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Locality

in online, dynamic, sequential, and distributed graph algorithms

Joint work with:

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- Navid Eslami
- Henrik Lievonen
- Darya Melnyk
- Joona Särkijärvi

arxiv.org/abs/2109.06593

Sorry, **no PRAM**

Four models of computing

SLOCAL
distributed,
sequential

LOCAL
distributed,
parallel

online LOCAL
centralized

dynamic LOCAL
centralized

LOCAL
distributed,
parallel

SLOCAL
distributed,
sequential

online LOCAL
centralized

dynamic LOCAL
centralized

LOCAL
distributed, parallel

Each node **in parallel**:

- looks at its radius-T neighborhood
- picks its output based on this information

(nodes have unique identifiers)

LOCAL
distributed,
parallel

SLOCAL
distributed,
sequential

online LOCAL
centralized

dynamic LOCAL
centralized

SLOCAL
distributed,
sequential

LOCAL
distributed,
parallel

online LOCAL
centralized

dynamic LOCAL
centralized

SLOCAL

distributed,
sequential

Each node in a **sequential, adversarial order**:

- looks at its radius-T neighborhood
- picks its output & state based on this information

SLOCAL
distributed,
sequential

LOCAL
distributed,
parallel

online LOCAL
centralized

dynamic LOCAL
centralized

SLOCAL
distributed,
sequential

LOCAL
distributed,
parallel

online LOCAL
centralized

dynamic LOCAL
centralized

Graph **constructed** by an adversary that adds nodes and edges one by one

We can **see everything**

We can **change** our output only within distance T from a point of change

dynamic
LOCAL
centralized

SLOCAL
distributed,
sequential

LOCAL
distributed,
parallel

online LOCAL
centralized

dynamic LOCAL
centralized

SLOCAL
distributed,
sequential

LOCAL
distributed,
parallel

online LOCAL
centralized

dynamic LOCAL
centralized

Some unknown input graph is **revealed** piece by piece:

- adversary points at a node v
- we can see the radius-T neighborhood of v
- we have to choose the label for v

