Distributed Algorithms 2022

10 Sinkless orientation
This week’s plan

• **Topic:** complexity of *sinkless orientation*
  • task: high-degree nodes must have outdegree \( \geq 1 \)
  • possible in \( O(\log n) \) rounds, not in \( o(\log n) \) rounds

• **Video:** *why* do we care about this?
  • e.g. hardness of graph coloring

• **Today:** how to *prove* it?
  • round elimination & fixed points
Sinkless orientation

• **Labels:** \{ O, I \}
  • O = “edge oriented away from the active node”
  • I = “edge oriented towards the active node”

• **Active:** [ O, ?, ? ]
  • “at least one outgoing edge”

• **Passive:** [ I, ?, ? ]
  • “at least one outgoing edge”
Sinkless orientation: $O, I$
  • active: [ $O, ?, ?$ ]
  • passive: [ $I, ?, ?$ ]

Output problem: $\{O\}, \{I\}, \{O,I\}$
  • active: [ $\{I\}, ?, ?$ ]
  • passive: [ $\{O\}, ?, ?$ ] or [ $\{O,I\}, ?, ?$ ]

Maximal problem: $\{I\}, \{O,I\}$
  • active: [ $\{I\}, \{O,I\}, \{O,I\}$ ]
  • passive: [ $\{O,I\}, ?, ?$ ]
Sinkless orientation: $O, I$
- active: $[O, ?, ?]$
- passive: $[I, ?, ?]$

Output problem: $\{O\}, \{I\}, \{O,I\}$
- active: $\{I\}$, $[\{\}, ?, ?]$  
- passive: $\{O\}$, $[\{\}, ?, ?]$ or $\{O,I\}$, $[\{\}, ?, ?]$

Maximal problem: $A, B$
- active: $[A, B, B]$
- passive: $[B, ?, ?]$
Output problem

• **Labels:** \{ A, B \}
  - A = “edge oriented away from the active node”
  - B = “edge oriented towards the active node”

• **Active:** [ A, B, B ]
  - “**exactly** one outgoing edge”

• **Passive:** [ B, ?, ? ]
  - “at least one outgoing edge”
Starting point: A, B
- active: [ A, B, B ]
- passive: [ B, ?, ? ]

Output problem: \{A\}, \{B\}, \{A,B\}
- active: [ \{B\}, ?, ? ]
- passive: ...

Maximal problem: \{B\}, \{A,B\}
- active: [ \{B\}, \{A,B\}, \{A,B\} ]
- passive: [ \{A,B\}, ?, ? ]
**Starting point:** A, B
  - active: [ A, B, B ]
  - passive: [ B, ?, ? ]

**Output problem:** \{A\}, \{B\}, \{A,B\}
  - active: [ \{B\}, ?, ? ]
  - passive: ...

**Maximal problem:** A, B
  - active: [ A, B, B ]
  - passive: [ B, ?, ? ]
Fixed points

• $X = \text{re}(X)$, and $X$ is not 0-round solvable

• "$X$ can be solved 1 round faster than $X$"
  • contradiction

• One of our assumptions fails — which one?
Fixed points

• $X = \text{re}(X)$, and $X$ is not 0-round solvable

• *$X$ cannot be solved in $o(\log n)$ rounds* in the deterministic PN model

• We can also derive hardness results for deterministic and randomized LOCAL model
Often used like this

• We are interested in problem $X$
• Find a suitable *relaxation* $Y$ of $X$
  • problem $Y$ is at most as hard as $X$
  • problem $Y$ is nontrivial
• Show that $Y = \text{re}(Y)$ or $Y = \text{re}(\text{re}(Y))$
  • $Y$ cannot be solved fast
  • $X$ cannot be solved fast
Sinkless and sourceless

- **Labels:** \{ O, I \}
  - O = “edge oriented away from the active node”
  - I = “edge oriented towards the active node”

- **Active:** [ O, I, ? ]
  - “at least one outgoing and one incoming edge”

- **Passive:** [ I, O, ? ]
  - “at least one outgoing and one incoming edge”