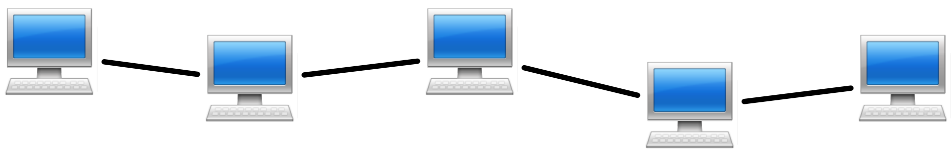
- **Weeks 1–2: informal introduction**
  - network = path 
- **Week 3: graph theory**
- **Weeks 4–7: models of computing**
  - what can be computed (efficiently)?
- **Weeks 8–11: lower bounds**
  - what cannot be computed (efficiently)?
- **Week 12: recap**

Week 7

– Randomised algorithms

Deterministic algorithms

- **init$_d$(…):** state
- **send$_d$(…):** message vector
- **receive$_d$(…):** state

Randomised algorithms

- **init$_d$(…):** *probability distribution* over states
- **send$_d$(…):** message vector
- **receive$_d$(…):** *probability distribution* over states

Randomised algorithms

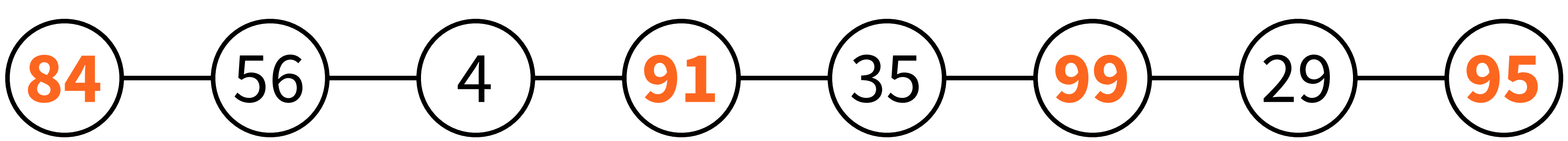
- You can always toss coins when you pick the new state

Randomised algorithms

- Randomised algorithm in PN model
- Randomised algorithm in LOCAL model
- Randomised algorithm in CONGEST model