We can **remember** everything

online LOCAL
centralized

SLOCAL
distributed,
sequential

LOCAL
distributed,
parallel

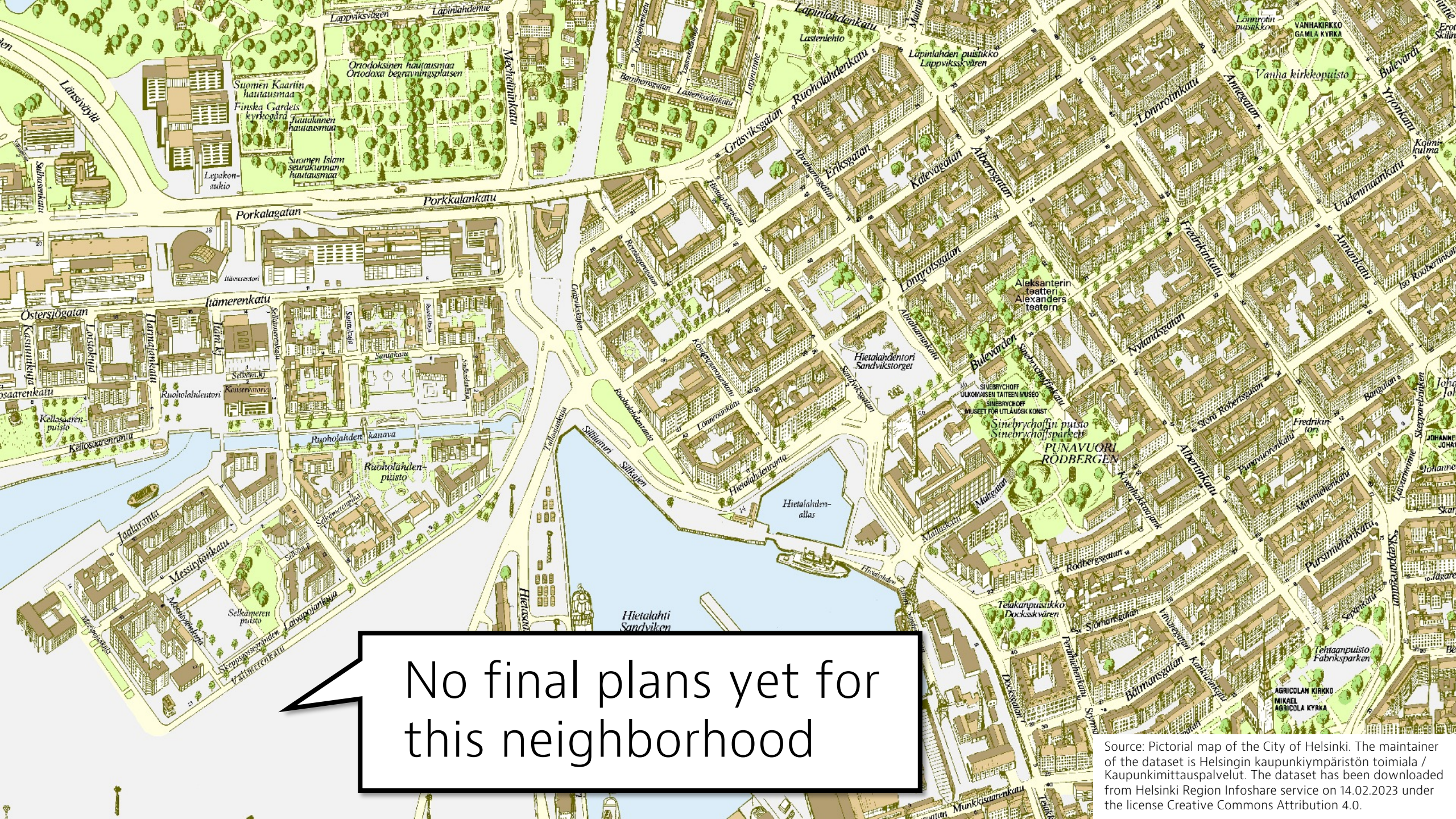
online LOCAL
centralized

dynamic LOCAL
centralized

Aim: capturing decision-making in the physical-world

Example: **urban planning**

How to maintain public services in a city that is rapidly growing?



Source: Pictorial map of the City of Helsinki. The maintainer of the dataset is Helsingin kaupunkiympäristön toimiala / Kaupunkimittauspalvelut. The dataset has been downloaded from Helsinki Region Infoshare service on 14.02.2023 under the license Creative Commons Attribution 4.0.

