Distributed Algorithms 2023

Local neighborhoods
High-level plan

Algorithm $A$ runs in $T$ rounds and solves problem $X$

$\rightarrow A$ is a mapping from radius-$T$ neighborhoods to local outputs

Such a mapping cannot solve $X$ correctly

$\rightarrow$ Problem $X$ is not solvable in $T$ rounds
Example: coloring

• **Problem:** find a vertex coloring with the smallest possible number of colors

• **Proof:** *three different approaches!*

Ex.
Example: coloring

- **Idea 1:** consider a path, *fix solutions in two neighborhoods*, construct another path
Example: coloring

- **Idea 2:** consider an odd cycle, *look at a node that outputs “3”*, construct a path.
Example: coloring

• **Idea 3:** if we can 2-color paths locally, *then we can also 2-color odd cycles*
What about...

• PN model?
• CONGEST model?
• Randomized algorithms?
Example: leaf distance

- **Graph family:** trees
- **Model:** LOCAL
- **Input:** unique IDs and value of $n$
- **Output:** distance to the nearest leaf node
Example: is it a forest?

- **Input is a forest:** all nodes output “yes”, otherwise: at least one node outputs “no”

- **Questions:**
  - is this solvable in PN, and how fast?
  - is this solvable in LOCAL, and how fast?
  - does it help if we know $n$?
Example: is it a forest?

• PN, $n$ is not known?
Example: is it a forest?

• PN, n is known?
Example: is it a forest?

• LOCAL, $n$ is not known?
Example: is it a forest?

• LOCAL, $n$ is known?