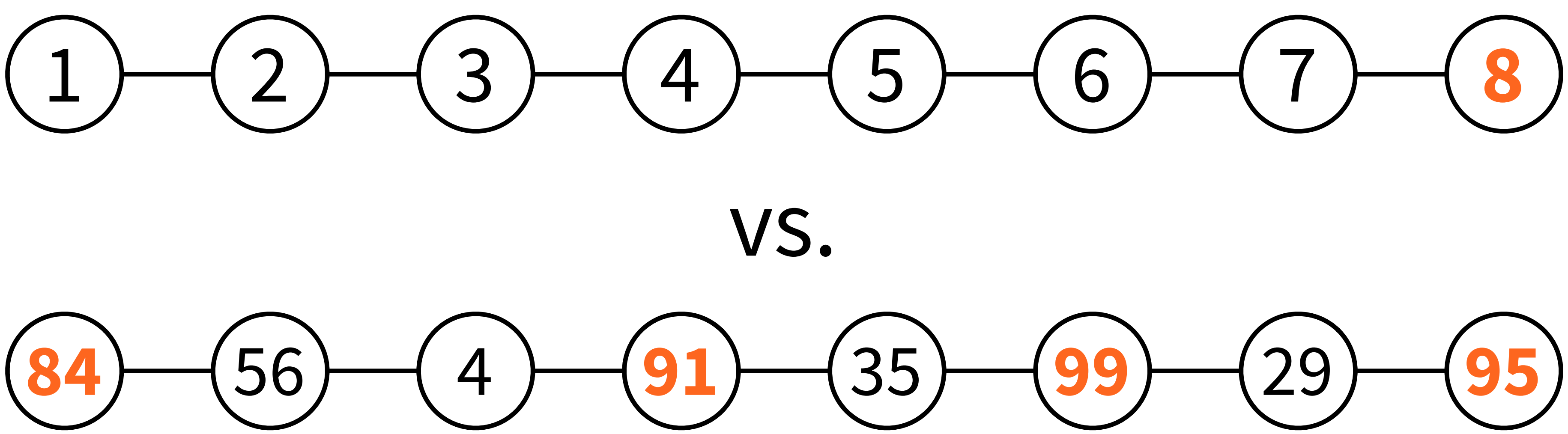
Uses of randomness

- **Break symmetry**
- **Similar to unique identifiers**



Uses of randomness

- **Better than unique identifiers: worst-case inputs unlikely?**

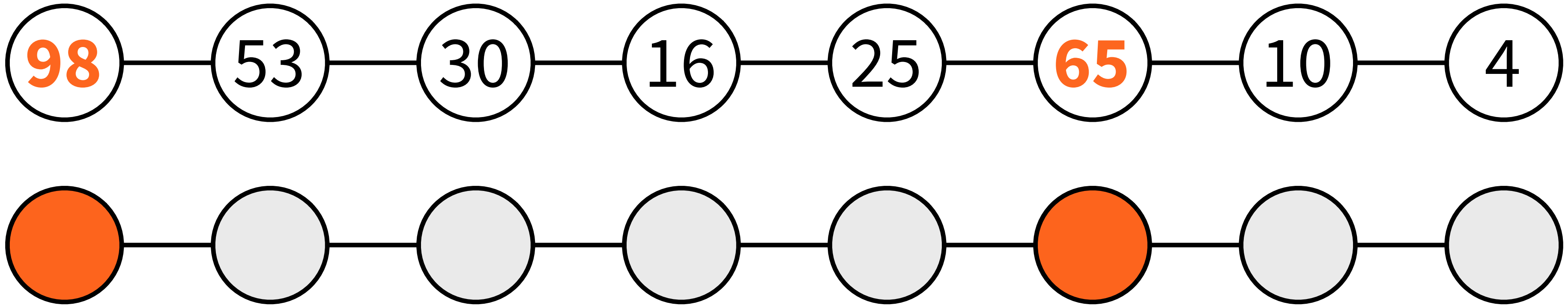


Guarantees

- *Monte Carlo*: **always fast**
  - running time deterministic
  - quality of output probabilistic
- *Las Vegas*: **always correct**
  - running time probabilistic
  - quality of output deterministic