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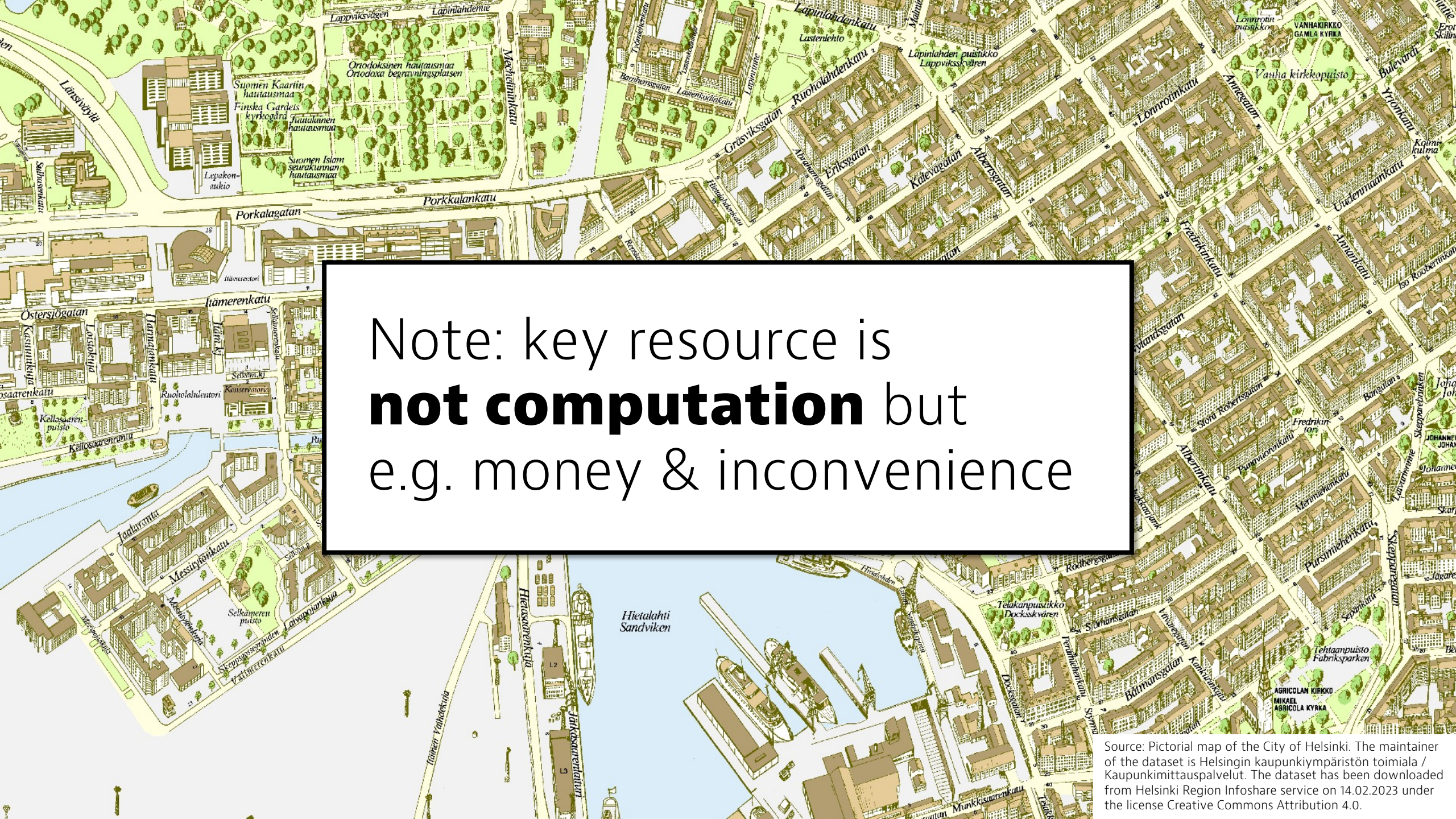

