Distributed Algorithms 2023

Sinkless orientation
This week’s plan

• **Topic:** complexity of *sinkless orientation*
  • task: high-degree nodes must have outdegree $\geq 1$
  • possible in $O(\log n)$ rounds, not in $o(\log n)$ rounds

• **Video:** *why* do we care about this?
  • e.g. hardness of graph coloring

• **Today:** how to *prove* it?
  • round elimination & fixed points
Sinkless orientation

• **Labels:** \{ O, I \}
  • O = “edge oriented away from the active node”
  • I = “edge oriented towards the active node”

• **Active:** [ O, ?, ? ]
  • “at least one outgoing edge”

• **Passive:** [ I, ?, ? ]
  • “at least one outgoing edge”
Sinkless orientation: $O, I$

- active: $[O, ?, ?]$
- passive: $[I, ?, ?]$

Output problem: $\{O\}, \{I\}, \{O, I\}$

- active: $\{I\}, ?, ?$
- passive: $\{O\}, ?, ?$ or $\{O, I\}, ?, ?$

Maximal problem: $\{I\}, \{O, I\}$

- active: $\{I\}, \{O, I\}, \{O, I\}$
- passive: $\{O, I\}, ?, ?$
Sinkless orientation: O, I
  • active: [ O, ?, ? ]
  • passive: [ I, ?, ? ]

Output problem: \{O\}, \{I\}, \{O,I\}
  • active: [ \{I\}, ?, ? ]
  • passive: [ \{O\}, ?, ? ] or [ \{O,I\}, ?, ? ]

Maximal problem: A, B
  • active: [ A, B, B ]
  • passive: [ B, ?, ? ]
Output problem

- **Labels**: \{ A, B \}
  - A = “edge oriented away from the active node”
  - B = “edge oriented towards the active node”

- **Active**: [ A, B, B ]
  - “exactly one outgoing edge”

- **Passive**: [ B, ?, ? ]
  - “at least one outgoing edge”
**Starting point:** A, B
- active: [ A, B, B ]
- passive: [ B, ?, ? ]

**Output problem:** \{A\}, \{B\}, \{A,B\}
- active: [ \{B\}, ?, ? ]
- passive: ...

**Maximal problem:** \{B\}, \{A,B\}
- active: [ \{B\}, \{A,B\}, \{A,B\} ]
- passive: [ \{A,B\}, ?, ? ]
**Starting point:** A, B
- active: [ A, B, B ]
- passive: [ B, ?, ? ]

**Output problem:** \{A\}, \{B\}, \{A,B\}
- active: [ \{B\}, ?, ? ]
- passive: ...

**Maximal problem:** A, B
- active: [ A, B, B ]
- passive: [ B, ?, ? ]
Fixed points

• \( X = \text{re}(X) \), and \( X \) is not 0-round solvable
• “\( X \) can be solved 1 round faster than \( X \)”
  • contradiction
• One of our assumptions fails — which one?
Fixed points

• $X = \text{re}(X)$, and $X$ is not 0-round solvable

• *X cannot be solved in $o(\log n)$ rounds* in the deterministic PN model

• We can also derive hardness results for deterministic and randomized LOCAL model
Often used like this

• We are interested in problem $X$
• Find a suitable $\text{relaxation} \ Y$ of $X$
  • problem $Y$ is at most as hard as $X$
  • problem $Y$ is nontrivial
• Show that $Y = \text{re}(Y)$ or $Y = \text{re}(\text{re}(Y))$
  • $Y$ cannot be solved fast
  • $X$ cannot be solved fast
Sinkless and sourceless

• **Labels:** \( \{ O, I \} \)
  - \( O \) = "edge oriented away from the active node"
  - \( I \) = "edge oriented towards the active node"

• **Active:** \[ O, I, ? \]
  - "at least one outgoing and one incoming edge"

• **Passive:** \[ I, O, ? \]
  - "at least one outgoing and one incoming edge"