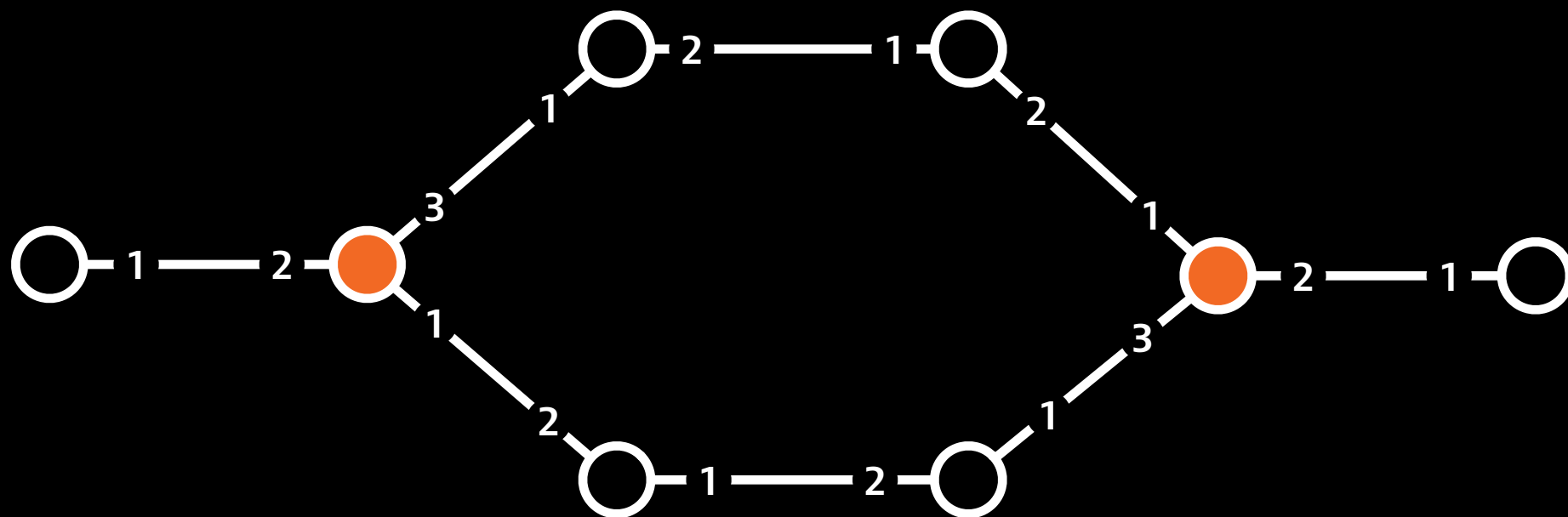
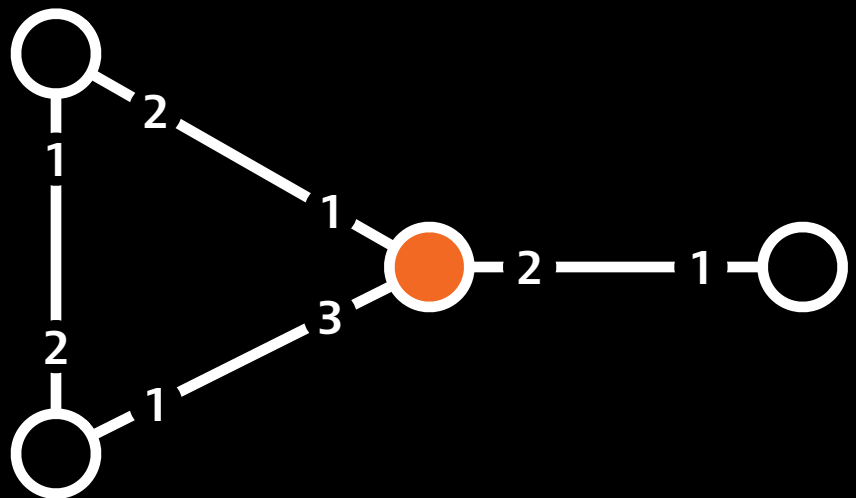