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Genuinely different models

SLOCAL
distributed,
sequential

LOCAL
distributed,
parallel

online LOCAL
centralized

dynamic LOCAL
centralized

coloring

SLOCAL
distributed,
sequential

LOCAL
distributed,
parallel

online LOCAL
centralized

dynamic LOCAL
centralized

coloring

SLOCAL
distributed,
sequential

LOCAL
distributed,
parallel

online LOCAL
centralized

cycle
detection

dynamic LOCAL
centralized

SLOCAL
distributed,
sequential

coloring

LOCAL
distributed,
parallel

online LOCAL
centralized

dynamic LOCAL
centralized

cycle detection

leader election

Closely related models

SLOCAL
distributed,
sequential

LOCAL
distributed,
parallel

online LOCAL
centralized

dynamic LOCAL
centralized

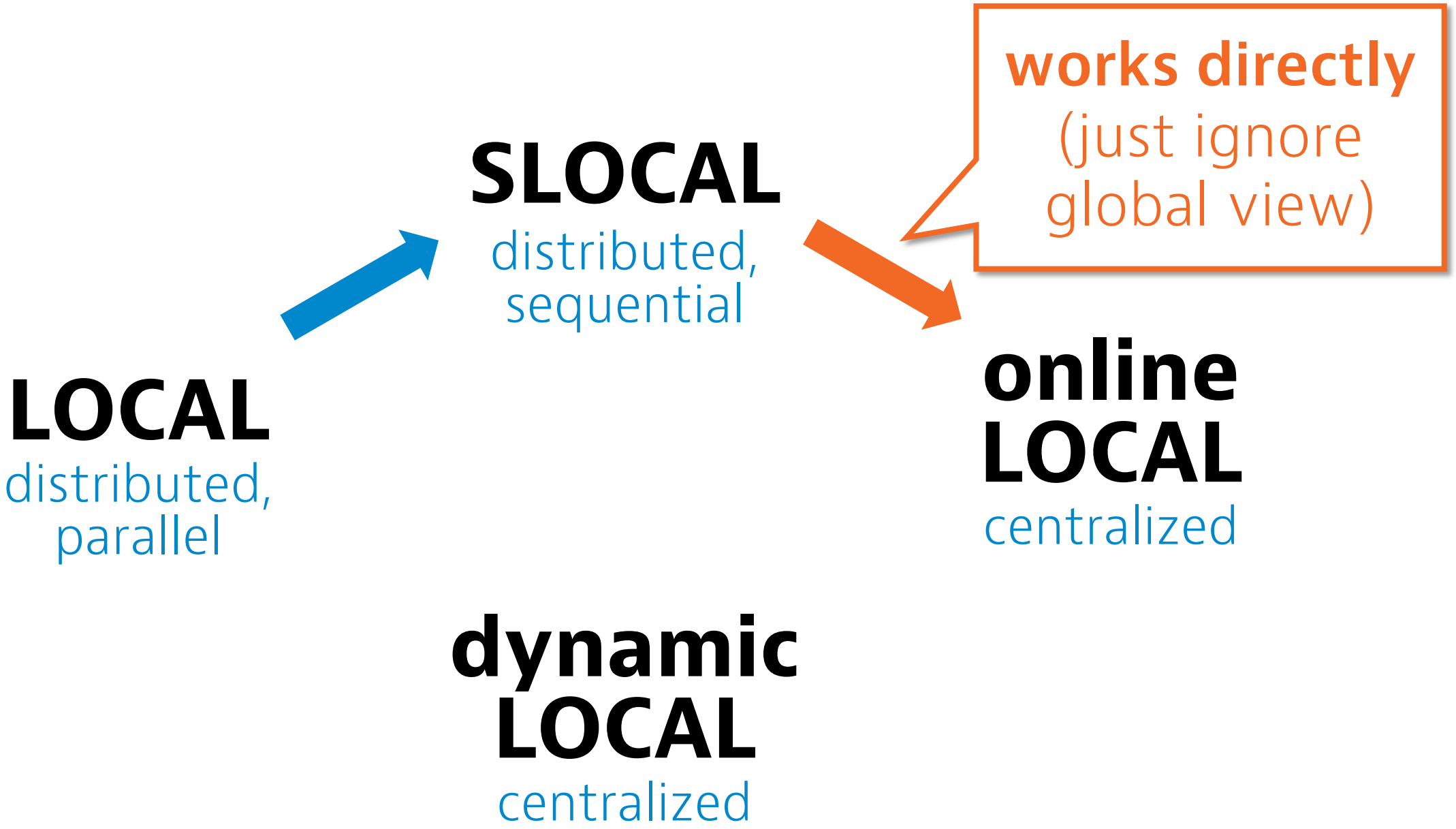
works directly
(just ignore
local states)

