Coordinating concurrent transmissions:
A constant-factor approximation of maximum-weight independent set in local conflict graphs

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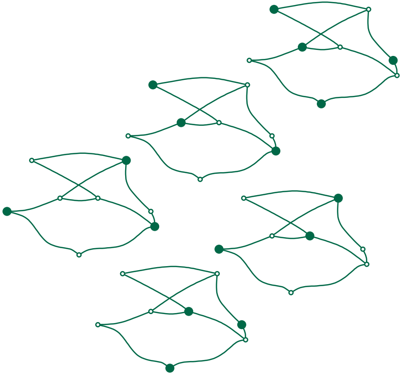
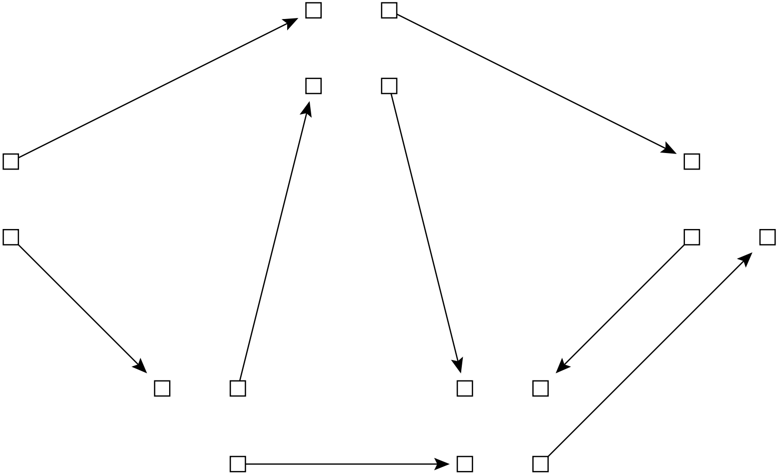
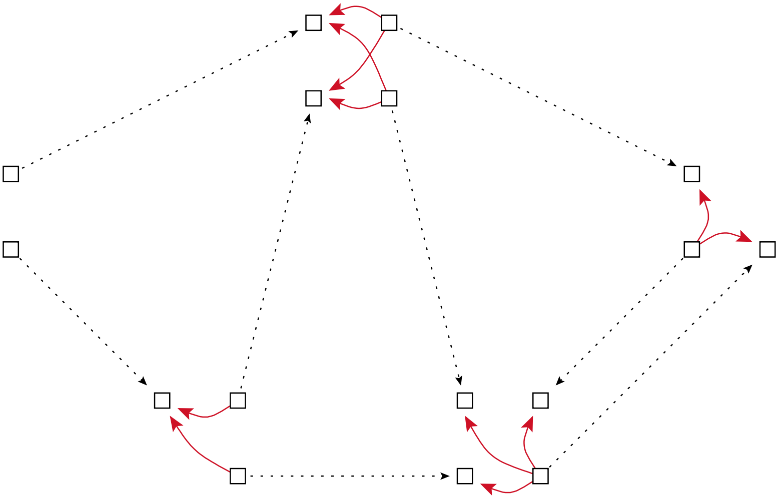