Distributed Algorithms 2025

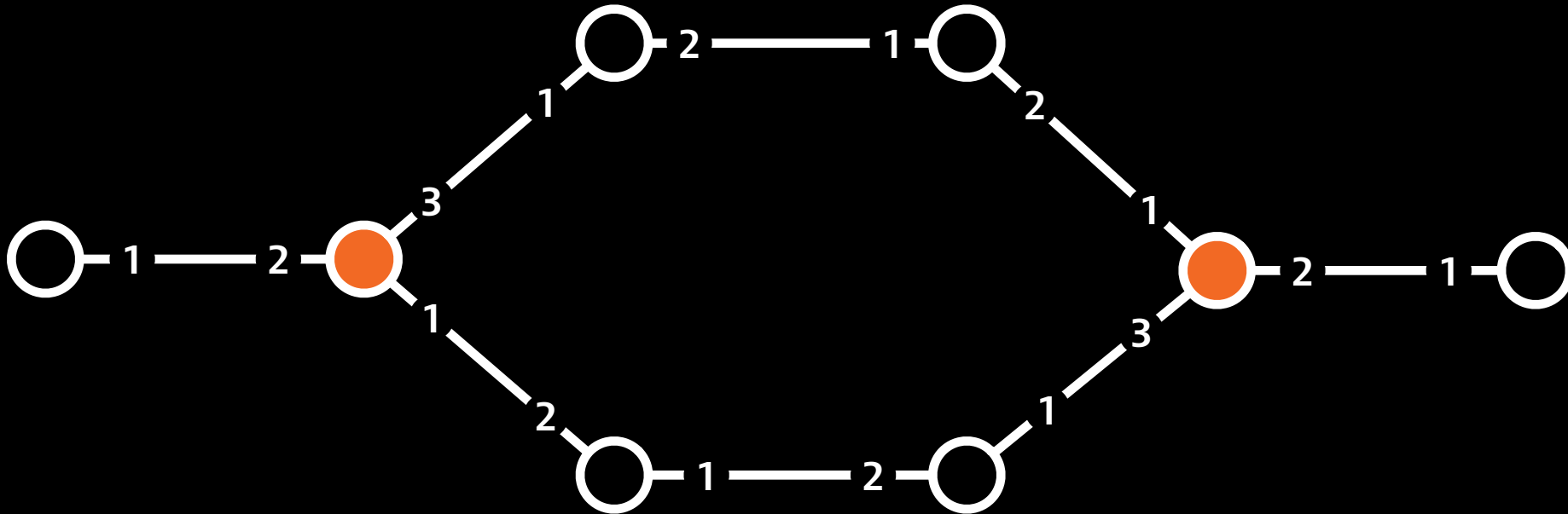
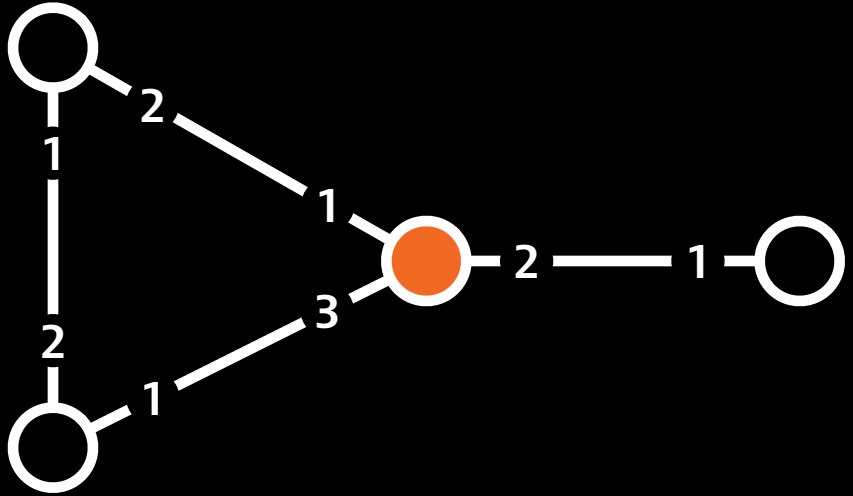
7 Covering maps



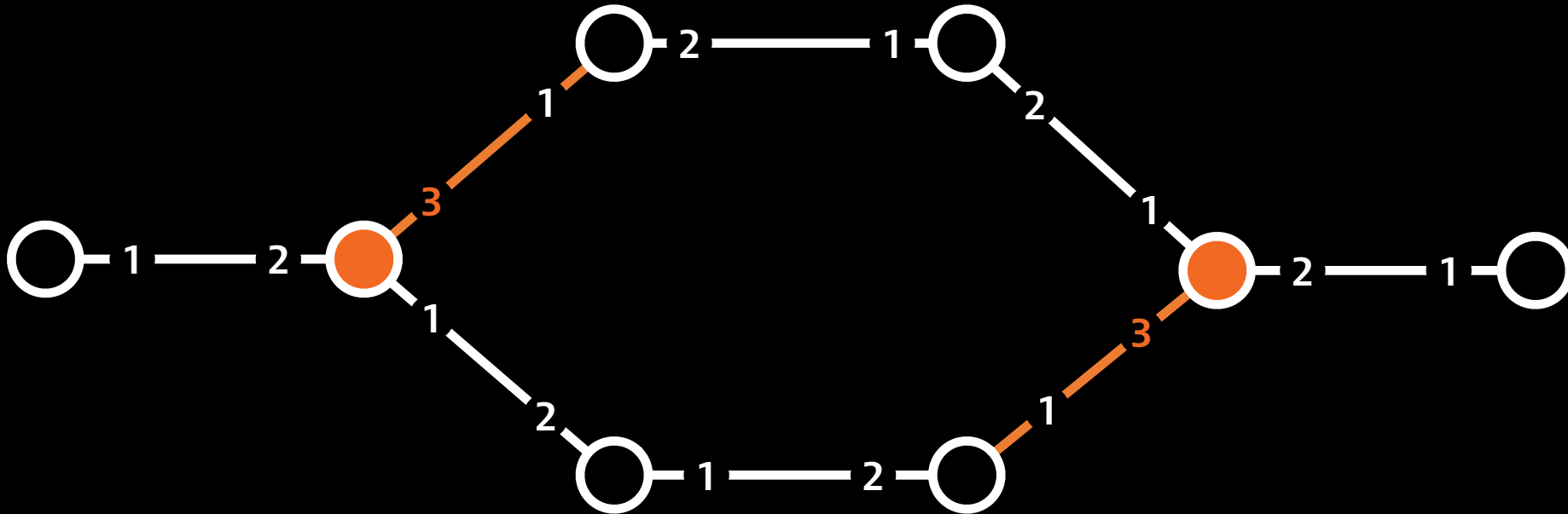
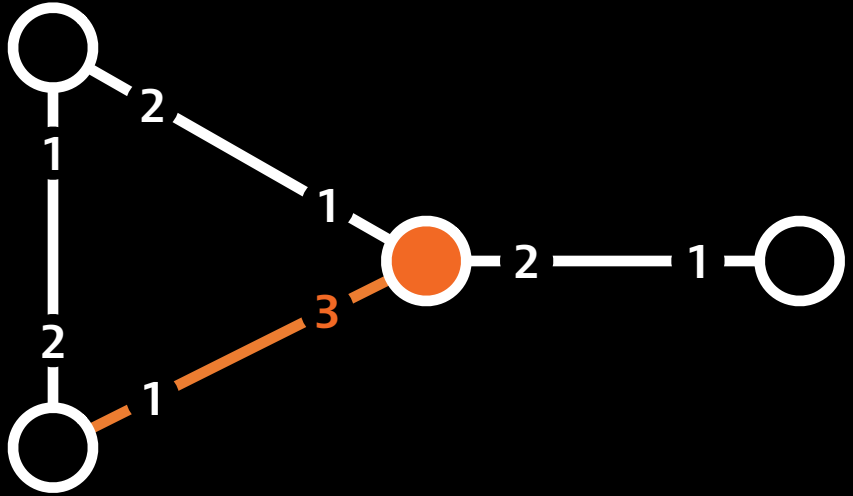
You are in a room
with three doors,
labeled 1, 2, and 3.