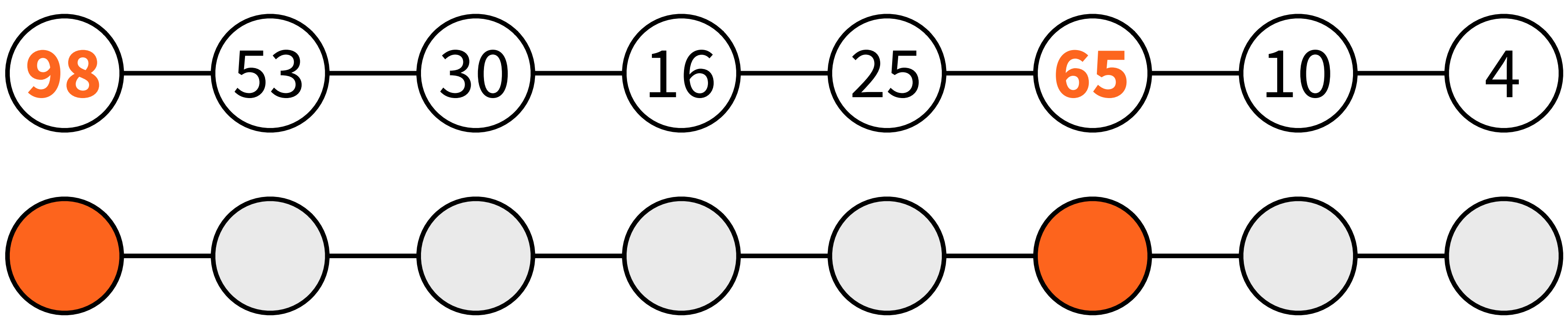
Monte Carlo

- **Example: large independent set**
- **Pick random values, local maxima join**



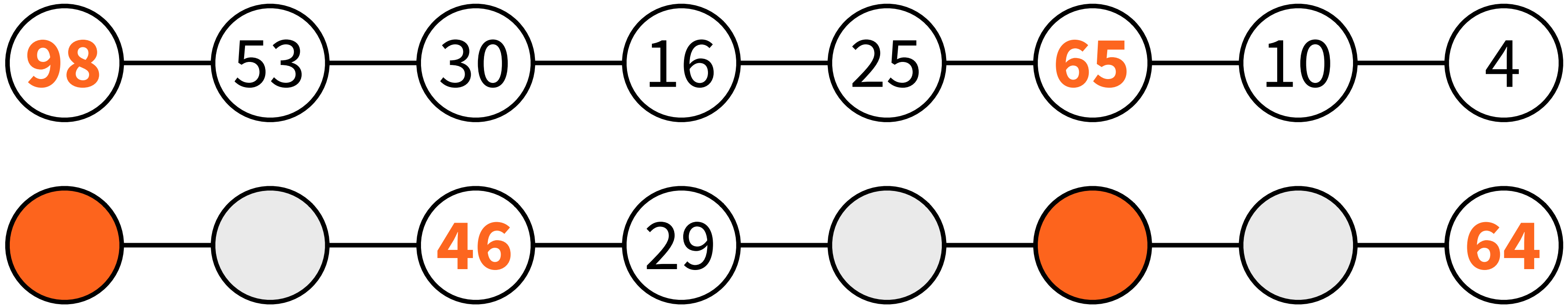
Monte Carlo

- Running time always $O(1)$
- Size of the set depends on random values



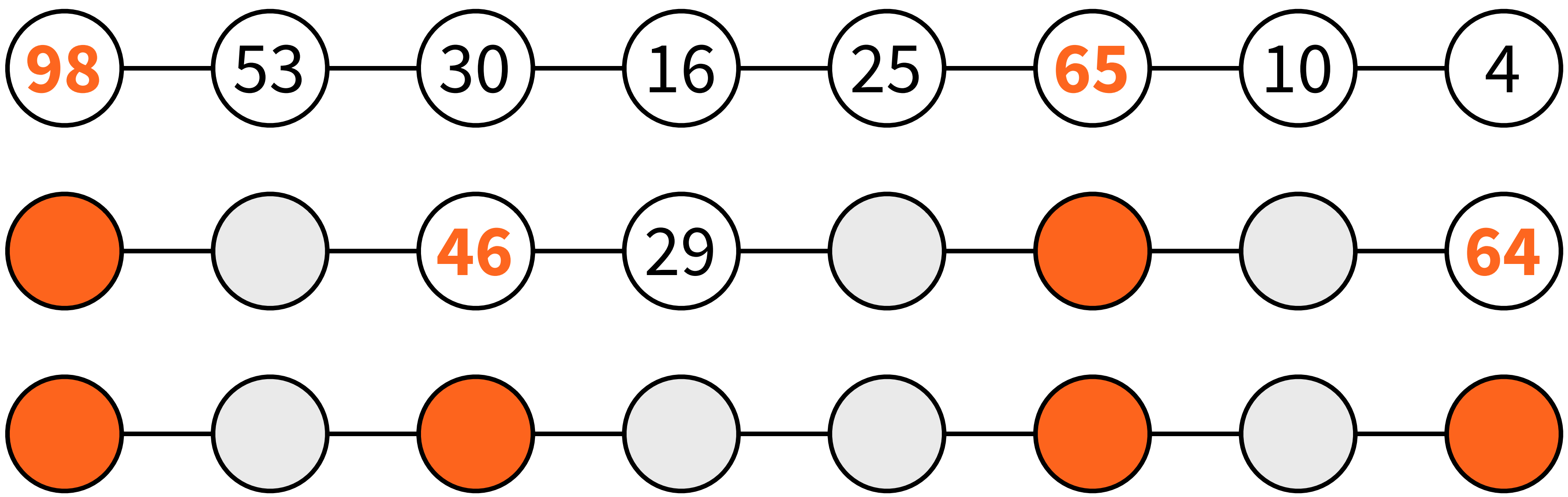
Las Vegas

- Pick random values, local maxima join, …



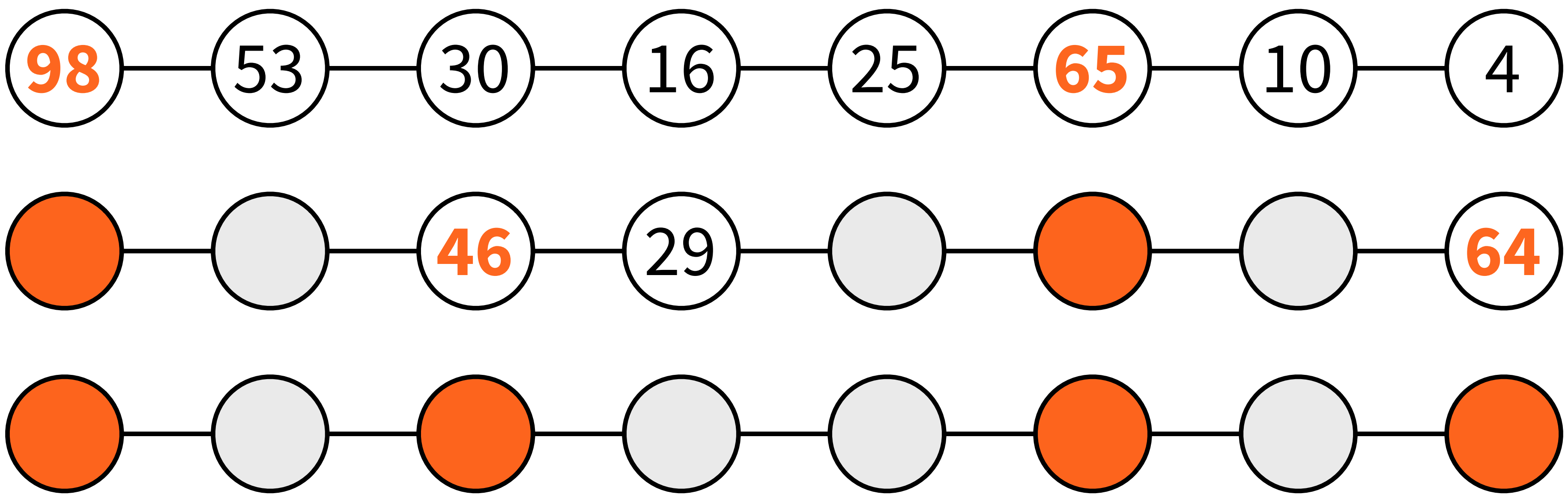
Las Vegas

- Pick random values, local maxima join, repeat until maximal



Las Vegas