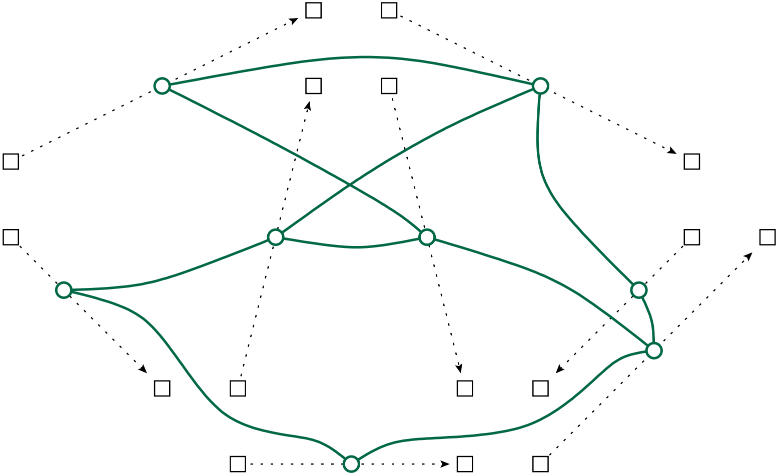
Wireless communication links



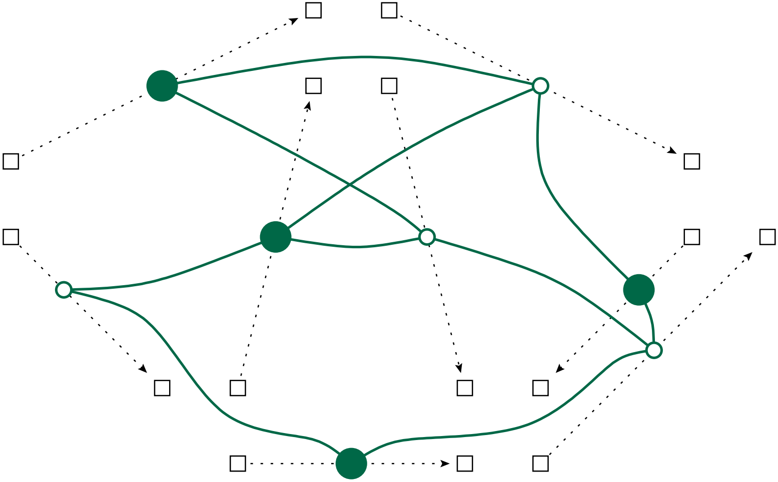
Interference — near-far effect



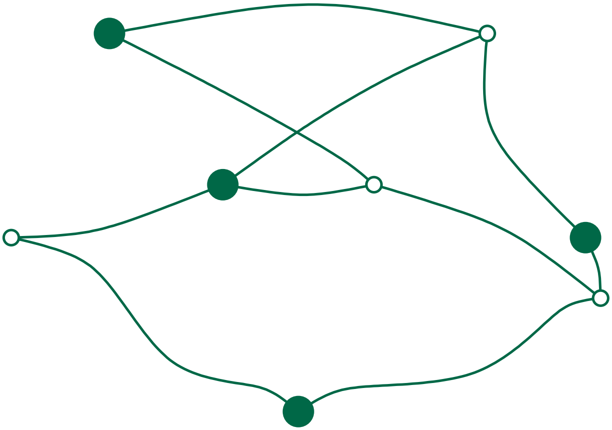
Conflict graph



Independent set in conflict graph



Independent set in conflict graph

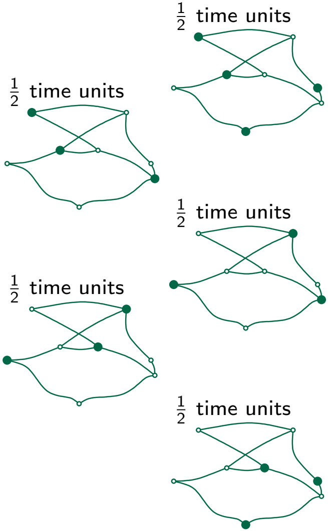


Problems associated with conflict graph

Problems:

A. Given per-link weights,
find maximum-weight
independent set
$=$ nonconflicting set of links

B. Given per-link data
transmission needs,
find a minimum-length
link schedule
(generalisation of
fractional graph colouring)



Known properties

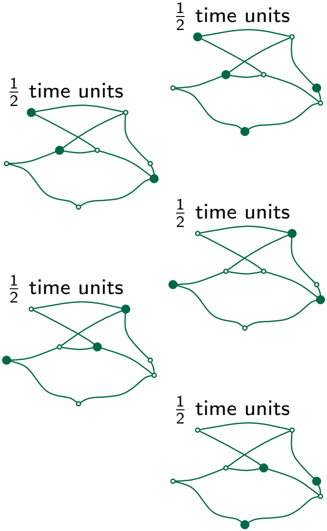
Problems:

A. max-weight independent set

B. min-length link schedule

Known properties:

- Approximation of **A** implies approximation of **B** (e.g., Young 2001, Jansen 2003)
- **A** and **B** hard to solve and approximate in general graphs (Lund–Yannakakis 1994, Håstad 1999, Khot 2001)



Assumptions on the problem structure

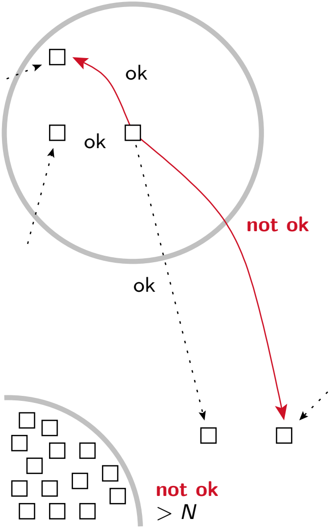
Define a new family of graphs:
***N*-local conflict graphs**

Assumptions:

- bounded **density of devices**:
  at most N devices in unit disk
- bounded **range of interference**:
  interfering transmitter must be close to interfered receiver

Non-assumptions:

- interference occurs if. . .