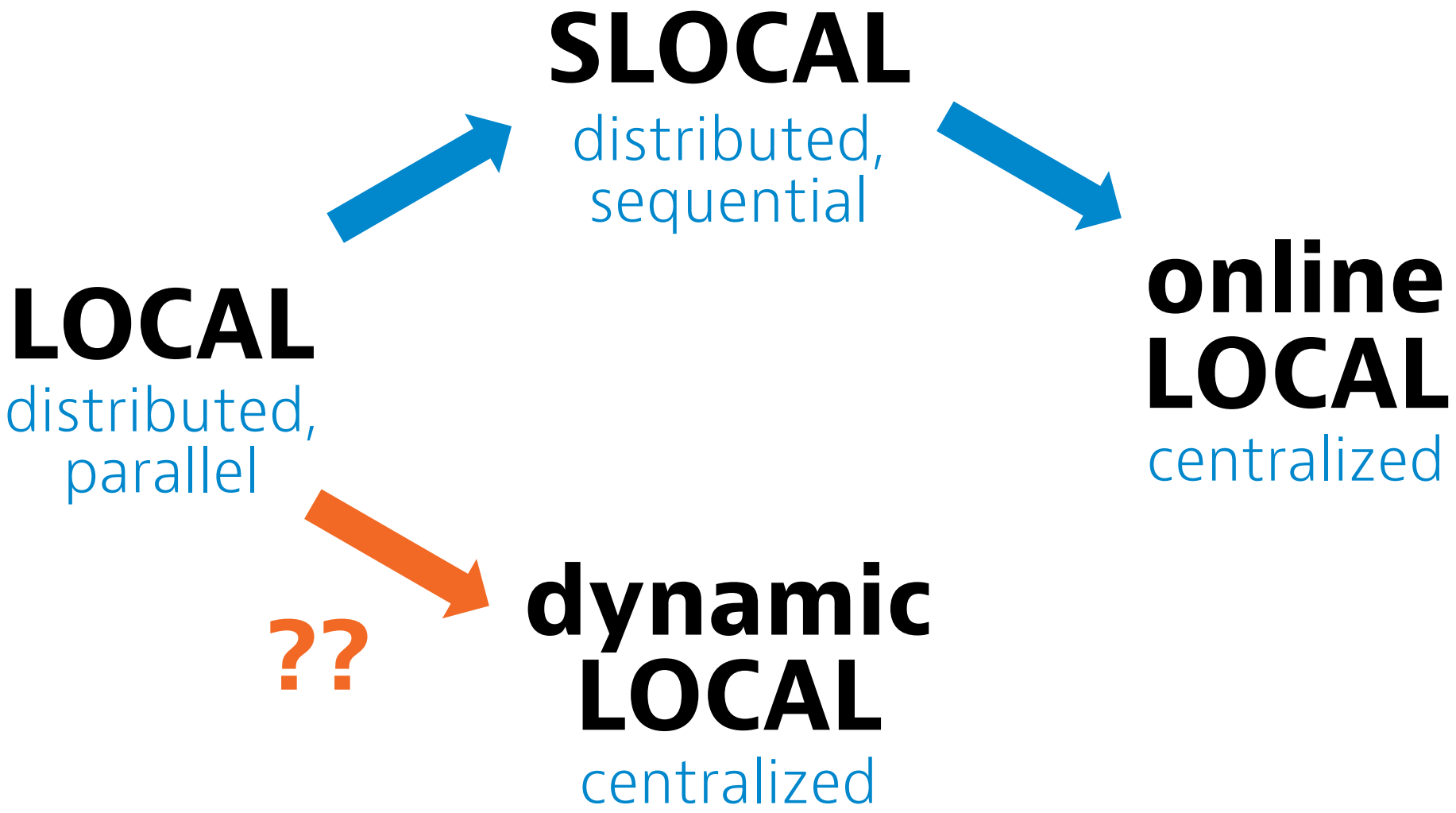
SLOCAL
distributed,
sequential

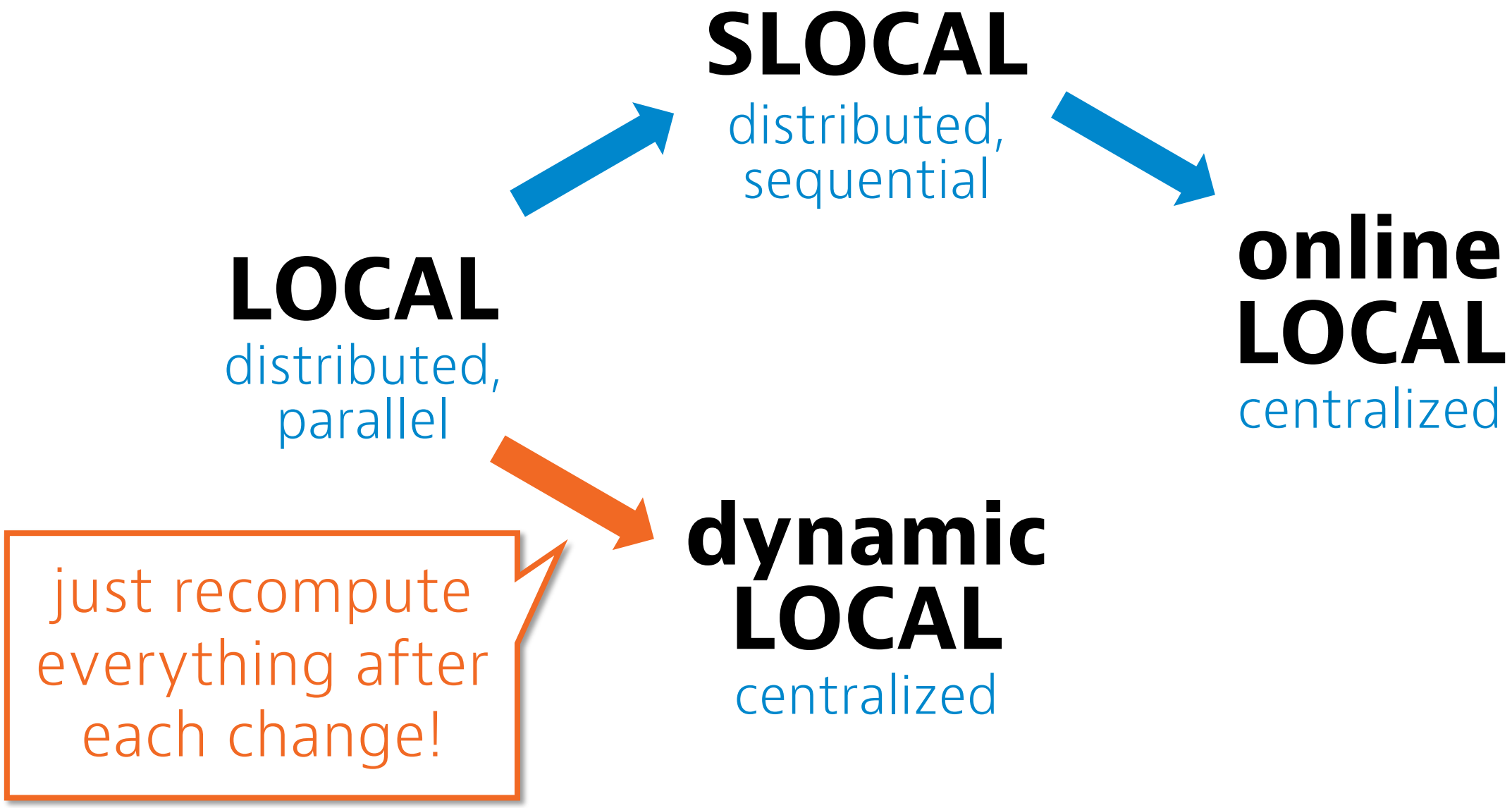
LOCAL
distributed,
