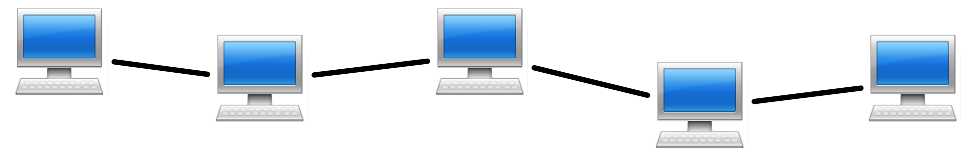
  - network = path 
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