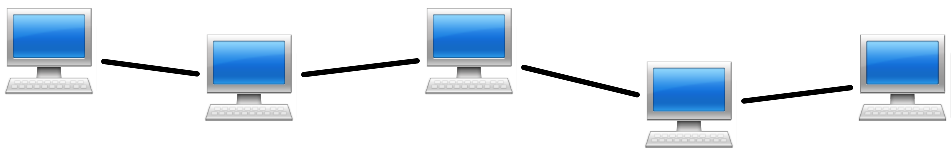
- **Weeks 1–2: informal introduction**
  - network = path 
- **Week 3: graph theory**
- **Weeks 4–7: models of computing**
  - what can be computed (efficiently)?
- **Weeks 8–11: lower bounds**
  - what cannot be computed (efficiently)?
- **Week 12: recap**

Mid-term exams

- **Mid-term exams:**
  - Thursday, 23 October 2014, 9:00am
  - Thursday, 11 December 2014, 9:00am
- **Register on time (*one week before*) in Oodi**

Mid-term exams

- **Topics:**
  - 1st exam: Chapters 1–6
  - 2nd exam: Chapters 1–12
- **See course web page for *learning objectives*!**

Week 6

– CONGEST model:
  bandwidth limitations

CONGEST model

- **LOCAL model: arbitrarily large messages**
- **CONGEST model: $O(\log n)$-bit messages**

CONGEST model

- **Any of these can be encoded in $O(\log n)$-bit messages:**
  - node identifier
  - number of nodes
  - number of edges
  - distance between two nodes …

CONGEST model

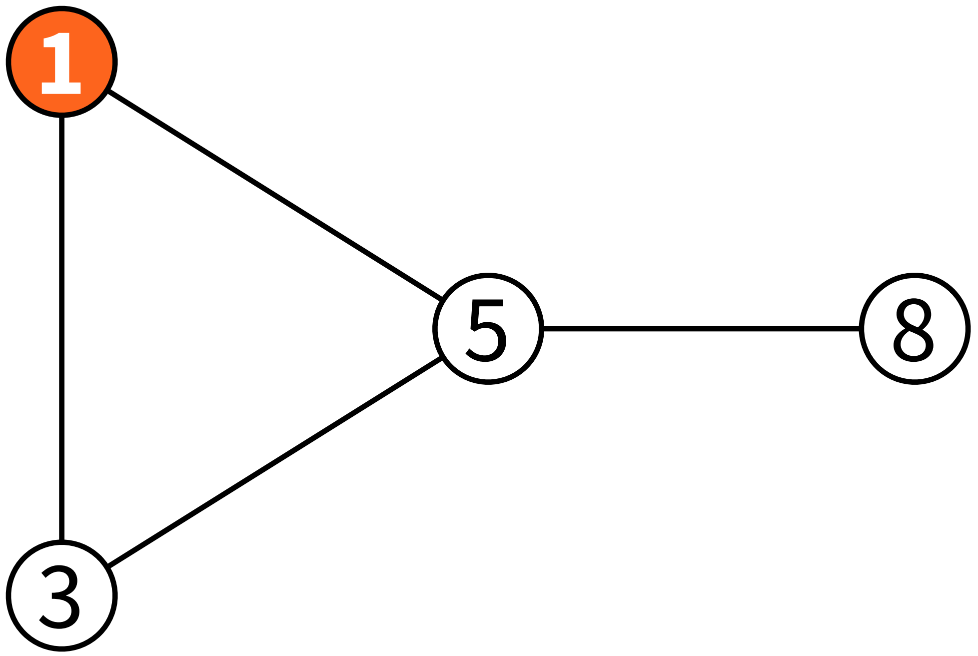
- **Many algorithms that we have seen only send small messages**
  - can be used directly in the CONGEST model
- **Exception: algorithm Gather**
  - may need to send $O(n^2)$-bit messages

CONGEST model

- **$O(n)$ time trivial in the LOCAL model**
  - brute force approach: Gather + solve locally
- **$O(n)$ time non-trivial in the CONGEST model**
- **Today: how to find all-pairs shortest paths in $O(n)$ time**

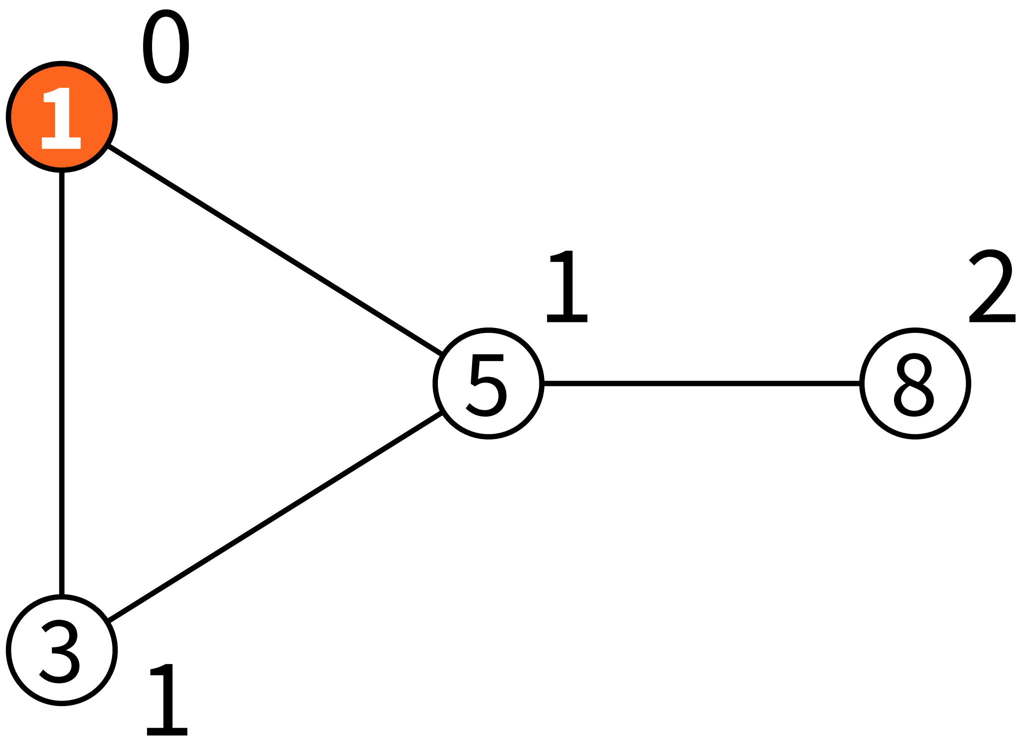
Single-source shortest paths

Input:



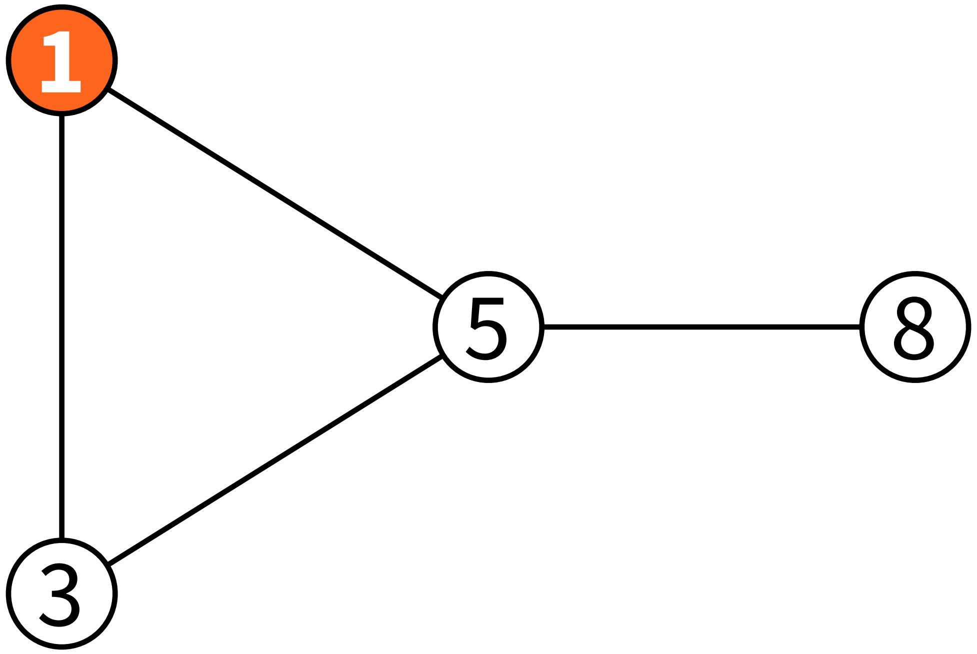
Single-source shortest paths

Output:



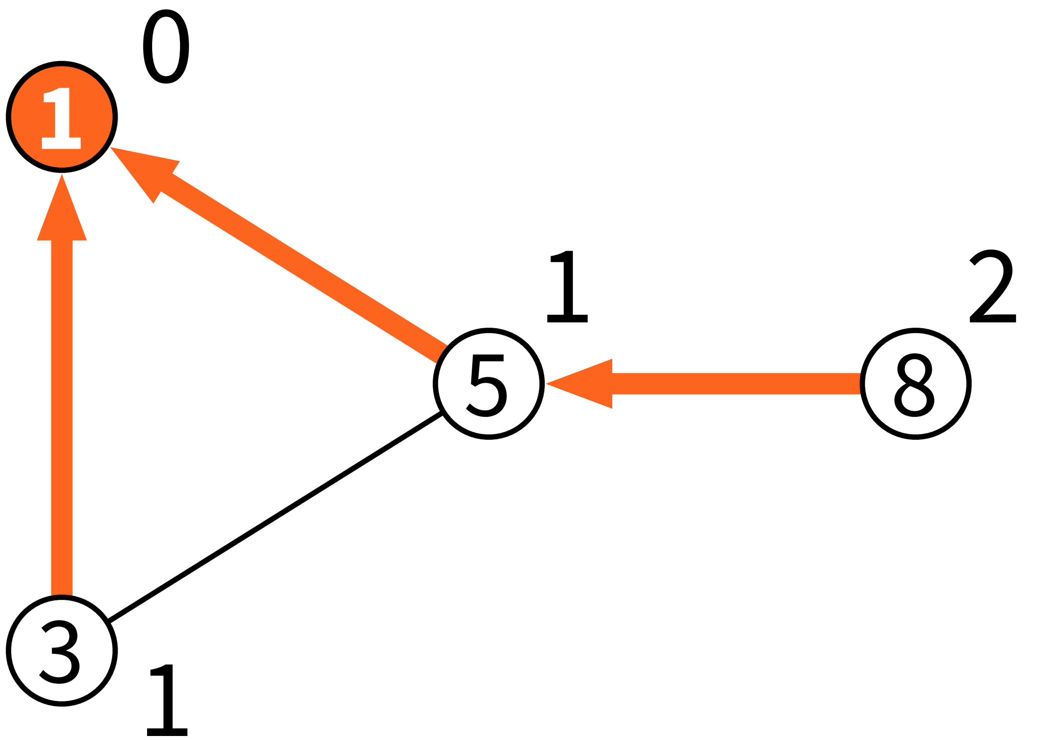
BFS tree

Input:



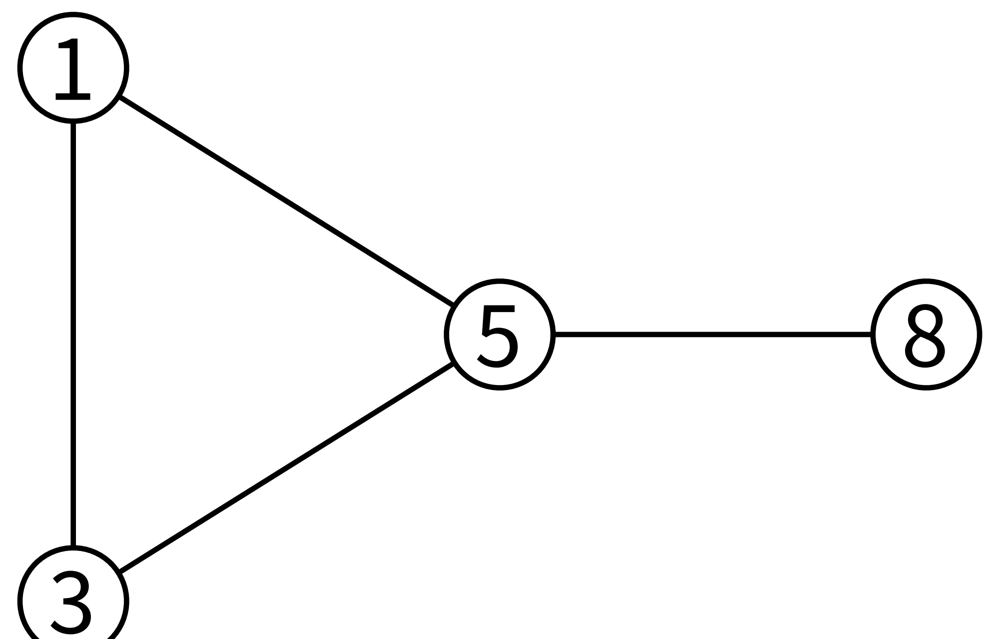
BFS tree

Output:



