Aalto University

Distributed Algorithms 2024

9 Round elimination

Can we automate our own work?

Meta-algorithmics

- **Normal algorithms** — example:
  - input: graph G
  - output: coloring of graph G
- **Meta-algorithms** — example:
  - input: ***computational problem*** P
  - output: ***algorithm*** for solving P

How to represent problems or algorithms?

Plan

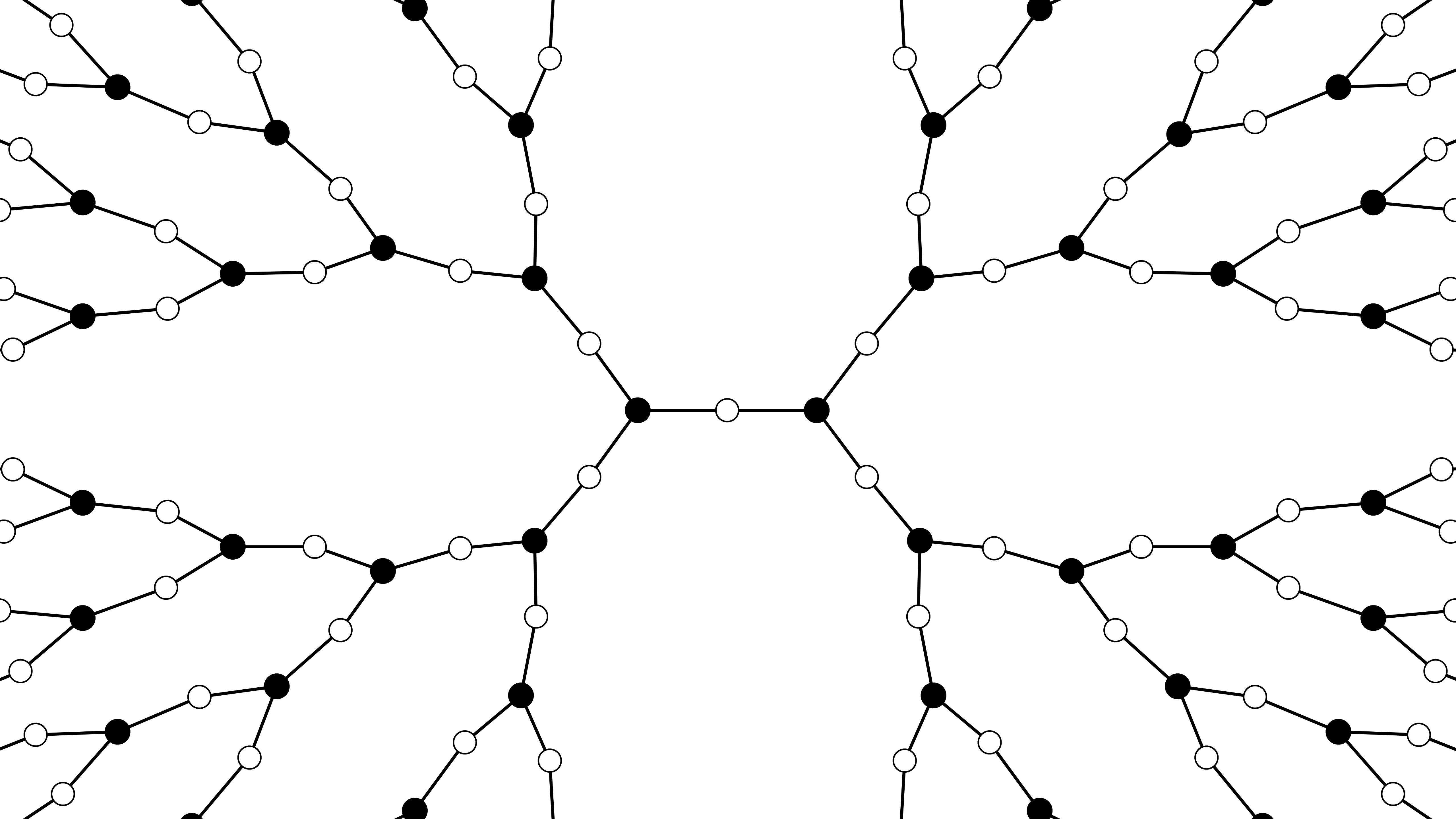
- **Topic:** ***round elimination***
  - function that maps problem X with complexity T to problem $X' = \mathrm{re}(X)$ with complexity $T - 1$
- **Video:** how to ***use*** round elimination
  - “re” was a black box
- **Today:** how to ***do*** round elimination
  - what happens inside the black box and why?