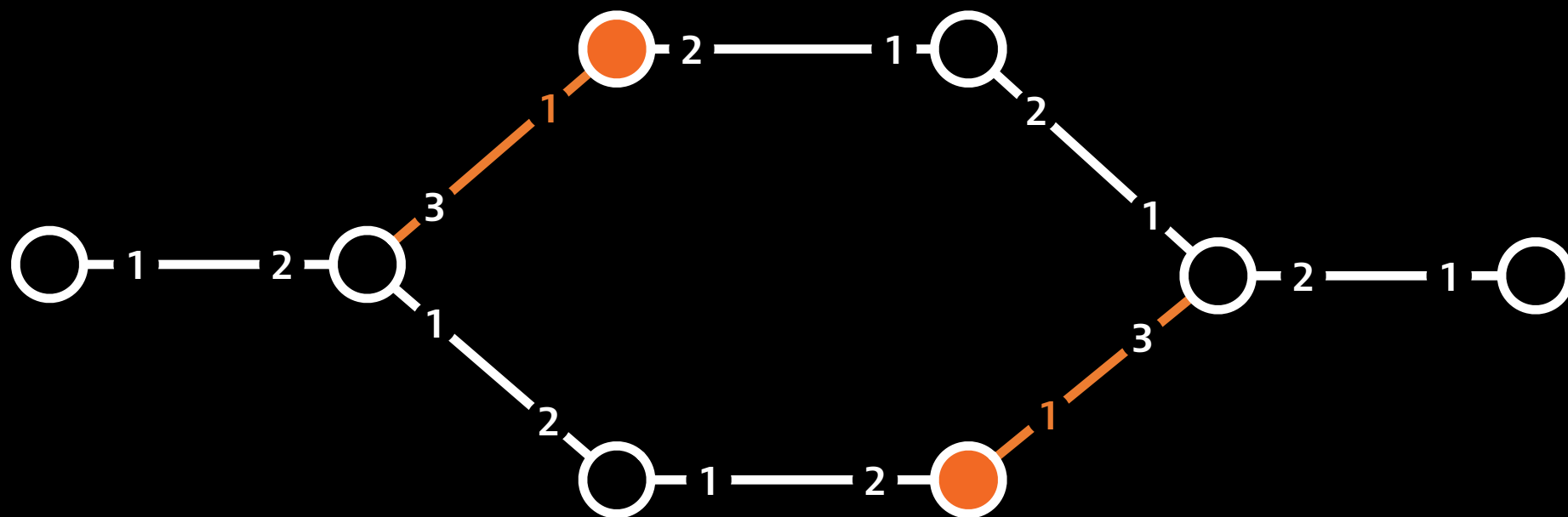
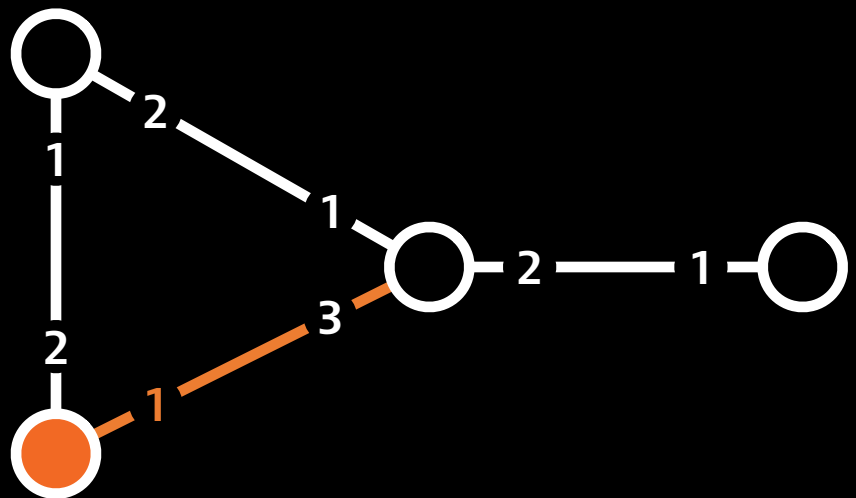
>