- **Output is always maximal independent set, running time probabilistic**



With high probability

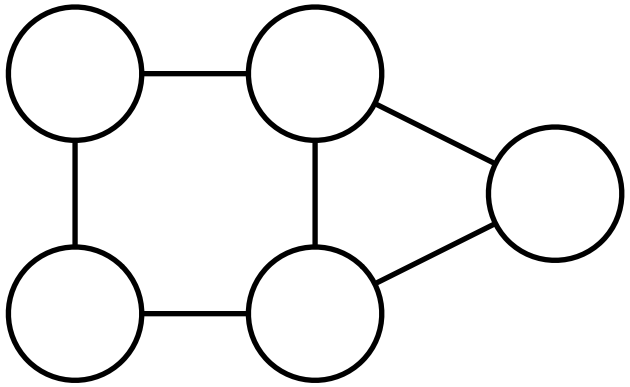
- **Success probability $1 - 1/n^c$**
  - I can choose any constant c
- **"algorithm A stops in time $O(\log n)$ with high probability"**
- **"running time is $O(\log n)$ w.h.p."**

Example: Graph colouring

- **Chapter 5: deterministic algorithm, $(\Delta + 1)$-colouring in $O(\Delta^2 + \log^* n)$ rounds**
- **Today: randomised algorithm, $(\Delta + 1)$-colouring in $O(\log n)$ rounds w.h.p.**