parallel

online LOCAL
centralized

dynamic LOCAL
centralized

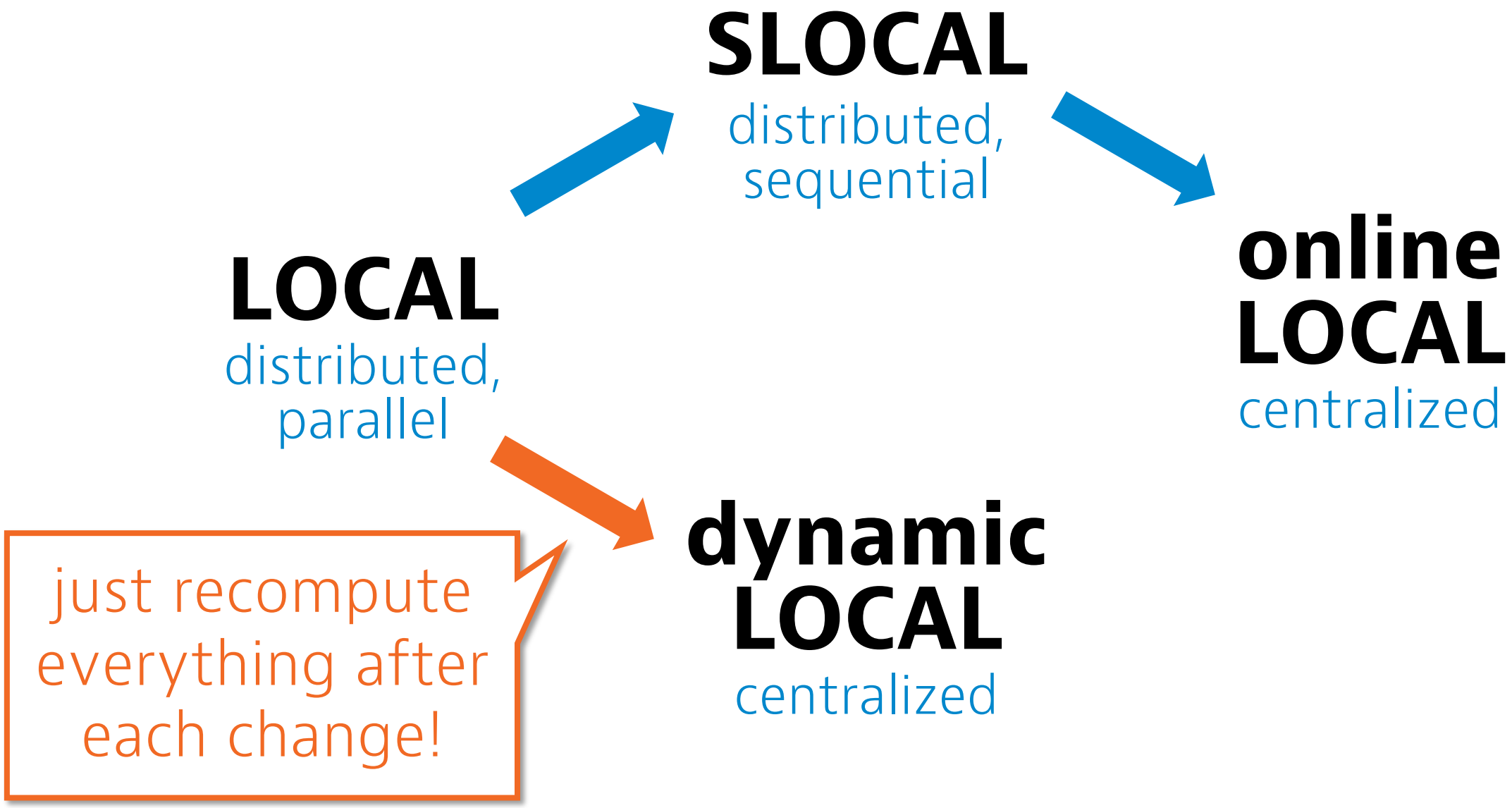

SLOCAL
distributed,
sequential
works directly
(just ignore
global view)
LOCAL
distributed,
parallel
online
LOCAL
centralized
dynamic
LOCAL
centralized

