

**Instructions.** There are four questions, each of them worth 15 points. For the minimum passing grade of 1/5, you will need approximately 30 points, and for the highest grade of 5/5, you will need approximately 50 points. You can answer in English, Finnish, or Swedish. In your answers, you can refer to the theorems that are presented in the textbook without proving them.

**Definitions.**

- The *counting problem*  $\Pi$  is defined as follows: if  $N = (V, P, p)$  is a port-numbered network, then  $g \in \Pi(N)$  if and only if  $g(v) = |V|$  for all  $v \in V$ . That is, in the counting problem each node has to output the value  $|V|$ , i.e., it has to indicate how many nodes there are in the network.
- Graph family  $F_C$  consists of all cycle graphs (i.e., connected 2-regular graphs).
- Graph family  $F_T$  consists of all trees (i.e., connected acyclic graphs).
- Graph family  $F_D$  consists of all graphs of maximum degree 2.

**Question 1.** Prove that the counting problem cannot be solved on  $F_C$  in the port-numbering model.

**Question 2.** Design an algorithm that solves the counting problem on  $F_T$  in the port-numbering model. An informal description of the algorithm is sufficient.

**Question 3.** Design an algorithm that solves the counting problem on  $F_C$  in the model of unique identifiers in time  $O(|V|)$ . An informal description of the algorithm is sufficient.

**Question 4.** (a) Prove that the counting problem cannot be solved on  $F_D$  in the model of unique identifiers, no matter how much time you have. (b) Prove that the counting problem cannot be solved in time  $o(|V|)$  on  $F_C$  in the model of unique identifiers.