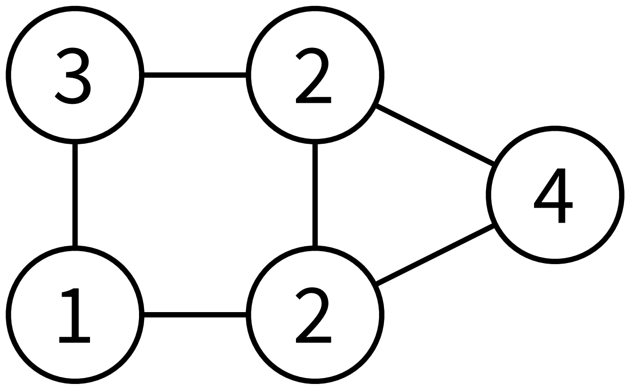
Algorithm idea

- Colour palette: $\{1, 2, \ldots, \Delta + 1\}$
- Pick a random colour
- Try again if conflicts…



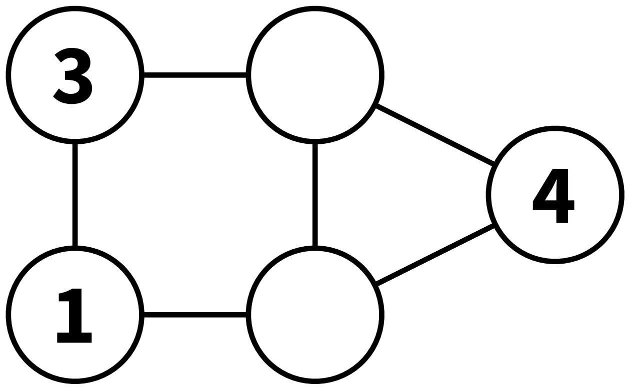
Algorithm idea

- Colour palette: $\{1, 2, \ldots, \Delta + 1\}$
- Pick a random colour
- Try again if conflicts…



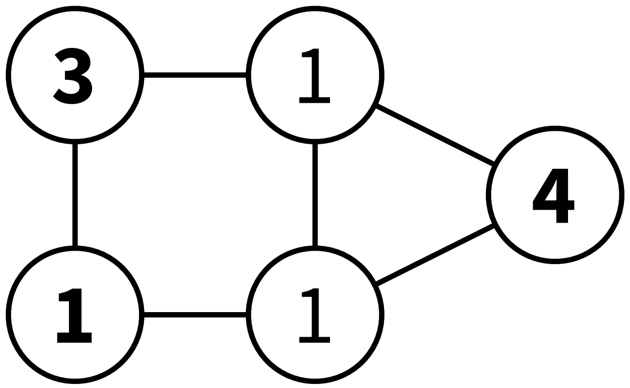
Algorithm idea

- Colour palette: $\{1, 2, \ldots, \Delta + 1\}$
- Pick a random colour
- Try again if conflicts…



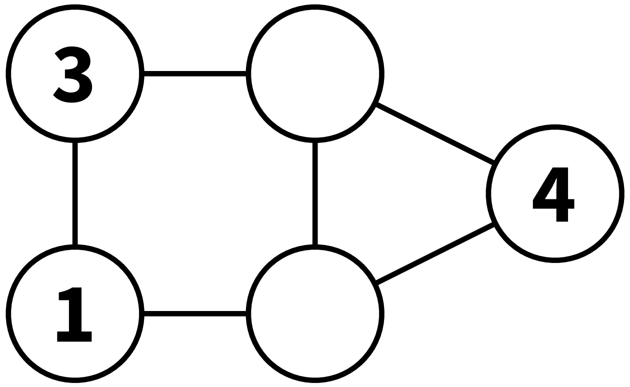
Algorithm idea

- Colour palette: $\{1, 2, \ldots, \Delta + 1\}$
- Pick a random colour
- Try again if conflicts…