—



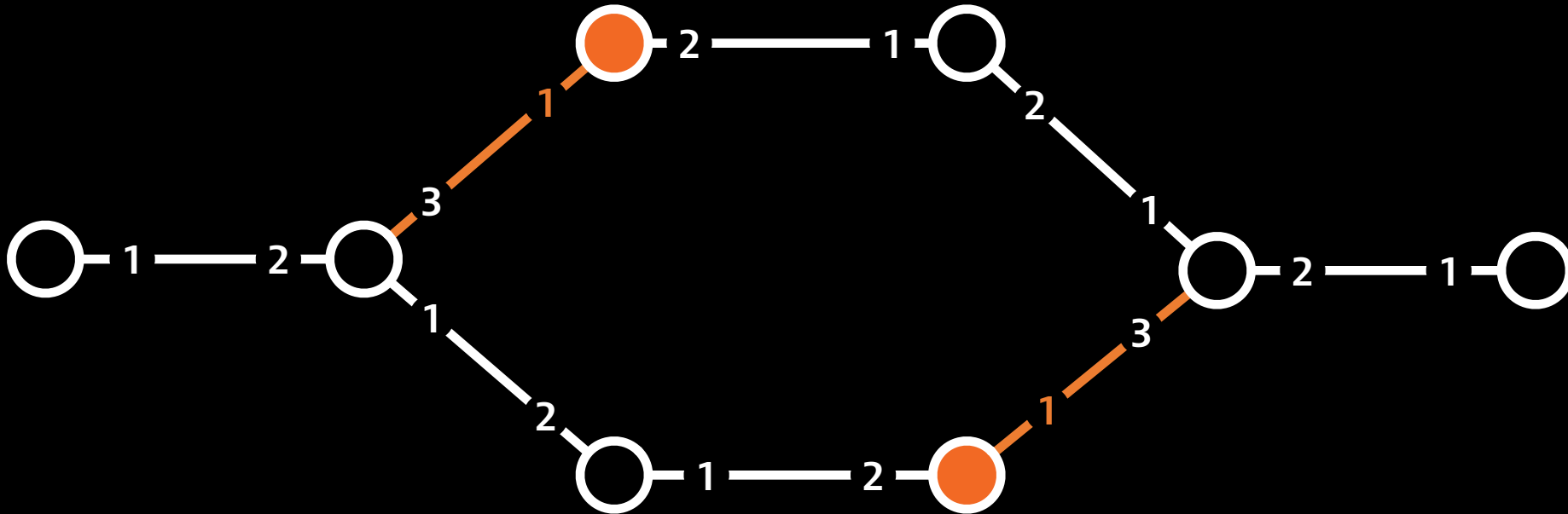
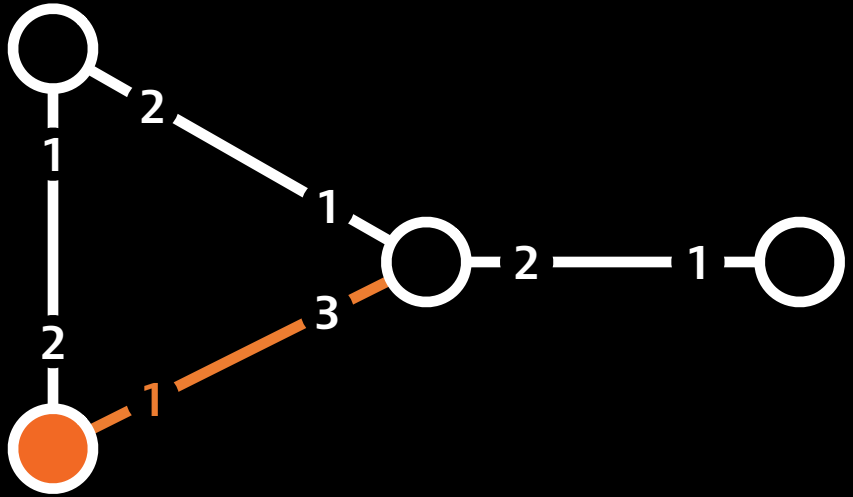
You are in a room
with three doors,
labeled 1, 2, and 3.
> open door 3



