Local approximation algorithms for scheduling problems in sensor networks

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Algosensors
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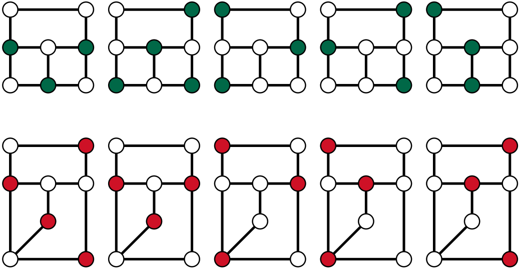
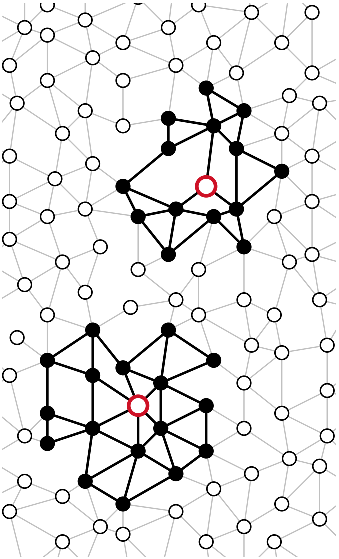


Local algorithms

- Operation of a node only depends on **input within its constant-size neighbourhood**
- Extreme scalability: **constant** amount of communication, memory and computation per node
- Weak model: 3-colouring a cycle impossible (Linial 1992)

Our result: local algorithms can be used to approximate nontrivial **scheduling problems**



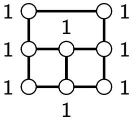
Sleep scheduling

Input: **redundancy graph**, battery capacities

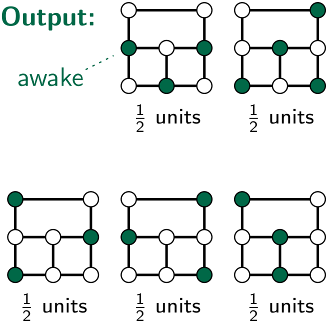
- Set of awake nodes = **dominating set** of redundancy graph
- Associate a time period with each dominating set
- **Maximise** total length
- Obey battery constraints

Motivation: maximising lifetime of a sensor network (pairwise redundancy)

Input:



Output:



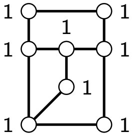
Activity scheduling

Input: **conflict graph**,
activity requirements

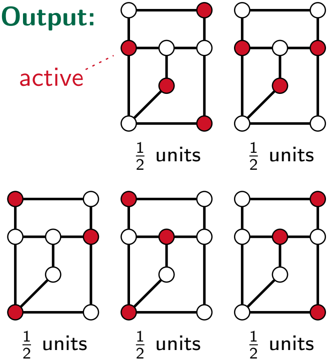
- Set of active nodes = **independent set** of conflict graph
- Associate a time period with each independent set
- **Minimise** total length
- Fulfil activity requirements

Motivation: minimising makespan of radio transmissions (pairwise interference)

Input:



Output:



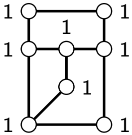
Scheduling problems

Sleep scheduling: generalisation of fractional domatic partition

Activity scheduling: generalisation of fractional graph colouring

- Linear programs
- The size of the LP can be exponential in the size of the graph
- **Hard to solve and approximate** in general graphs

Input:



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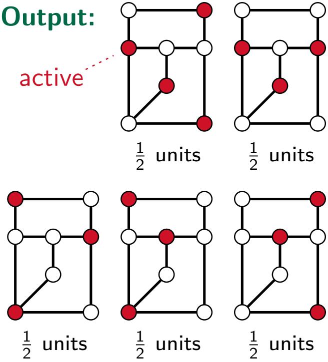
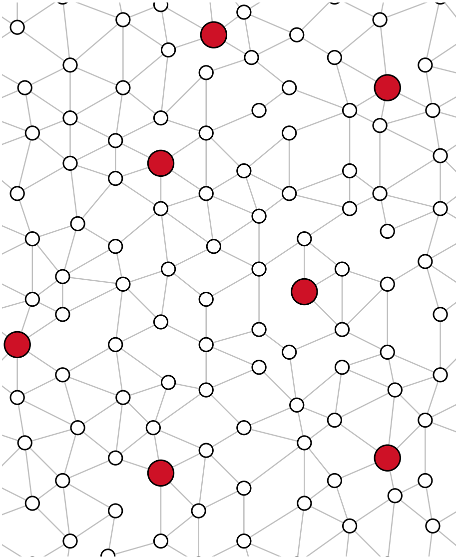