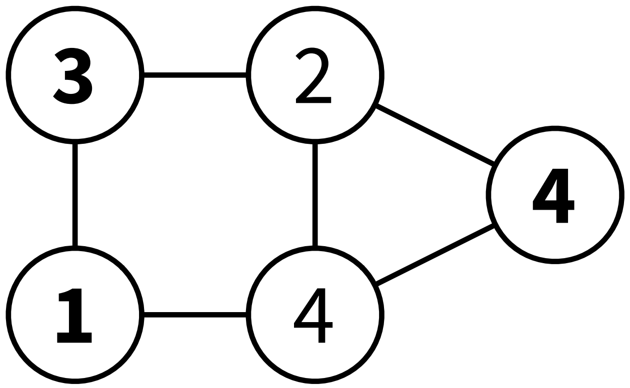
Algorithm idea

- Colour palette: $\{1, 2, \ldots, \Delta + 1\}$
- Pick a random colour
- Try again if conflicts…



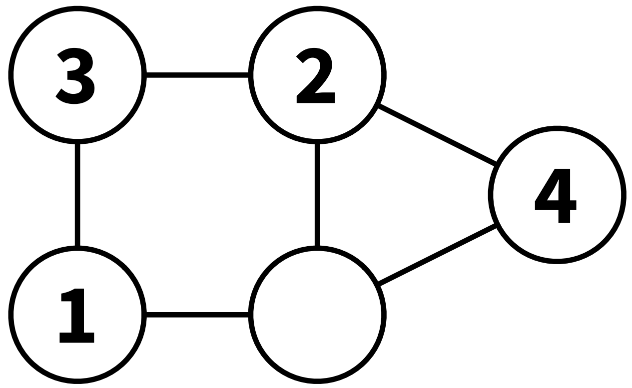
Algorithm idea

- Colour palette: $\{1, 2, \ldots, \Delta + 1\}$
- Pick a random colour
- Try again if conflicts…



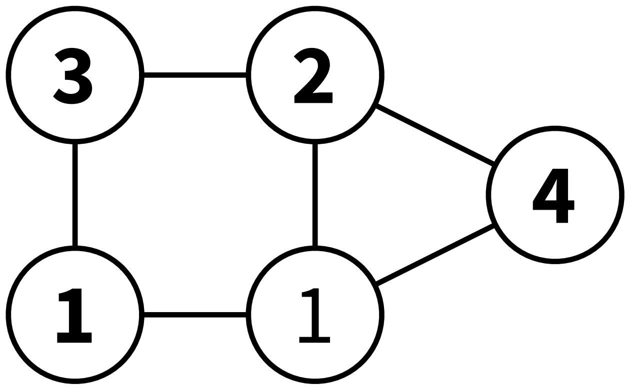
Algorithm idea

- Colour palette: $\{1, 2, \ldots, \Delta + 1\}$
- Pick a random colour
- Try again if conflicts…



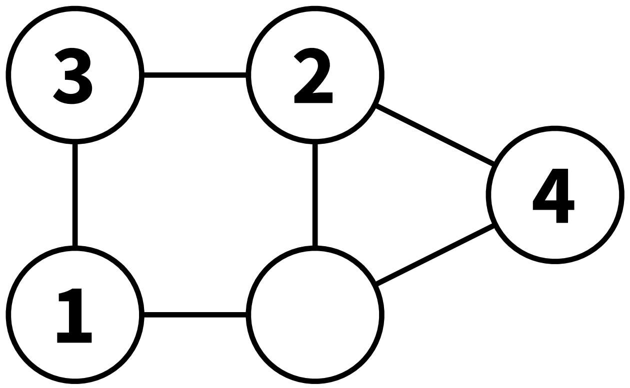
Algorithm idea

- Colour palette: $\{1, 2, \ldots, \Delta + 1\}$
- Pick a random colour
- Try again if conflicts…