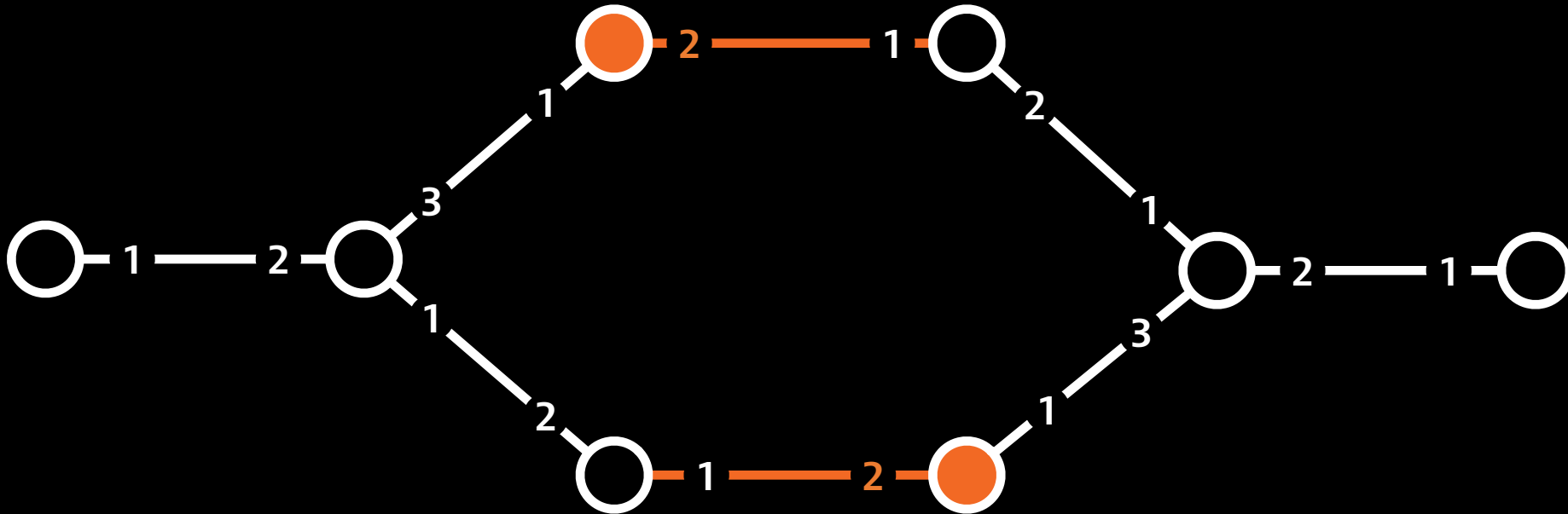
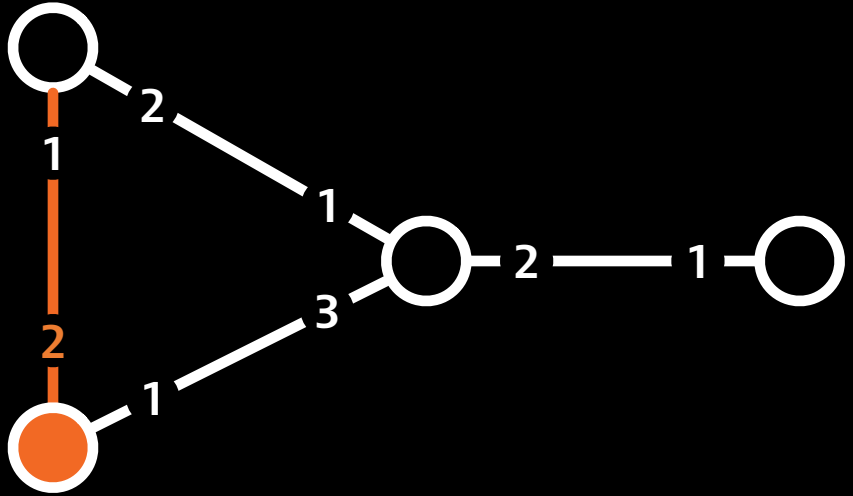


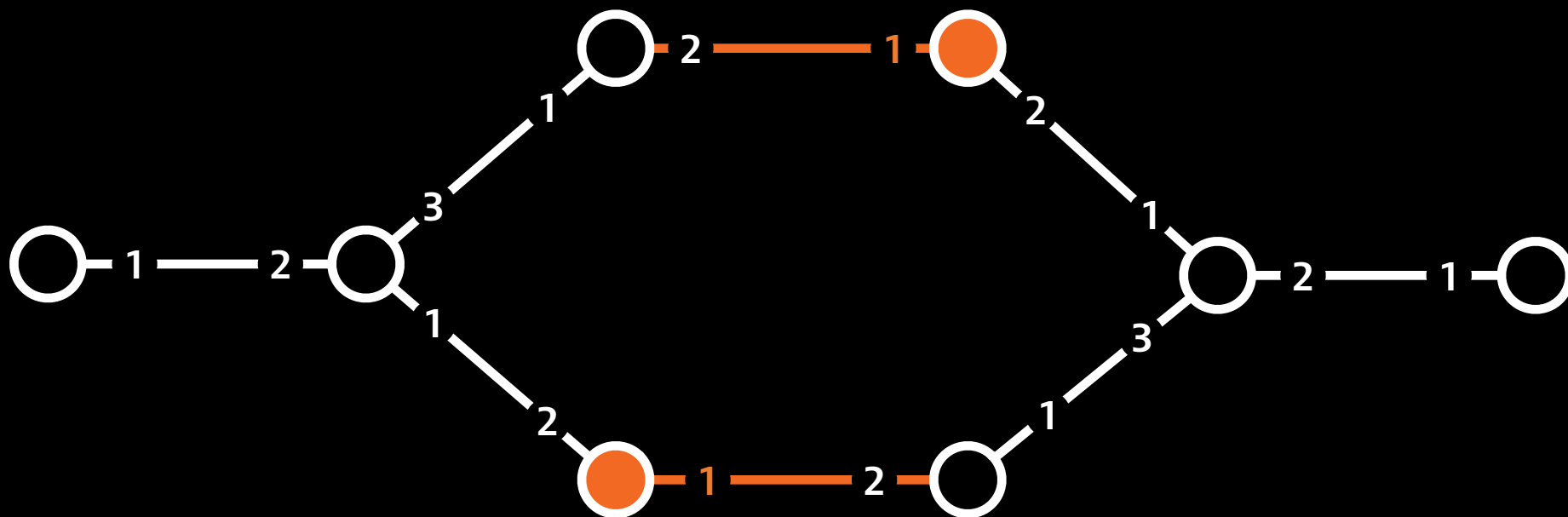
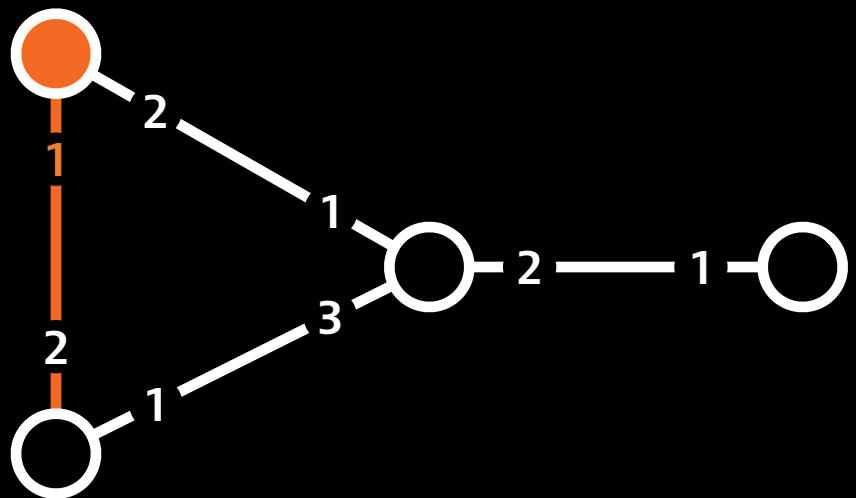
You enter a room with two doors, labeled 1 and 2. You just came in through doorway 1.