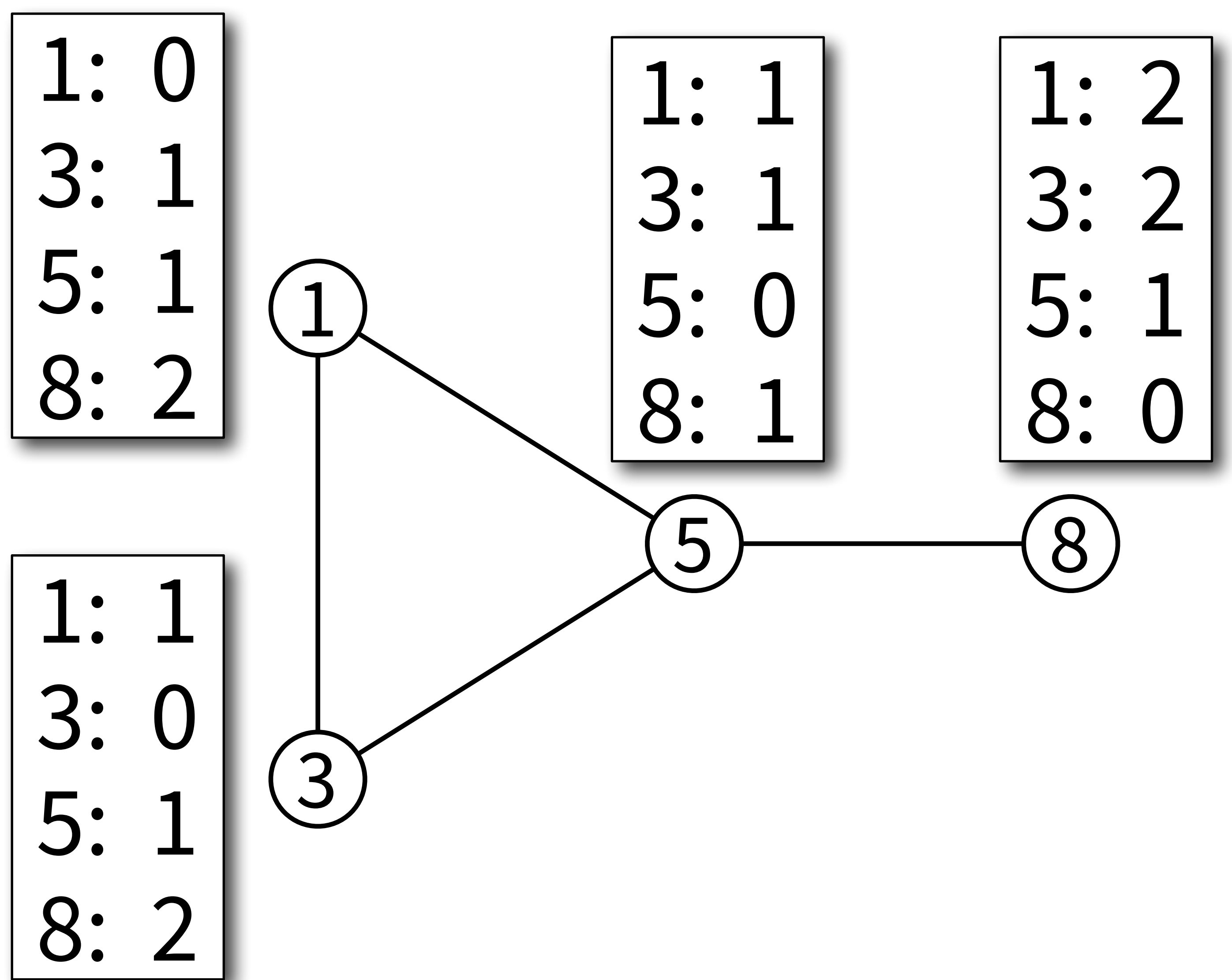
All-pairs shortest paths

Input:



All-pairs shortest paths

Output:

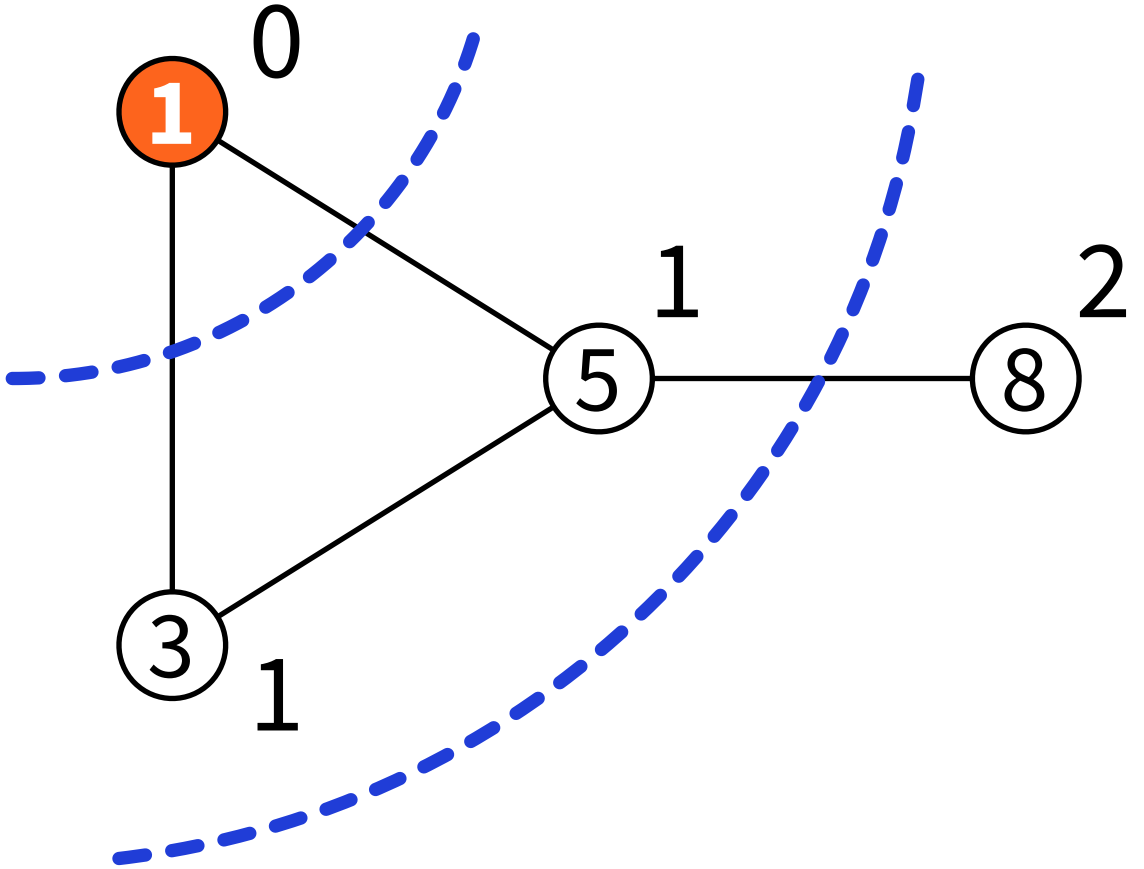


Algorithm Wave

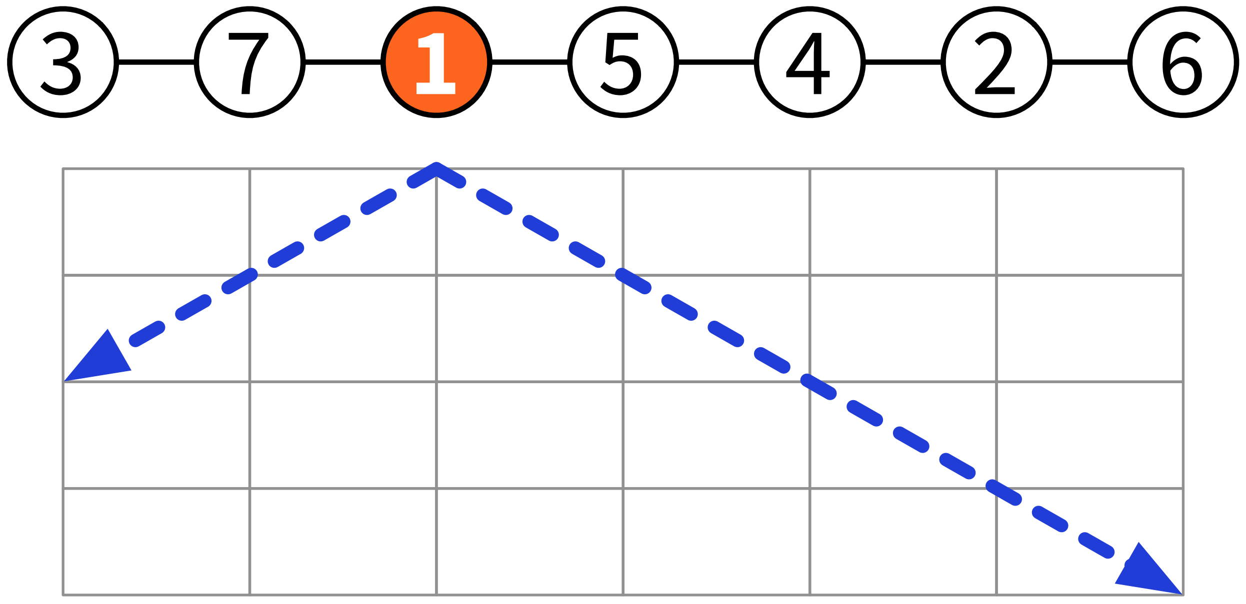
- **Solves single-source shortest paths in time $O(\mathrm{diam}(G))$**
- **Leader creates a 'wave', other nodes propagate it**
- **Wave first received in round t: distance to leader is t**

Algorithm Wave

Output:



Algorithm Wave

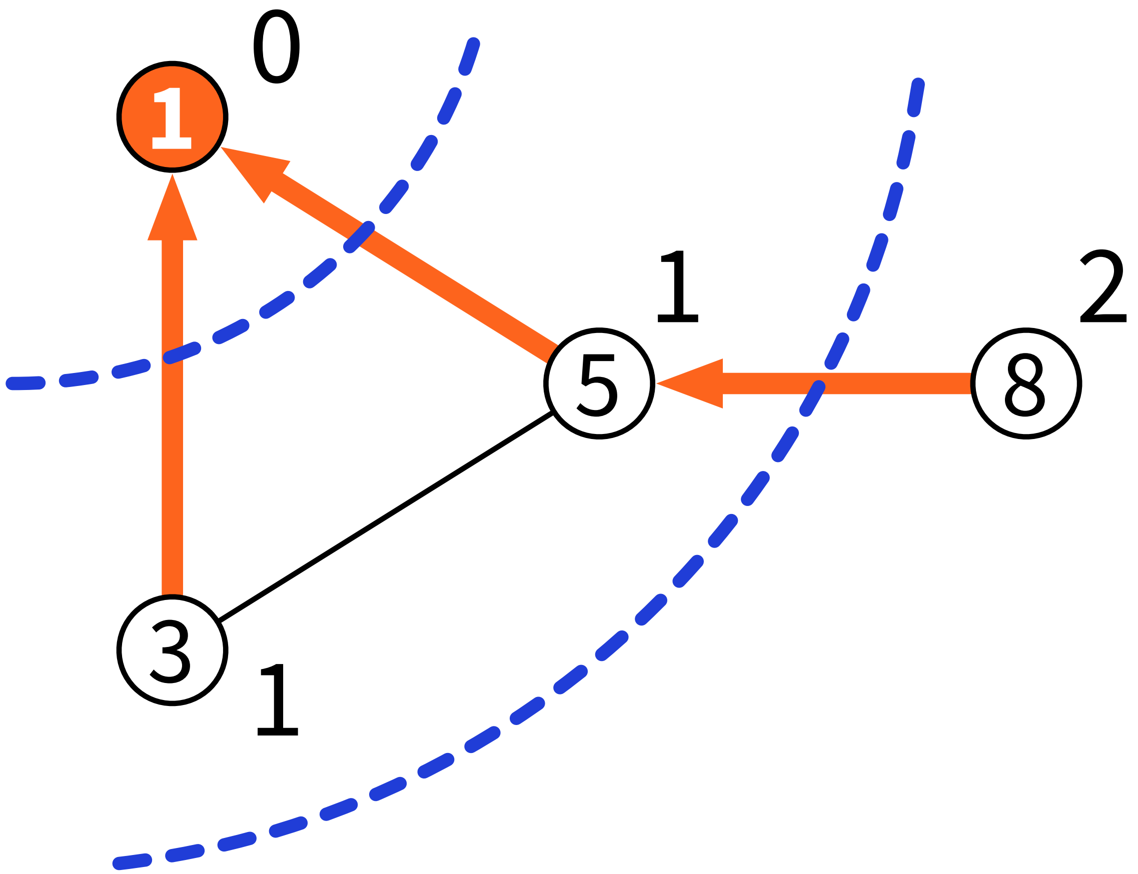


Algorithm BFS

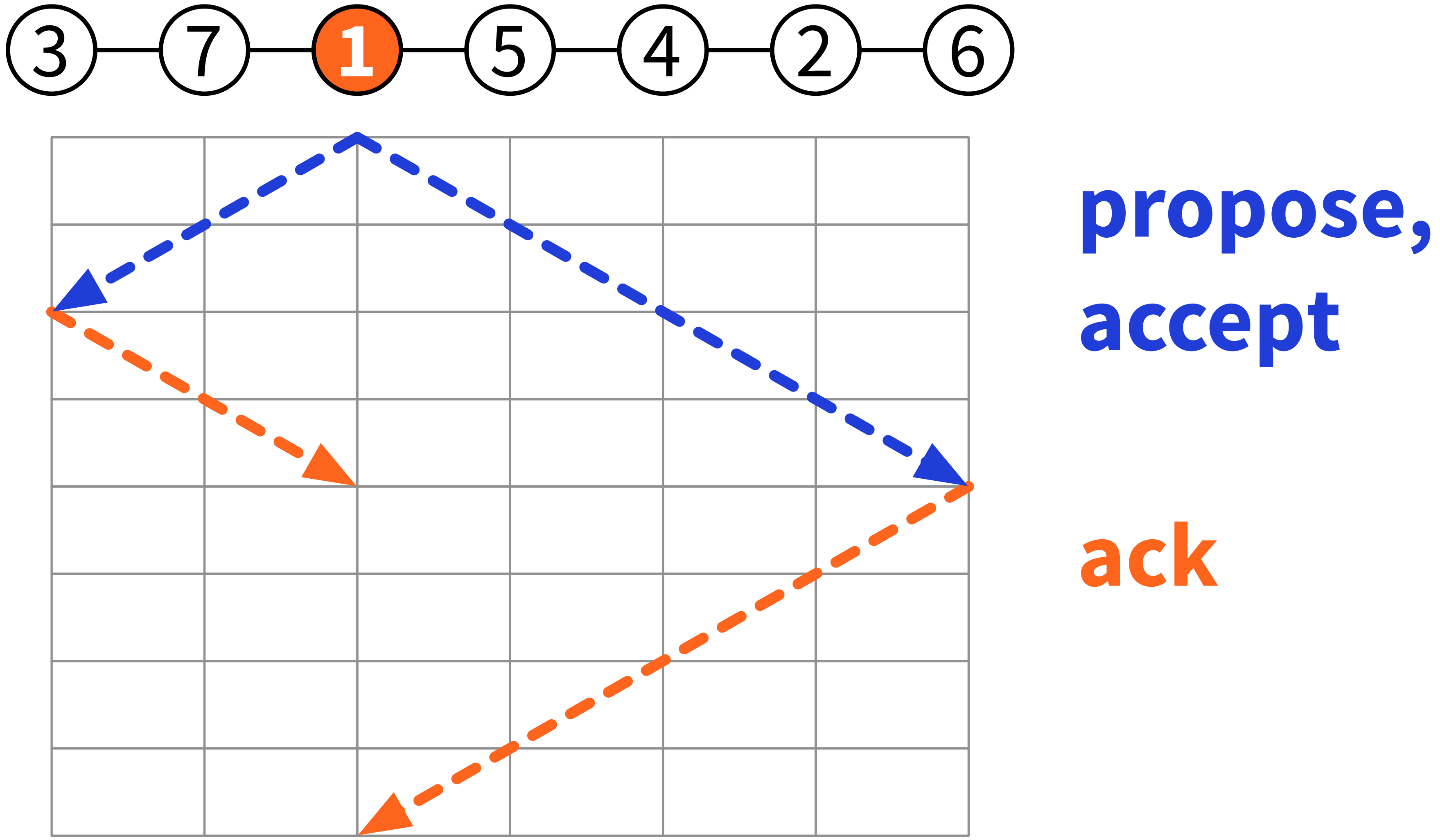
- **Wave + handshakes**
- **Tree construction:**
  - “proposal” + “accept”
  - everyone knows their parent & children
- **Acknowledgements back from leaf nodes**

Algorithm BFS

Output:



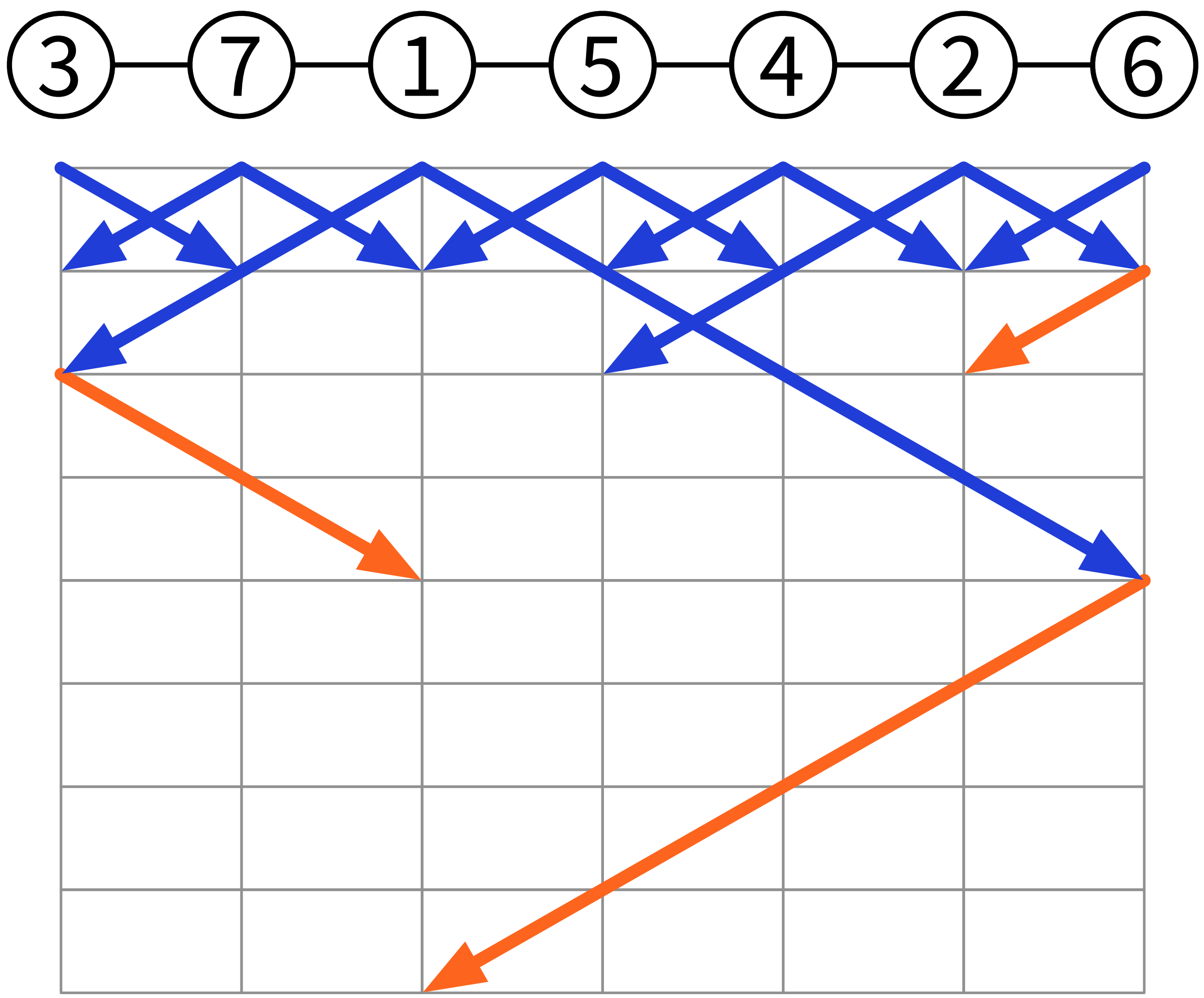
Algorithm BFS



Algorithm Leader

- **Each node creates a separate BFS process**
- **When two BFS processes “collide”, the one with the smaller root “wins”**
  - each node only needs to send messages related to *one BFS process*
- **One tree wins everyone else → leader**

