Distributed Algorithms 2022

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Local neighborhoods

High-level plan

Algorithm A runs in **T rounds** and solves problem X

➡ A is a **mapping from radius-T neighborhoods** to local outputs

Such a mapping cannot solve X correctly

➡ Problem X is not solvable in T rounds

Example: coloring

- **Problem:** find a vertex coloring with the smallest possible number of colors

- **Proof:** *three different approaches!*

Example: coloring

- **Idea 1:** consider a path, *fix solutions in two neighborhoods*, construct another path

Example: coloring

- **Idea 2:** consider an odd cycle, *look at a node that outputs "3"*, construct a path

Example: coloring

- **Idea 3:** if we can 2-color paths locally,
  then we can also 2-color odd cycles

What about…

- PN model?

- CONGEST model?

- Randomized algorithms?

Example: leaf distance

- **Graph family:** trees
- **Model:** LOCAL
- **Input:** unique IDs and value of n
- **Output:** distance to the nearest leaf node

Example: is it a forest?

- **Input is a forest:** *all* nodes output *"yes"*, **otherwise:** *at least one* node outputs *"no"*

- **Questions:**
  - is this solvable in PN, and how fast?
  - is this solvable in LOCAL, and how fast?
  - does it help if we know n?

Example: is it a forest?

- PN, n is not known?

Example: is it a forest?

- PN, n is known?

Example: is it a forest?

- LOCAL, n is not known?

Example: is it a forest?

- LOCAL, n is known?