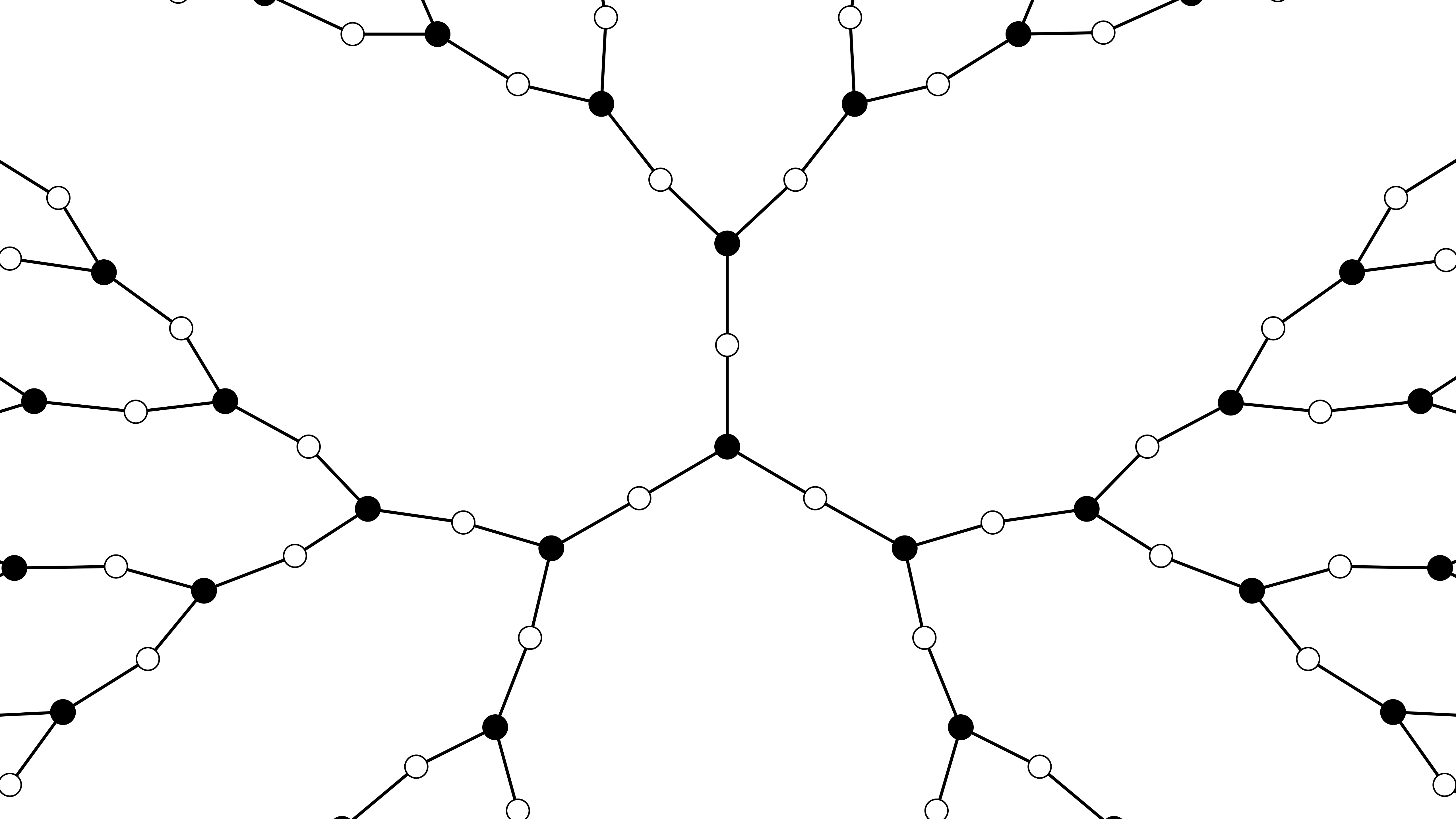
Round elimination

- Basic idea already used by Linial (1987)
  - *"it is not possible to 3-color cycles in* $o(\log^* n)$ *rounds"*
- Until 2015 it was thought this is an ad-hoc trick that only works for graph coloring
- **Lots** of new applications since 2016
- General idea formalized in 2019