> _



You enter a room with two doors, labeled 1 and 2. You just came in through doorway 1.
> open door 2

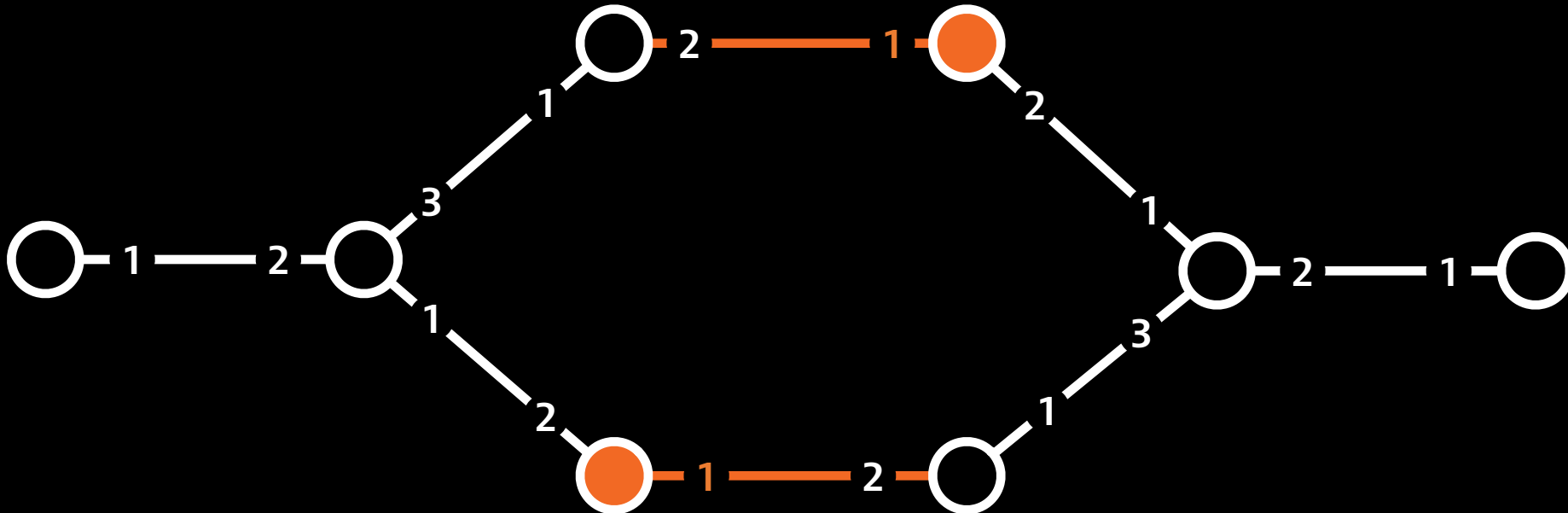
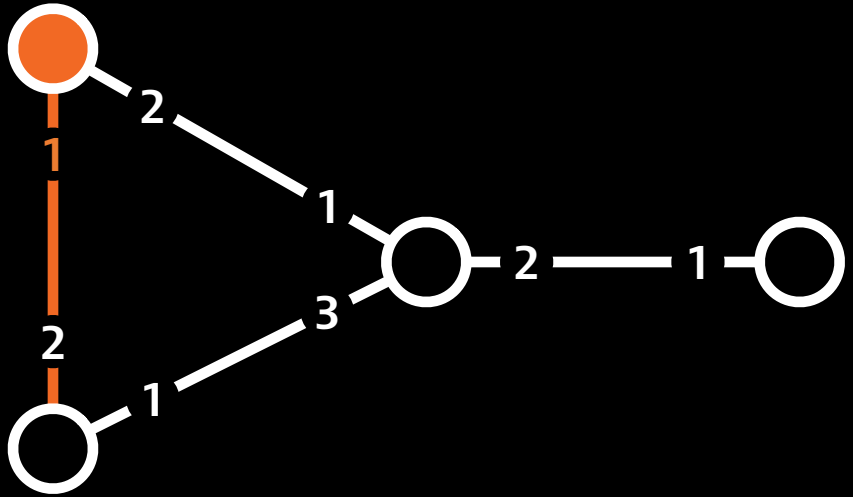


