Sinkless orientation
This week’s plan

• **Topic:** complexity of *sinkless orientation*
  • task: high-degree nodes must have outdegree ≥ 1
  • possible in $O(\log n)$ rounds, not in $o(\log n)$ rounds

• **Video:** *why* do we care about this?
  • e.g. hardness of graph coloring

• **Today:** how to *prove* it?
  • round elimination & fixed points
Sinkless orientation

• **Labels:** \{ O, I \}
  - \(O = \) “edge oriented away from the active node”
  - \(I = \) “edge oriented towards the active node”

• **Active:** [ O, ?, ? ]
  - “at least one outgoing edge”

• **Passive:** [ I, ?, ? ]
  - “at least one outgoing edge”
active: [O, ?, ?]

passive: [I, ?, ?]
Sinkless orientation: \( O, I \)
- active: \([ O, ?, ? ]\)
- passive: \([ I, ?, ? ]\)

Output problem: \{O\}, \{I\}, \{O,I\}
- active: \([ \{I\}, ?, ? ]\)
- passive: \([ \{O\}, ?, ? ]\) or \([ \{O,I\}, ?, ? ]\)

Maximal problem: \{I\}, \{O,I\}
- active: \([ \{I\}, \{O,I\}, \{O,I\} ]\)
- passive: \([ \{O,I\}, ?, ? ]\)
Sinkless orientation: \(O, I\)
- active: \([O, ?, ?]\)
- passive: \([I, ?, ?]\)

Output problem: \(\{O\}, \{I\}, \{O,I\}\)
- active: \([\{I\}, ?, ?]\)
- passive: \([\{O\}, ?, ?]\) or \([\{O,I\}, ?, ?]\)

Maximal problem: \(A, B\)
- active: \([A, B, B]\)
- passive: \([B, ?, ?]\)
Output problem

• **Labels:** \{ A, B \}
  - A = “edge oriented away from the active node”
  - B = “edge oriented towards the active node”

• **Active:** [ A, B, B ]
  - “exactly one outgoing edge”

• **Passive:** [ B, ?, ? ]
  - “at least one outgoing edge”
**Starting point:** A, B
- active: [ A, B, B ]
- passive: [ B, ?, ? ]

**Output problem:** \{A\}, \{B\}, \{A,B\}
- active: [ \{B\}, ?, ? ]
- passive: ...

**Maximal problem:** \{B\}, \{A,B\}
- active: [ \{B\}, \{A,B\}, \{A,B\} ]
- passive: [ \{A,B\}, ?, ? ]
Starting point: A, B
  • active: [ A, B, B ]
  • passive: [ B, ?, ? ]

Output problem: \{A\}, \{B\}, \{A,B\}
  • active: [ \{B\}, ?, ? ]
  • passive: ...

Maximal problem: A, B
  • active: [ A, B, B ]
  • passive: [ B, ?, ? ]
Fixed points

• $X = \text{re}(X)$, and $X$ is not 0-round solvable
• “$X$ can be solved 1 round faster than $X$”
  • contradiction
• One of our assumptions fails — which one?
Fixed points

• $X = \text{re}(X)$, and $X$ is not 0-round solvable

• *X cannot be solved in $o(\log n)$ rounds* in the deterministic PN model

• We can also derive hardness results for deterministic and randomized LOCAL model
Often used like this

• We are interested in problem $X$
• Find a suitable relaxation $Y$ of $X$
  • problem $Y$ is at most as hard as $X$
  • problem $Y$ is nontrivial
• Show that $Y = \text{re}(Y)$ or $Y = \text{re}(\text{re}(Y))$
  • $Y$ cannot be solved fast
  • $X$ cannot be solved fast
Sinkless and sourceless

• **Labels:** \{ O, I \}
  - \( O = \) "edge oriented away from the active node"
  - \( I = \) "edge oriented towards the active node"

• **Active:** [ O, I, ? ]
  - "at least one outgoing and one incoming edge"

• **Passive:** [ I, O, ? ]
  - "at least one outgoing and one incoming edge"