Solution

- Hard problems
- Weak model of computation

Solution: **markers**

1. Markers break symmetry
2. Characterisation of marker distribution constrains the family of graphs

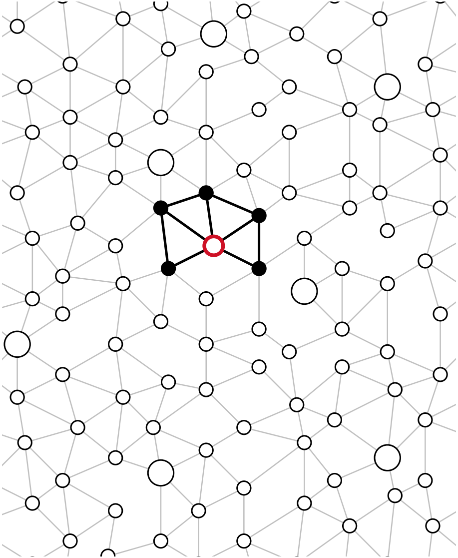


Marked graphs

$(\Delta, \ell_1, \ell_\mu, \mu)$-marked graph:

- Degree $\leq \Delta$
- ≥ 1 marker within ℓ_1 hops from any node
- $\leq \mu$ markers within ℓ_μ hops from any node

Intuition: bounded growth, symmetry-breakers nearby



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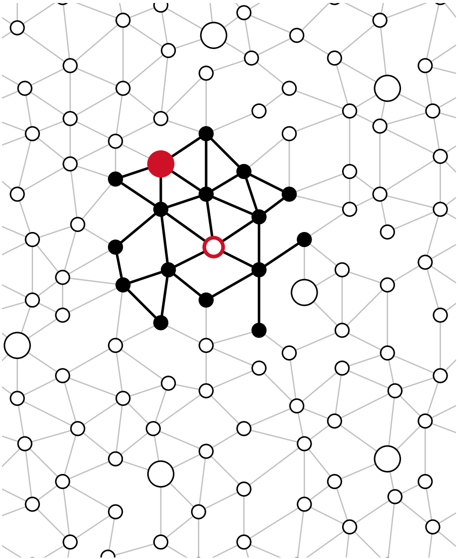

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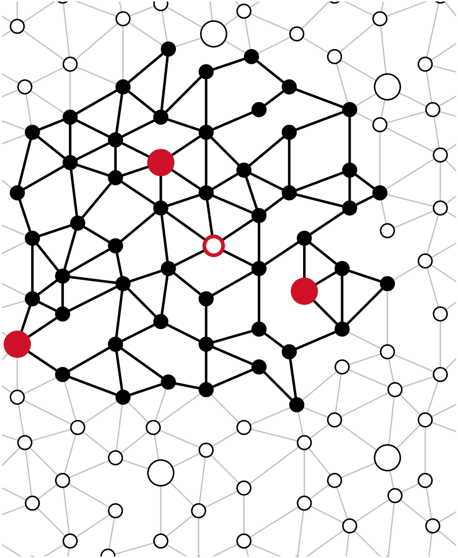
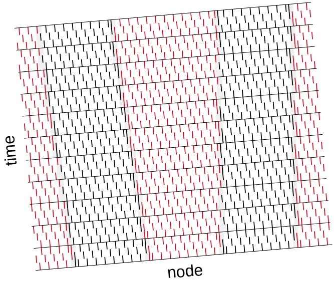
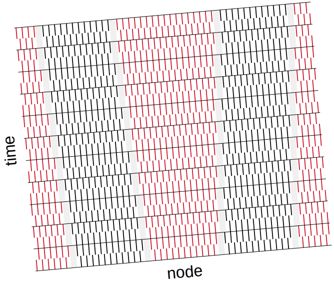


Main results

Local $(1+\epsilon)$-approximation
algorithm for sleep scheduling
in $(\Delta, \ell_1, \ell_\mu, \mu)$-marked graphs
for any $\epsilon > 4\Delta/\lfloor(\ell_\mu - \ell_1)/\mu\rfloor$

Local $1/(1-\epsilon)$-approximation
algorithm for activity scheduling
in $(\Delta, \ell_1, \ell_\mu, \mu)$-marked graphs
for any $\epsilon > 4/\lfloor(\ell_\mu - \ell_1)/\mu\rfloor$





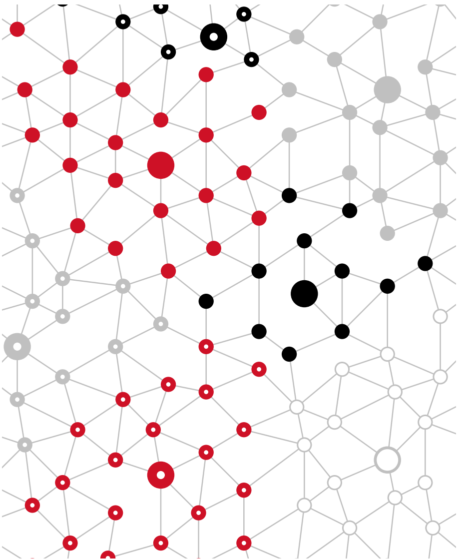
- Markers are enough:
  no coordinates needed
- Markers are necessary
- Cannot improve ϵ by factor 9