Algorithm Leader



propose, accept

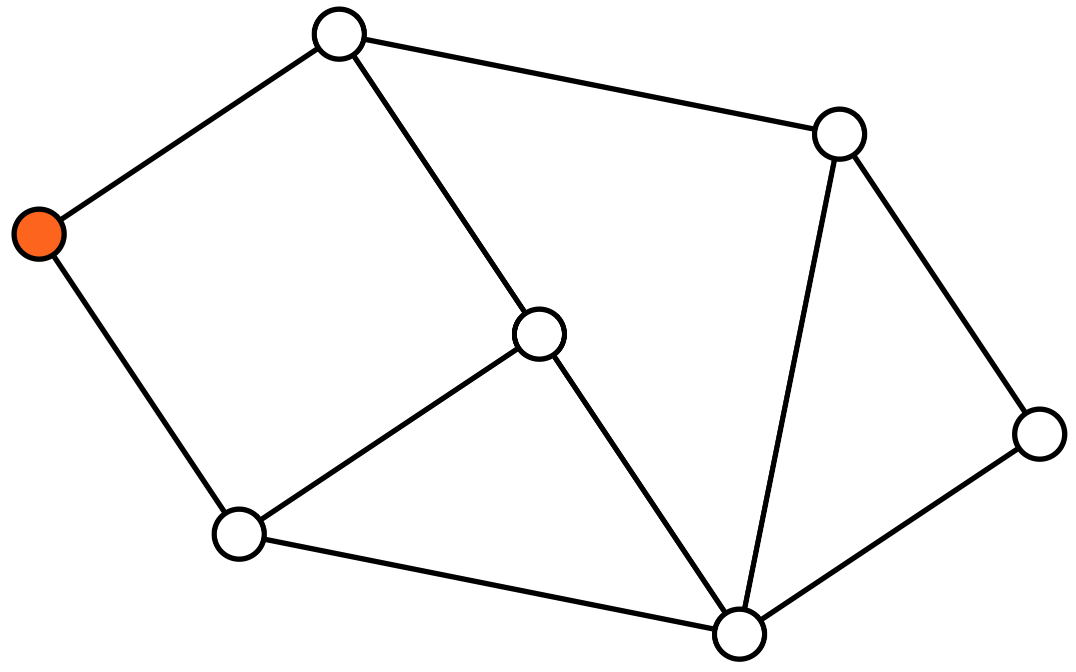
ack

Algorithm APSP

- **Basic idea: run Wave from each node**
- **Challenge: congestion**
  - all waves parallel → too many bits per edge
  - all waves sequentially → takes too long
- **Solution: pipelining**