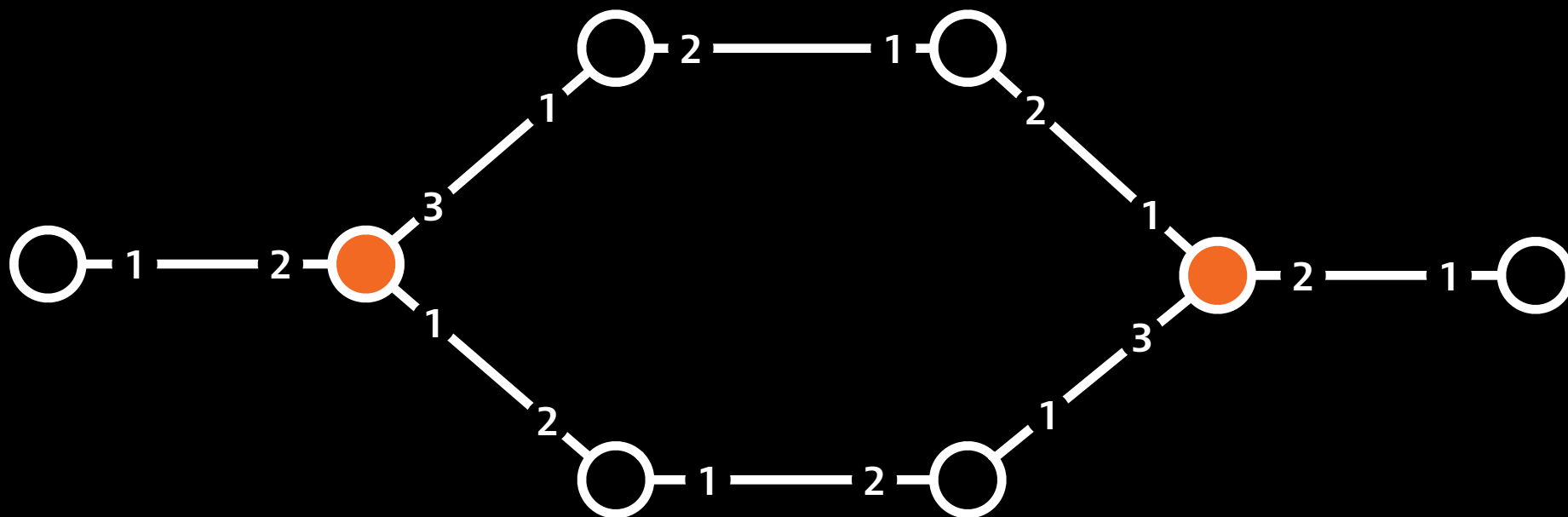
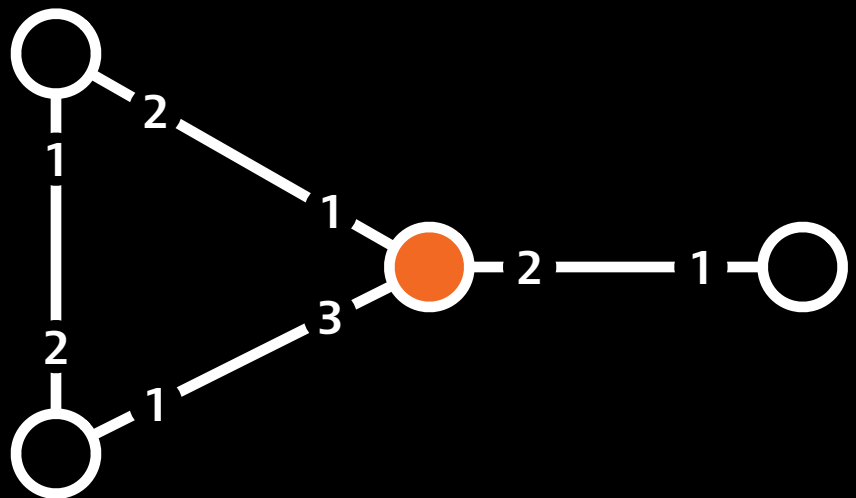


You enter a room with two doors, labeled 1 and 2. You just came in through doorway 1.

>

—





High-level plan

- **Goal:**

- show that problem X cannot be solved in the port-numbering model

High-level plan

- **Goal:**

- show that problem X cannot be solved in the port-numbering model

- **General approach:**

- construct port-numbered networks so that some nodes $u, v, \dots$ will always produce the *same output*
- show that if $u, v, \dots$ have the same output, then it is *not a feasible solution* for X