Weak 3-labeling

- **Labels:** 1, 2, 3
- **Active nodes:**
  - degree 3
  - not all labels same
- **Passive nodes:**
  - degree 2
  - both labels same





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10 Sinkless orientation

Plan

- **Topic:** complexity of ***sinkless orientation***
  - task: high-degree nodes must have outdegree ≥ 1
  - possible in $O(\log n)$ rounds, not in $o(\log n)$ rounds
- **Video:** ***why*** do we care about this?
  - e.g. hardness of graph coloring
- **Today:** how to ***prove*** it?
  - round elimination & fixed points

Sinkless orientation

- **Labels:** { O, I }
  - O = *"edge oriented away from the active node"*
    I = *"edge oriented towards the active node"*
- **Active:** [O, ?, ?]
  - *"at least one outgoing edge"*
- **Passive:** [I, ?, ?]
  - *"at least one outgoing edge"*

Sinkless orientation: O, I

- active: [O, ?, ?]
- passive: [I, ?, ?]

Output problem: {O}, {I}, {O,I}

- active: [{I}, ?, ?]
- passive: [{O}, ?, ?] or [{O,I}, ?, ?]

Maximal problem: {I}, {O,I}

- active: [{I}, {O,I}, {O,I}]
- passive: [{O,I}, ?, ?]

Sinkless orientation: O, I

- active: [O, ?, ?]
- passive: [I, ?, ?]

Output problem: {O}, {I}, {O,I}

- active: [{I}, ?, ?]
- passive: [{O}, ?, ?] or [{O,I}, ?, ?]

Maximal problem: A, B

- active: [A, B, B]
- passive: [B, ?, ?]

Output problem

- **Labels:** { A, B }
  - A = *“edge oriented away from the active node”*
  - B = *“edge oriented towards the active node”*
- **Active:** [A, B, B]
  - *“**exactly** one outgoing edge”*
- **Passive:** [B, ?, ?]
  - *“at least one outgoing edge”*

Starting point: A, B

- active: [A, B, B]
- passive: [B, ?, ?]

Output problem: {A}, {B}, {A,B}

- active: [{B}, ?, ?]
- passive: …

Maximal problem: {B}, {A,B}

- active: [{B}, {A,B}, {A,B}]
- passive: [{A,B}, ?, ?]

Starting point: A, B

- active: [A, B, B]
- passive: [B, ?, ?]

Output problem: {A}, {B}, {A,B}

- active: [{B}, ?, ?]
- passive: ...

Maximal problem: **A**, **B**

- active: [**A**, **B**, **B**]
- passive: [**B**, ?, ?]

Fixed points

- ***X* = re(*X*)**, and *X* is not 0-round solvable
- "*X* can be solved 1 round faster than *X*"
  - contradiction
- One of our assumptions fails — which one?

Fixed points

- $\boldsymbol{X} = \mathbf{re}(\boldsymbol{X})$, and X is not 0-round solvable
- ***X* cannot be solved in *o*(log *n*) rounds** in the deterministic PN model
- We can also derive hardness results for deterministic and randomized LOCAL model

Often used like this

- We are interested in problem X
- Find a suitable ***relaxation*** Y of X
  - problem Y is at most as hard as X
  - problem Y is nontrivial
- Show that Y = re(Y) or Y = re(re(Y))
  - Y cannot be solved fast
  - X cannot be solved fast

Sinkless and sourceless

- **Labels:** { O, I }
  - O = *"edge oriented away from the active node"*
    I = *"edge oriented towards the active node"*
- **Active:** [O, I, ?]
  - *"at least one outgoing and one incoming edge"*
- **Passive:** [I, O, ?]
  - *"at least one outgoing and one incoming edge"*