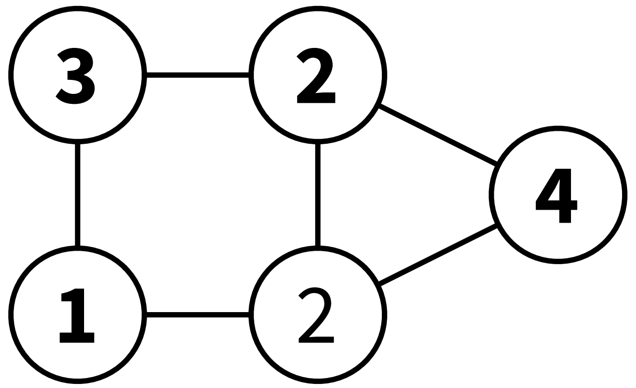
Algorithm idea

- Colour palette: $\{1, 2, \ldots, \Delta + 1\}$
- Pick a random colour
- Try again if conflicts…



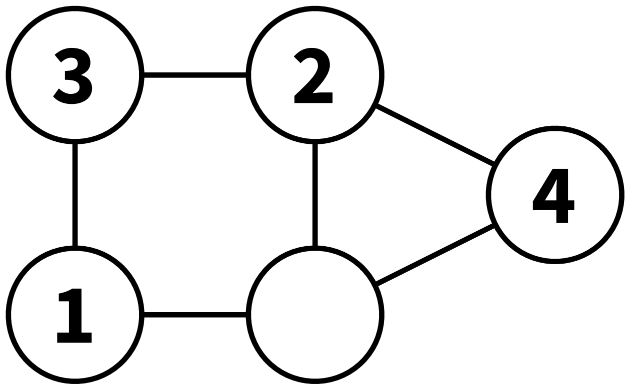
Algorithm idea

- Colour palette: $\{1, 2, \ldots, \Delta + 1\}$
- Pick a random colour
- Try again if conflicts…



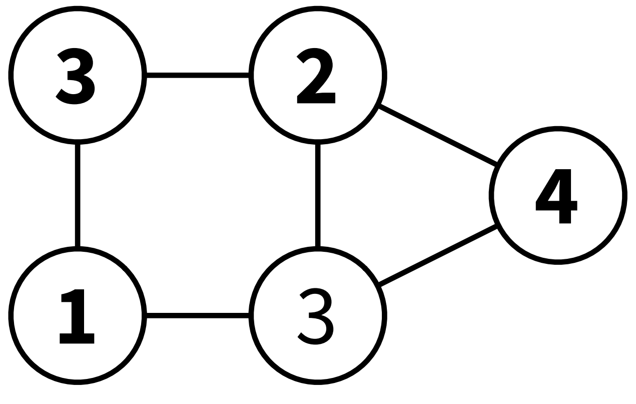
Algorithm idea

- Colour palette: $\{1, 2, \ldots, \Delta + 1\}$
- Pick a random colour
- Try again if conflicts…



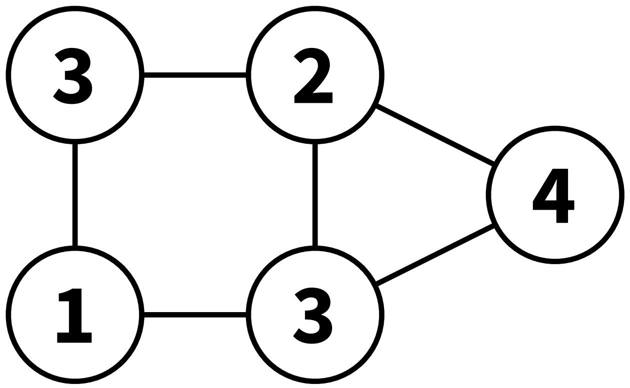
Algorithm idea

- Colour palette: $\{1, 2, \ldots, \Delta + 1\}$
- Pick a random colour
- Try again if conflicts...



Algorithm idea

- Colour palette: $\{1, 2, \ldots, \Delta + 1\}$
- Pick a random colour
- Try again if conflicts...