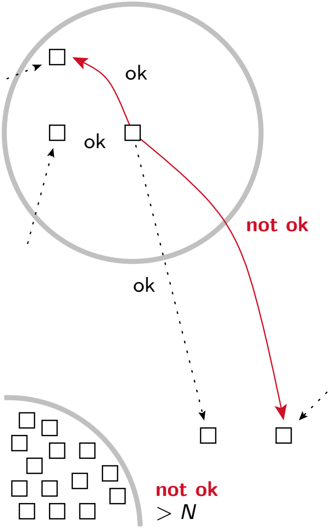


N-local conflict graphs: large family

Contains, for example:

- bipartite graphs
- cycles
- complete graph on N^2 vertices
- subgraphs of N-local conflict graphs
- $(N-1)$-local conflict graphs

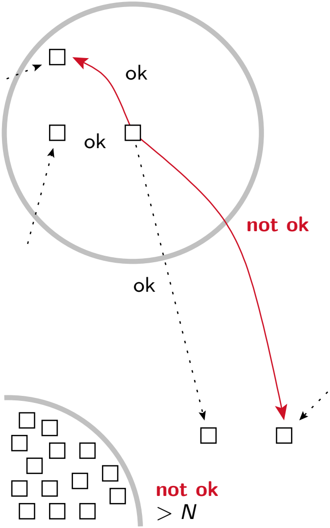
Generalisation of **local graphs** and **civilised graphs**



N-local conflict graphs: new family

Not contained in:

- bounded-degree graphs
  (proof: $K_{1,n}$)
- bounded-density graphs
  (proof: $K_{n,n}$)
- planar graphs (proof: $K_{3,3}$)
- disk graphs (proof: $K_{3,3}$)
- bipartite graphs (proof: K_3)
- graphs closed under taking minors
  (proof: split edges of a large clique)



Our results

Problems:

A. max-weight independent set

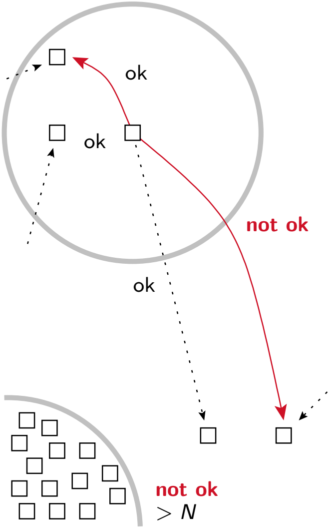
B. min-length link schedule

Assumptions:

- bounded density of devices
- bounded range of interference

Our results:

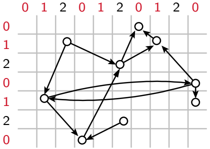
- **A** & **B**: $(5+\epsilon)$-approximation
- **A** & **B**: no PTAS

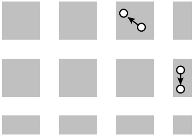


Algorithm sketch

Apply a **shifting strategy**
(Hochbaum–Maass 1985)

- Use a modular grid
- Try several locations
- Choose the best



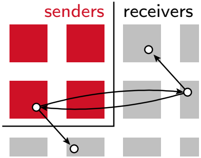


Short links (within a grid cell):

- Exhaustive search for each cell

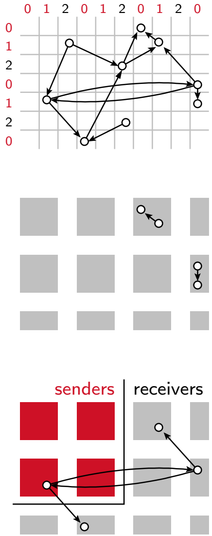
Long links (between grid cells):

- Find a large directed cut



Summary

- *N*-local conflict graphs:
  - bounded density of devices, bounded range of interference
- Max-weight independent set, min-length link schedule
- $(5 + \epsilon)$-approximation, no PTAS
- Open problems: distributed, coordinate-free algorithms?



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