Algorithm sketch

Several partitions of communication graph

- Configuration 0: Voronoi cells for markers
- Configuration 1: shift cell borders
- Configuration *i*: shift *i* units

Solve the scheduling problem locally for each cell, interleave the solutions



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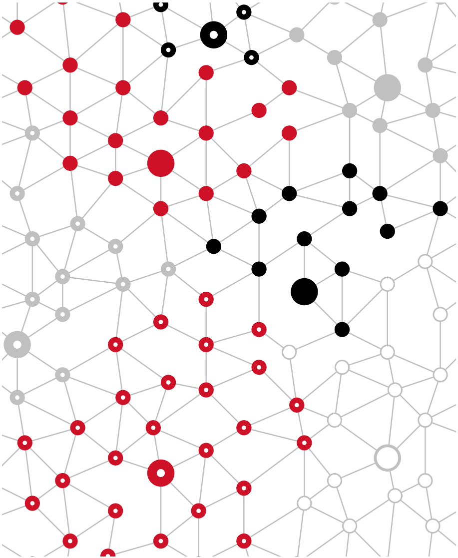

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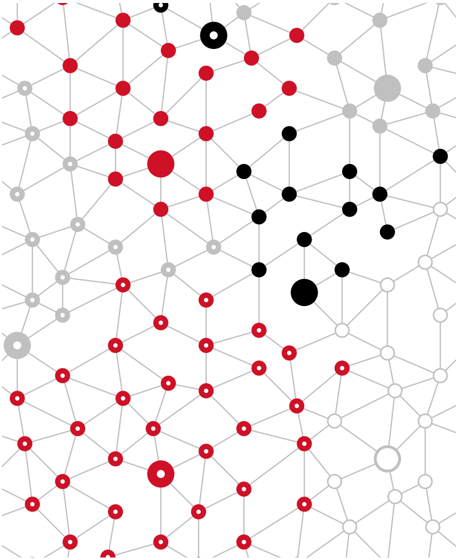

Summary

- **Local** approximation scheme: constant effort per node
- Fractional scheduling problems, both **packing** and **covering**
- Can be extended beyond pairwise redundancy/conflicts as long as there is "locality"
- **Markers** are enough, coordinates not needed
- Constants are not practical, more work needed

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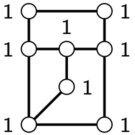
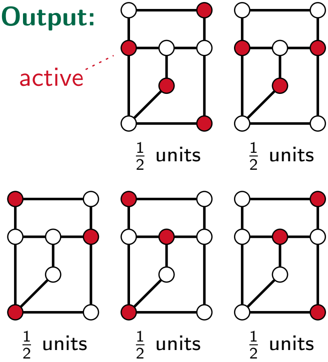
Input:



Output:



Appendix: Examples of marked graphs

- 2-dimensional grid of nodes
  - Use a sparser grid to place the markers
  - "Local approximation scheme":
    any approximation ratio by using a sparse enough grid
    (cost: higher computational complexity)
- "Coarse grids", graphs quasi-isometric to 2-dimensional grids
  - Arbitrary small-scale structure
- Cutting parts of coarse grids, with $L + 1$ hop margins
  - Arbitrary small-scale and large-scale structure
  - **Medium-scale structure has similarities with low-dimensional grids**

Appendix: Sleep scheduling LP

Input:

– communication graph $\mathcal{G}$
– redundancy graph $\mathcal{R}$, subgraph of $\mathcal{G}$
– battery capacity $b(v) \geq 0$ for each node $v \in V_\mathcal{R}$

Task:

maximise $\sum_D x(D)$

subject to $\sum_D D(v)x(D) \leq b(v)$ and $x(D) \geq 0$

v ranges over $V_\mathcal{R}$
D ranges over dominating sets of $\mathcal{R}$

$D(v) = 1$ if $v \in D$ and $D(v) = 0$ if $v \notin D$
$x(D) =$ the length of the time period associated with D

Appendix: Activity scheduling LP

Input:

– communication graph $\mathcal{G}$
– conflict graph $\mathcal{C}$, subgraph of $\mathcal{G}$
– activity requirement $a(v) \geq 0$ for each node $v \in V_{\mathcal{C}}$

Task:

minimise $\sum_I x(I)$

subject to $\sum_I I(v)x(I) \geq a(v)$ and $x(I) \geq 0$

v ranges over $V_{\mathcal{C}}$
I ranges over independent sets of $\mathcal{C}$

$I(v) = 1$ if $v \in I$ and $I(v) = 0$ if $v \notin I$
$x(I) =$ the length of the time period associated with I