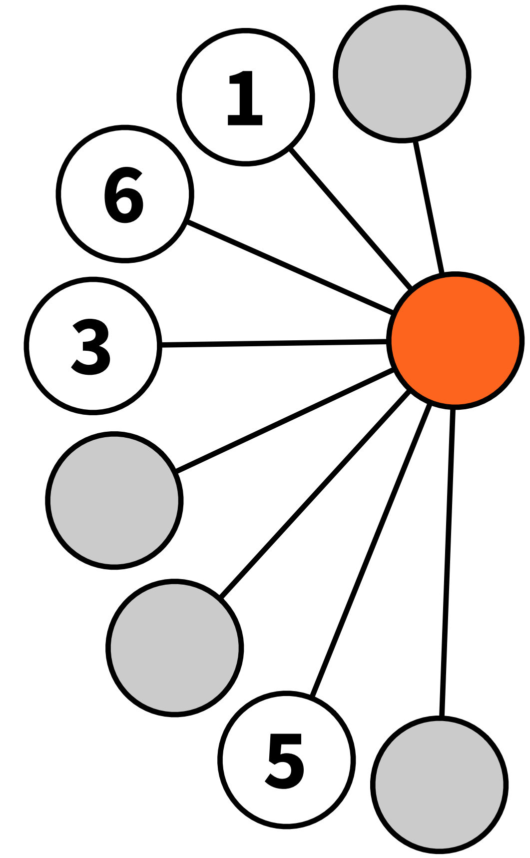
Algorithm idea 2

- **Colour palette: $\{1, 2, \ldots, \Delta + 1\}$**
- **Pick a random *free* colour**
  - not used by any neighbour that has stopped
- **Try again if conflicts…**

Algorithm idea 3

- **Colour palette: $\{1, 2, \ldots, \Delta + 1\}$**
- **Active with probability 1/2**
- **If *active*, pick a random *free* colour**
  - not used by any neighbour that has stopped
- **Try again if conflicts…**

Algorithm idea 3



- **Active with probability 1/2**
- **Intuition:**
  - assume: d neighbours still running
  - roughly $d/2$ of them active
  - at least $d + 1$ colours in my palette
  - easy to pick a colour without conflicts (?)

- $s = 1$, $c \neq \perp$:
    - stopping state; **output** c
- $s = 1$, $c = \perp$:
    - probability 1/2: $c \leftarrow \perp$
    - probability 1/2: $c \leftarrow$ **random free colour**
    - $s \leftarrow 0$
- $s = 0$:
    - if conflicts: $c \leftarrow \perp$
    - $s \leftarrow 1$

Algorithm DBRand: **Randomised colouring**

- **Lemma 1:** A running node succeeds with probability 1/4

Algorithm DBRand: **Randomised colouring**

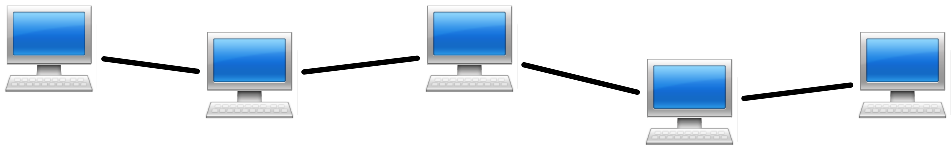
- **Lemma 1:** A running node succeeds with probability 1/4
- $\boldsymbol{T(n) = 2(c+1)\log_{4/3} n = O(\log n)}$
- **Lemma 2:** For any v, node v stops in time $T(n)$ with probability $\boldsymbol{1 - 1/n^{c+1}}$

Algorithm DBRand: **Randomised colouring**

- $T(n) = 2(c + 1) \log_{4/3} n = O(\log n)$
- **Lemma 2:** For any v, node v stops in time $T(n)$ with probability $1 - 1/n^{c+1}$
- **Theorem 3:** All nodes stop in time $T(n)$ with probability $1 - 1/n^{c}$

Summary

- **Randomness may help**
- **Common idea: each node makes random trials until successful**
- **Typical running time: $O(\log n)$ w.h.p.**
  - proof technique: in each round, each node successful with some constant probability

- **Weeks 1–2: informal introduction**
  - network = path 
- **Week 3: graph theory**
- **Weeks 4–7: models of computing**
  - what can be computed (efficiently)?
- **Weeks 8–11: lower bounds**
  - what cannot be computed (efficiently)?
- **Week 12: recap**