Distributed Algorithms 2021

10 Sinkless orientation
This week’s plan

• **Topic:** complexity of *sinkless orientation*
  • task: high-degree nodes must have outdegree $\geq 1$
  • possible in $O(\log n)$ rounds, not in $o(\log n)$ rounds

• **Video:** *why* do we care about this?
  • e.g. hardness of graph coloring

• **Today:** how to *prove* it?
  • round elimination & fixed points
Sinkless orientation

• **Labels:** \{ O, I \}
  • O = “edge oriented away from the active node”
  • I = “edge oriented towards the active node”

• **Active:** [ O, ?, ? ]
  • “at least one outgoing edge”

• **Passive:** [ I, ?, ? ]
  • “at least one outgoing edge”
**Sinkless orientation:** \( O, I \)
- active: \([ O, ?, ? ]\)
- passive: \([ I, ?, ? ]\)

**Output problem:** \( \{O\}, \{I\}, \{O,I\} \)
- active: \([ \{I\}, ?, ? ]\)
- passive: \([ \{O\}, ?, ? ]\) or \([ \{O,I\}, ?, ? ]\)

**Maximal problem:** \( \{I\}, \{O,I\} \)
- active: \([ \{I\}, \{O,I\}, \{O,I\} ]\)
- passive: \([ \{O,I\}, ?, ? ]\)
Sinkless orientation: \( O, I \)
- active: \[ O, ?, ? \]
- passive: \[ I, ?, ? \]

Output problem: \{O\}, \{I\}, \{O,I\}
- active: \[ \{I\}, ?, ? \]
- passive: \[ \{O\}, ?, ? \] or \[ \{O,I\}, ?, ? \]

Maximal problem: \( A, B \)
- active: \[ A, B, B \]
- passive: \[ B, ?, ? \]
Output problem

• **Labels:** \{ A, B \}
  - A = “edge oriented away from the active node”
  - B = “edge oriented towards the active node”

• **Active:** [ A, B, B ]
  - “*exactly* one outgoing edge”

• **Passive:** [ B, ?, ? ]
  - “at least one outgoing edge”
Starting point: A, B
  • active: [ A, B, B ]
  • passive: [ B, ?, ? ]

Output problem: \{A\}, \{B\}, \{A,B\}
  • active: [ \{B\}, ?, ? ]
  • passive: ...

Maximal problem: \{B\}, \{A,B\}
  • active: [ \{B\}, \{A,B\}, \{A,B\} ]
  • passive: [ \{A,B\}, ?, ? ]
Starting point: A, B
  • active: [ A, B, B ]
  • passive: [ B, ?, ? ]

Output problem: {A}, {B}, {A,B}
  • active: [ {B}, ?, ? ]
  • passive: ...

Maximal problem: A, B
  • active: [ A, B, B ]
  • passive: [ B, ?, ? ]
Fixed points

• $X = \text{re}(X)$, and $X$ is not 0-round solvable

• “$X$ can be solved 1 round faster than $X$”
  • contradiction

• One of our assumptions fails — which one?
Fixed points

• \( X = \text{re}(X) \), and \( X \) is not 0-round solvable

• \( X \) cannot be solved in \( o(\log n) \) rounds in the deterministic PN model

• We can also derive hardness results for deterministic and randomized LOCAL model
Often used like this

- We are interested in problem $X$
- Find a suitable relaxation $Y$ of $X$
  - problem $Y$ is at most as hard as $X$
  - problem $Y$ is nontrivial
- Show that $Y = \text{re}(Y)$ or $Y = \text{re}(\text{re}(Y))$
  - $Y$ cannot be solved fast
  - $X$ cannot be solved fast
Sinkless and sourceless

**Labels:** \{ O, I \}
- O = “edge oriented away from the active node”
- I = “edge oriented towards the active node”

**Active:** [ O, I, ? ]
- “at least one outgoing and one incoming edge”

**Passive:** [ I, O, ? ]
- “at least one outgoing and one incoming edge”