High-level plan

- **Goal:**

- show that problem X cannot be solved in the port-numbering model



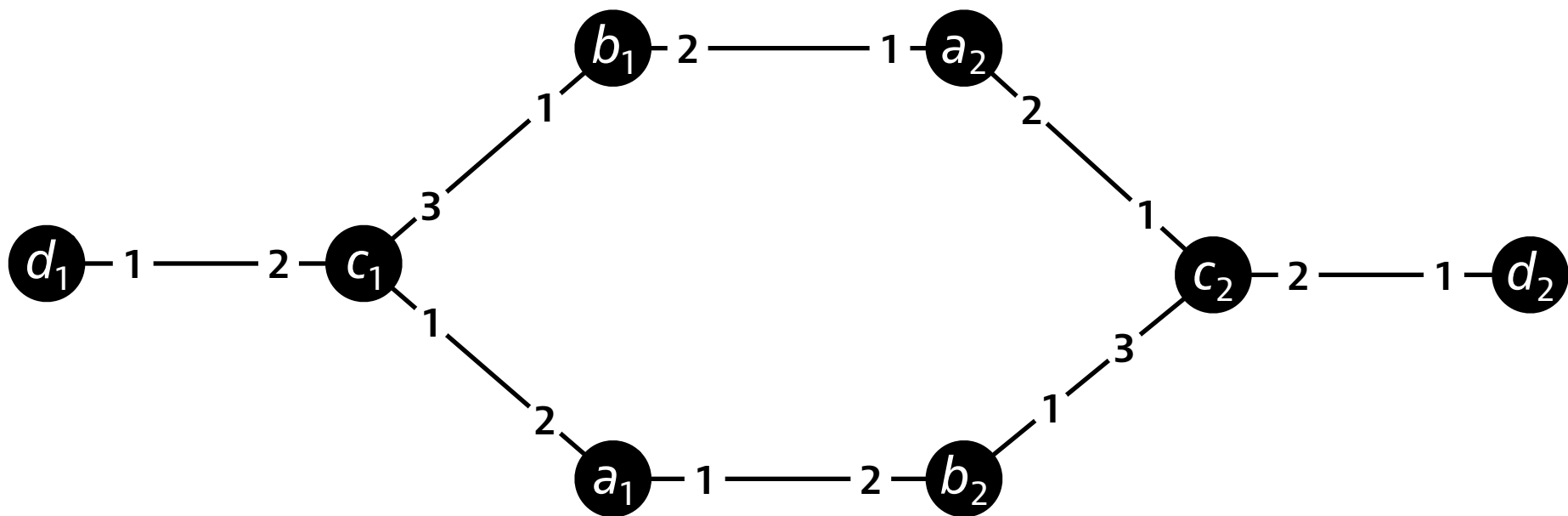
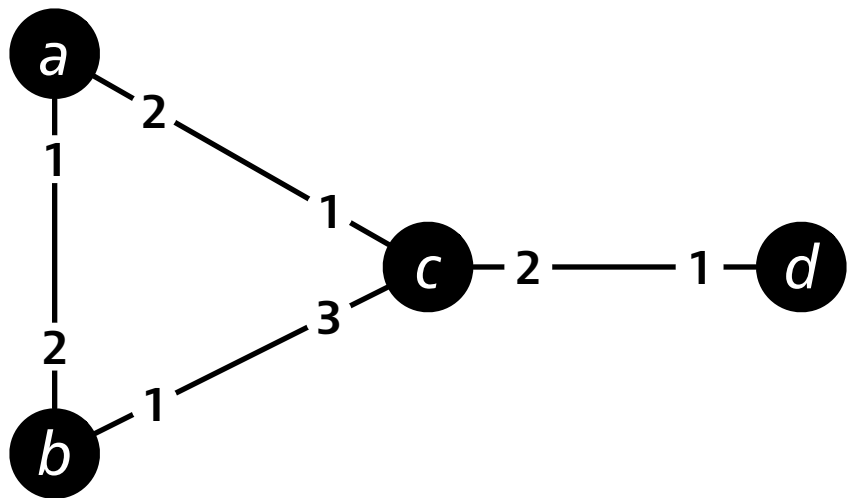
Covering
maps used
here

- **General approach:**

- construct port-numbered networks so that some nodes $u, v, \dots$ will always produce the *same output*
- show that if $u, v, \dots$ have the same output, then it is *not a feasible solution* for X

Covering map

- Two port-numbered networks:
 - $N = (V, P, p)$
 - $N' = (V', P', p')$
- Surjection $f: V \rightarrow V'$ that preserves:
 - inputs
 - degrees
 - connections
 - port numbers



Covering map

- “Fools” any deterministic algorithm
- If f is a covering map from N to N' , then:
 - v and $f(v)$ have the same state *before* round 1
 - v and $f(v)$ send the same messages in round 1
 - v and $f(v)$ receive the same messages in round 1
 - v and $f(v)$ have the same state *after* round 1

Covering map

- “Fools” any deterministic algorithm
- If f is a covering map from N to N' , then:
 - v and $f(v)$ have the same state *before* round T
 - v and $f(v)$ send the same messages in round T
 - v and $f(v)$ receive the same messages in round T
 - v and $f(v)$ have the same state *after* round T

Common steps

- Starting point: graph problem X
- Which graph G would be a “hard instance”?
- How to choose a port numbering N of G ?
- How to choose the other network N' ?
- How to construct mapping from N to N' ?

Example: 2-node path

Example: 4-node path

Common setup

- ***N is the network we care about***
 - simple port-numbered network
 - well-defined and interesting underlying graph
- ***N' is something strange***
 - not necessarily a simple port-numbered network
 - running A in N' makes no sense
 - **introduced only to analyze what happens when we run A in N**

Observations

- We can use covering maps to construct **universal** counterexamples
 - **adaptive:** *“for any given algorithm A we can find a hard input N ”*
 - **universal:** *“there is an input N that is hard for any algorithm A ”*

Week 8 (October 27–31)

- **DISC 2025 conference**
- **No course meetings**
 - no lecture, no exercise session
- **Other course activities as usual**
 - self-study as usual
 - quiz as usual, exercise deadlines as usual
 - help available in Zulip