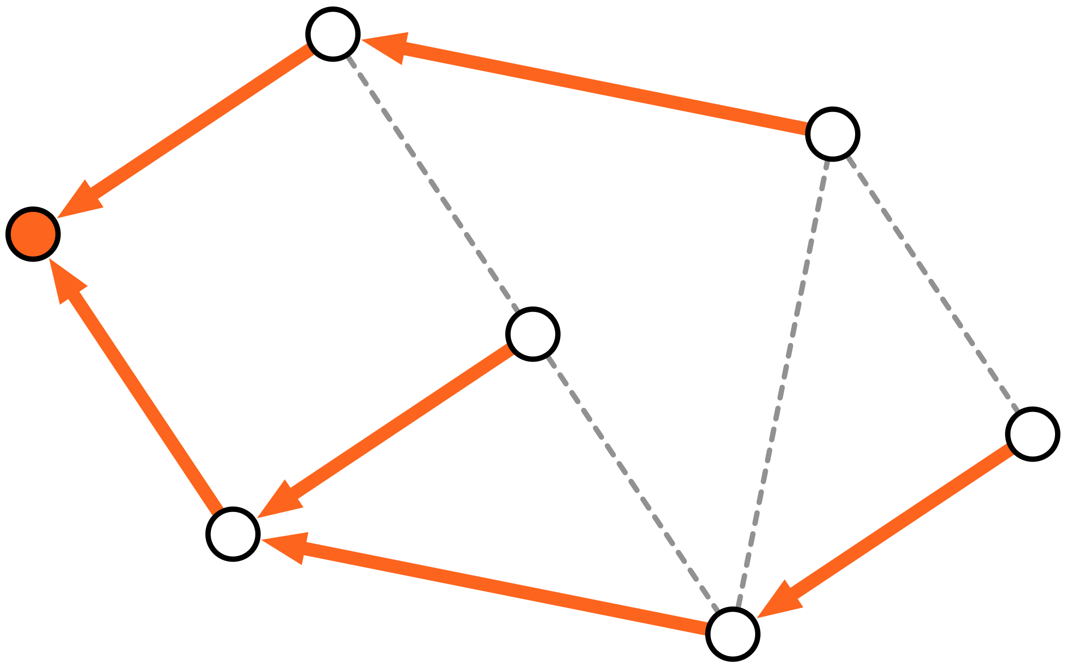
Algorithm APSP

- **Elect leader**



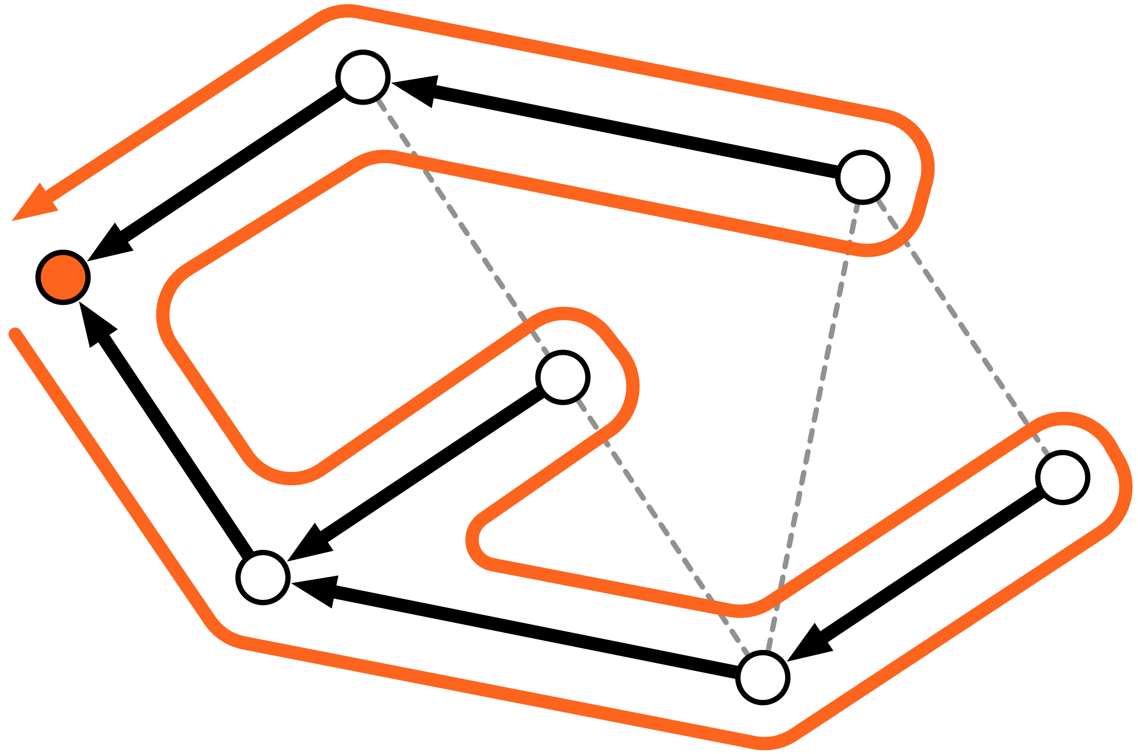
Algorithm APSP

- Elect leader, construct BFS tree



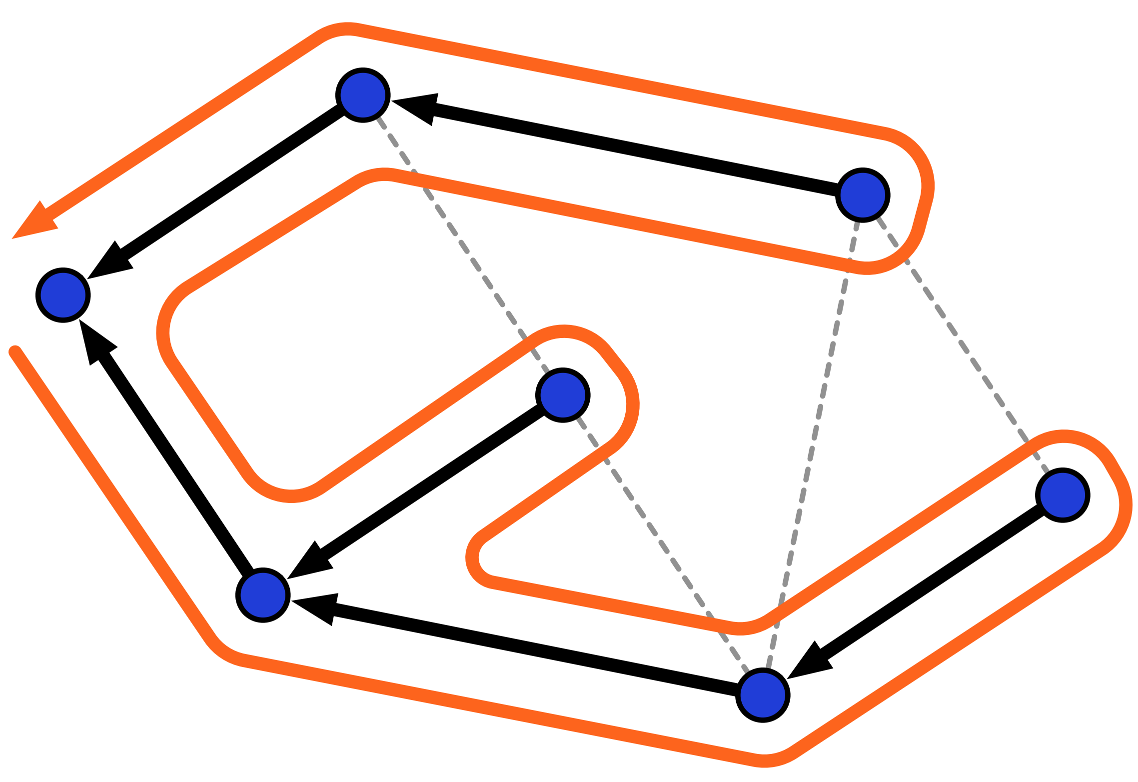
Algorithm APSP

- **Move token along BFS tree slowly**

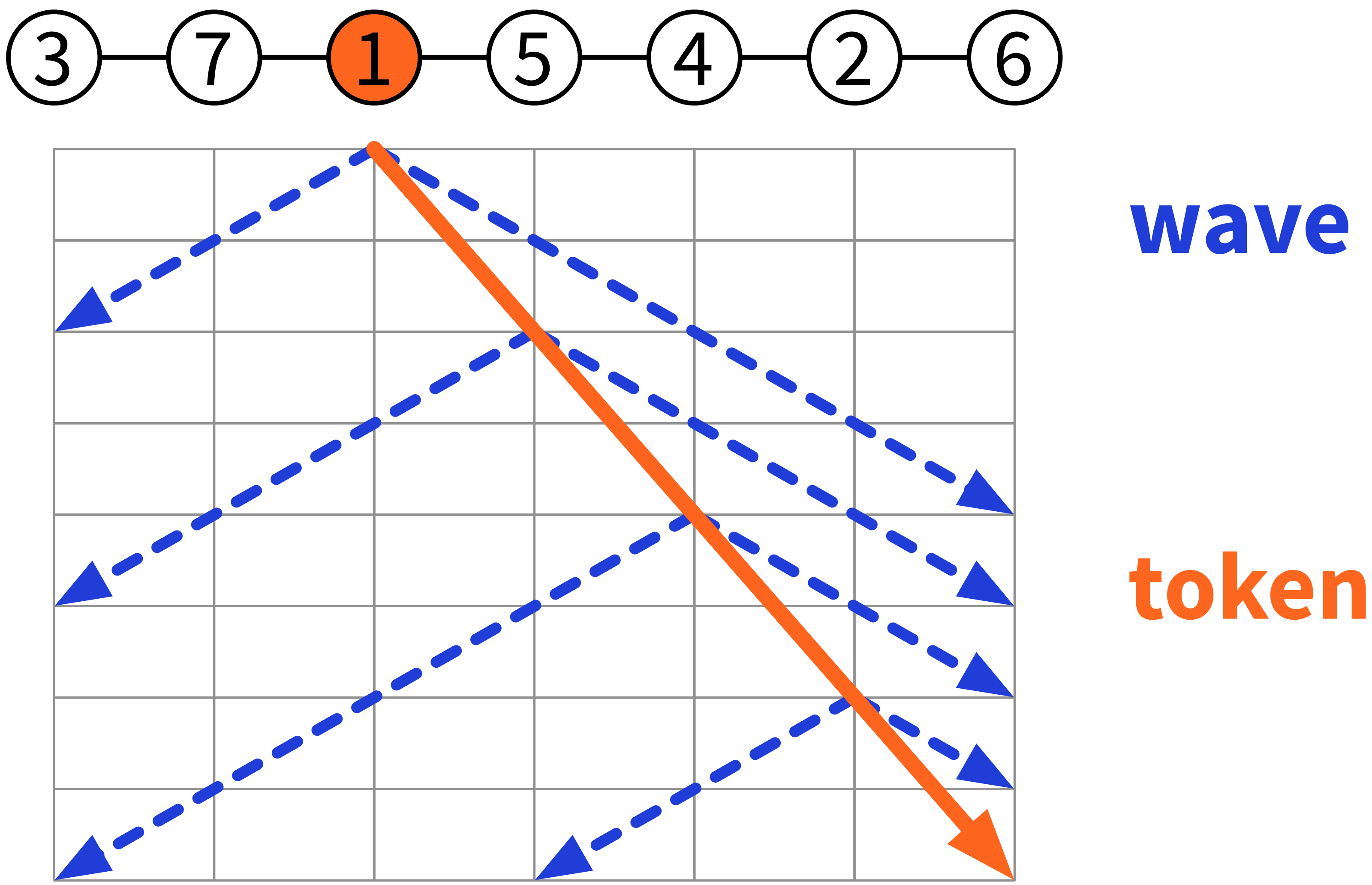


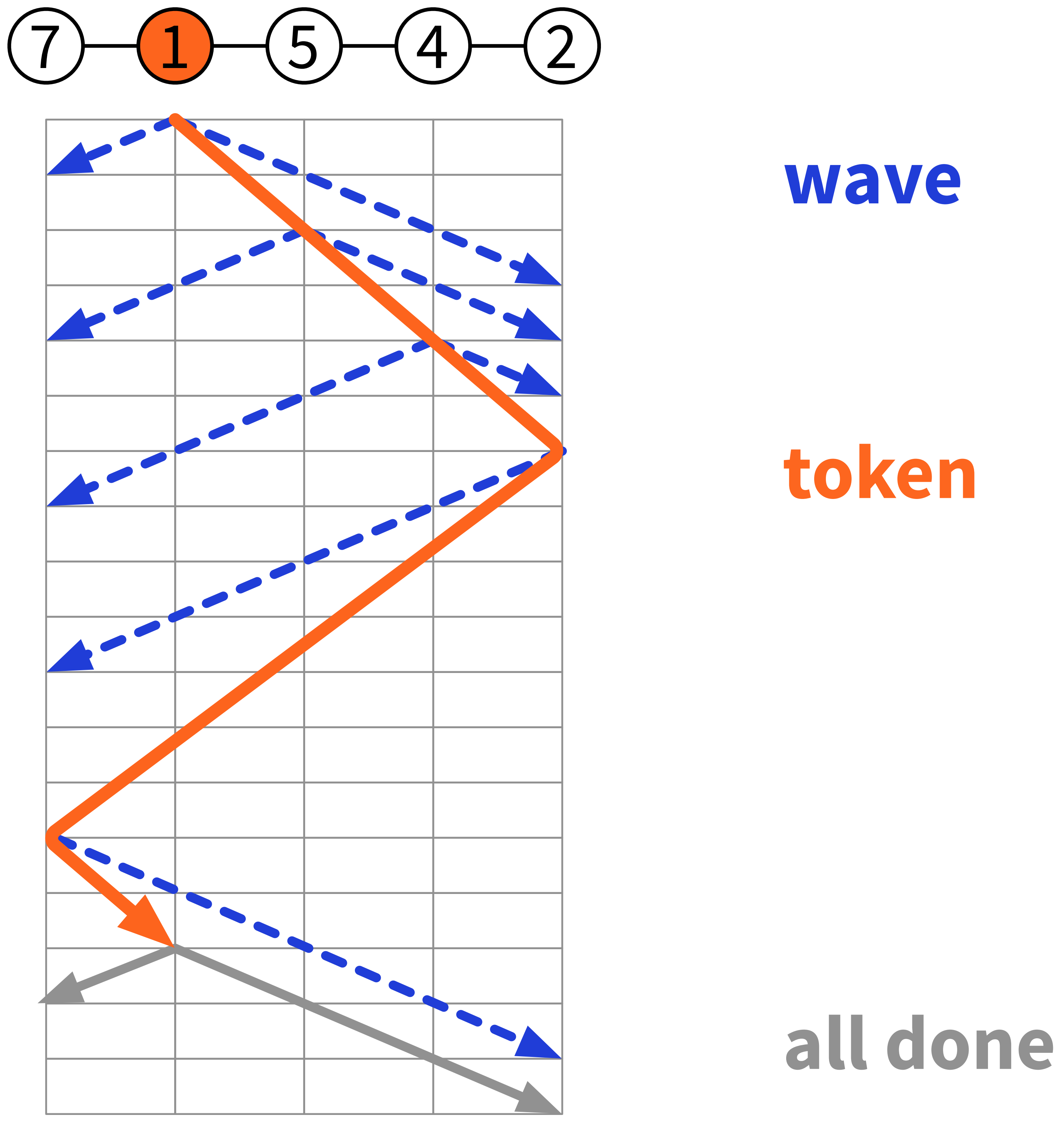
Algorithm APSP

- **Create *wave* every time we visit a new node**



Algorithm APSP





7
1
5
4
2
wave
token
all done

Algorithm APSP

- **Algorithm animation:**
  http://users.ics.aalto.fi/suomela/apsp/

Pipelining

- n operations, each operation takes time t
- **Parallel:** t rounds, bad congestion
- **Sequential:** nt rounds, no congestion
- **Pipelining:** $n + t$ rounds, no congestion

Summary

- **LOCAL model: unlimited bandwidth**
- **CONGEST model: $O(\log n)$ bandwidth**
- **$O(n)$ or $O(\text{diam}(G))$ time is no longer trivial**
- **Example: all-pairs shortest paths in time $O(n)$, pipelining helps**

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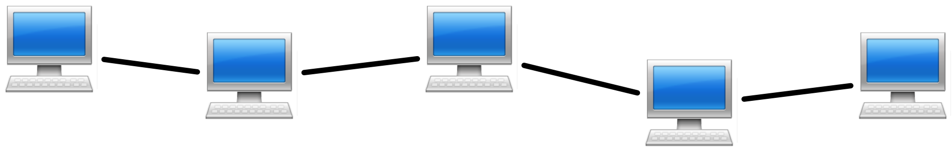
  - network = path 
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