CS-E4510 Distributed Algorithms / Jukka Suomela
exam, 16 October 2025

Material. You can bring one A4 paper (two-sided), with any content you want. No other material or equipment is allowed in the exam.

Instructions. There are three questions; please ***try to answer something in each of them***. If you cannot solve a problem entirely, please try to solve at least some special case or simpler version of the problem (see the hints for suggestions), or failing that, please at least explain what you tried and what went wrong.

For question 1, we expect a ***mathematical proof*** (it can be short and informal). For questions 2–3, it is sufficient to give a ***brief, informal description*** of the algorithm, and a ***brief, informal explanation*** of why it solves the problem correctly. You do not need to specify algorithms in the state-machine formalism, or give a complete proof of correctness. You are free to refer to algorithms and results that we discussed in the lectures, course material, and exercises; there is no need to repeat any of their details. In all questions it is enough that a friendly, cooperative reader can understand your idea correctly and see why it makes sense.

Definitions. Let $G = (V, E)$ be a graph. As usual, n denotes the number of nodes in the input graph. For each node $v \in V$, let

$$N(v) = \{u \in V : \mathrm{dist}(u, v) \leq 1\} = \mathrm{ball}_G(v, 1)$$

denote the set of nodes within the radius-1 neighborhood of v. That is, $N(v)$ consists of v itself and its neighbors. For a set $X \subseteq V$ and a node $v \in V$, let

$$\mathrm{weight}(v, X) = |N(v) \cap X|$$

denote the number of nodes in X that are within radius-1 neighborhoods of v. Finally, we define that $X \subseteq V$ is a *one-two-happy set* if

$$1 \leq \mathrm{weight}(v, X) \leq 2$$

holds for all nodes $v \in V$. That is, every node must have exactly 1 or 2 nodes of X in its radius-1 neighborhood.

Recall also the usual definitions:

- G is a *tree* if it is connected and acyclic.
- G is a *path graph* if it is a tree where all nodes have degree at most 2.
- G is a *cycle graph* if it is connected and 2-regular.

Question 1: Graph theory. Prove: there exists a tree $G = (V, E)$ such that no $X \subseteq V$ is a one-two-happy set. That is, there exists a tree where it is impossible to find a one-two-happy set.

Question 2: PN. Give a deterministic distributed algorithm that finds a one-two-happy set in the following setting in the PN model (any running time is fine):

- Graph family: ***path graphs***.
- Local inputs: nothing.
- Local outputs: a one-two-happy set.

Question 3: LOCAL. Give a deterministic distributed algorithm that finds a one-two-happy set in the following setting in the LOCAL model in $\boldsymbol{O(\log^* n)}$ rounds:

- Graph family: ***cycle graphs***.
- Local inputs: unique identifier and the value of n. You can assume that the unique identifiers are chosen from $\{1, 2, \ldots, n^2\}$.
- Local outputs: a one-two-happy set.

Hints: If you cannot design a deterministic algorithm, try to design a randomized algorithm. If you cannot solve the general case, please try to solve the problem in directed cycles. If you cannot achieve the running time of $O(\log^* n)$ rounds, try to achieve any nontrivial running time, i.e., $o(n)$ rounds.