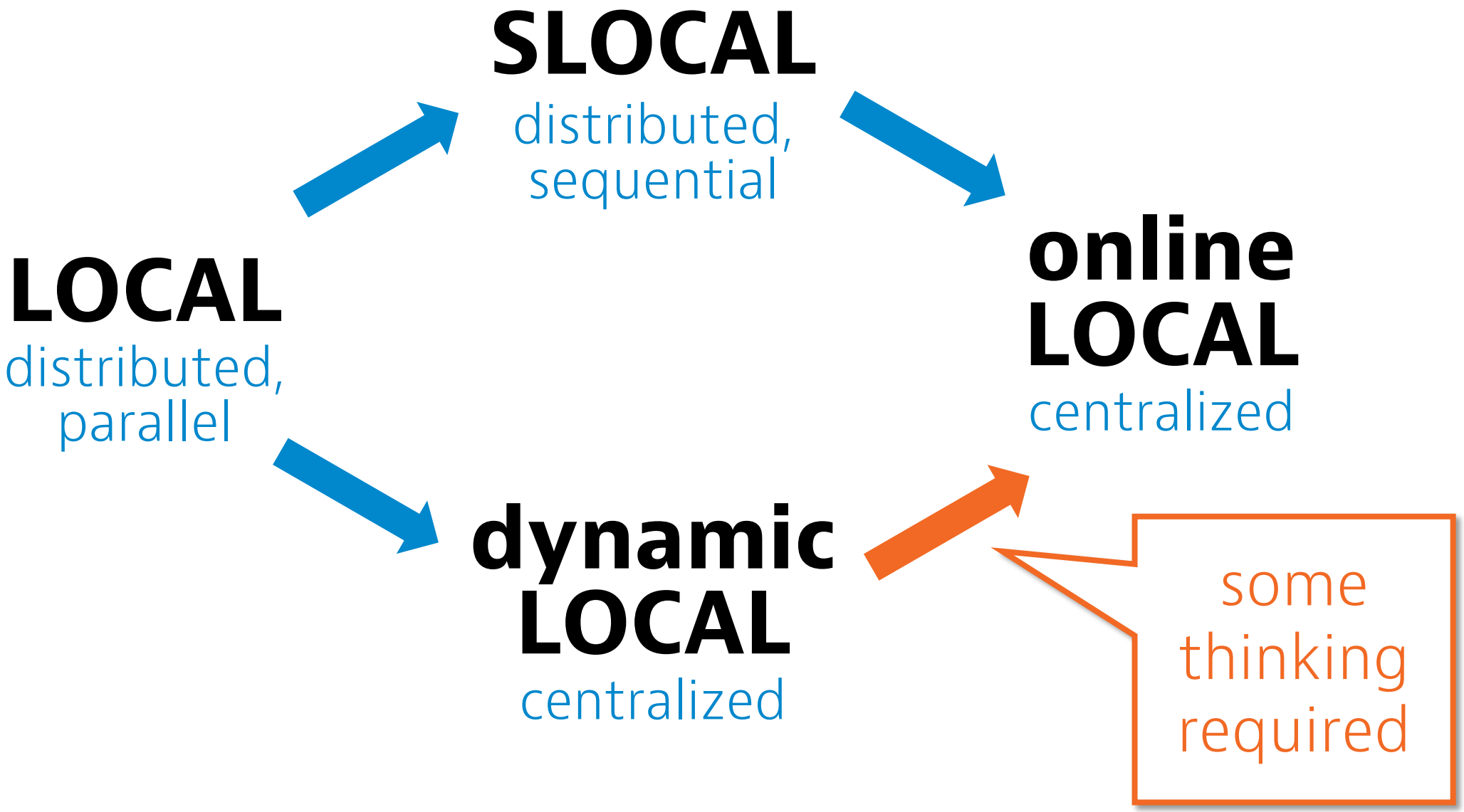

SLOCAL
distributed,
sequential
online
LOCAL
centralized
LOCAL
distributed,
parallel
??
dynamic
LOCAL
centralized

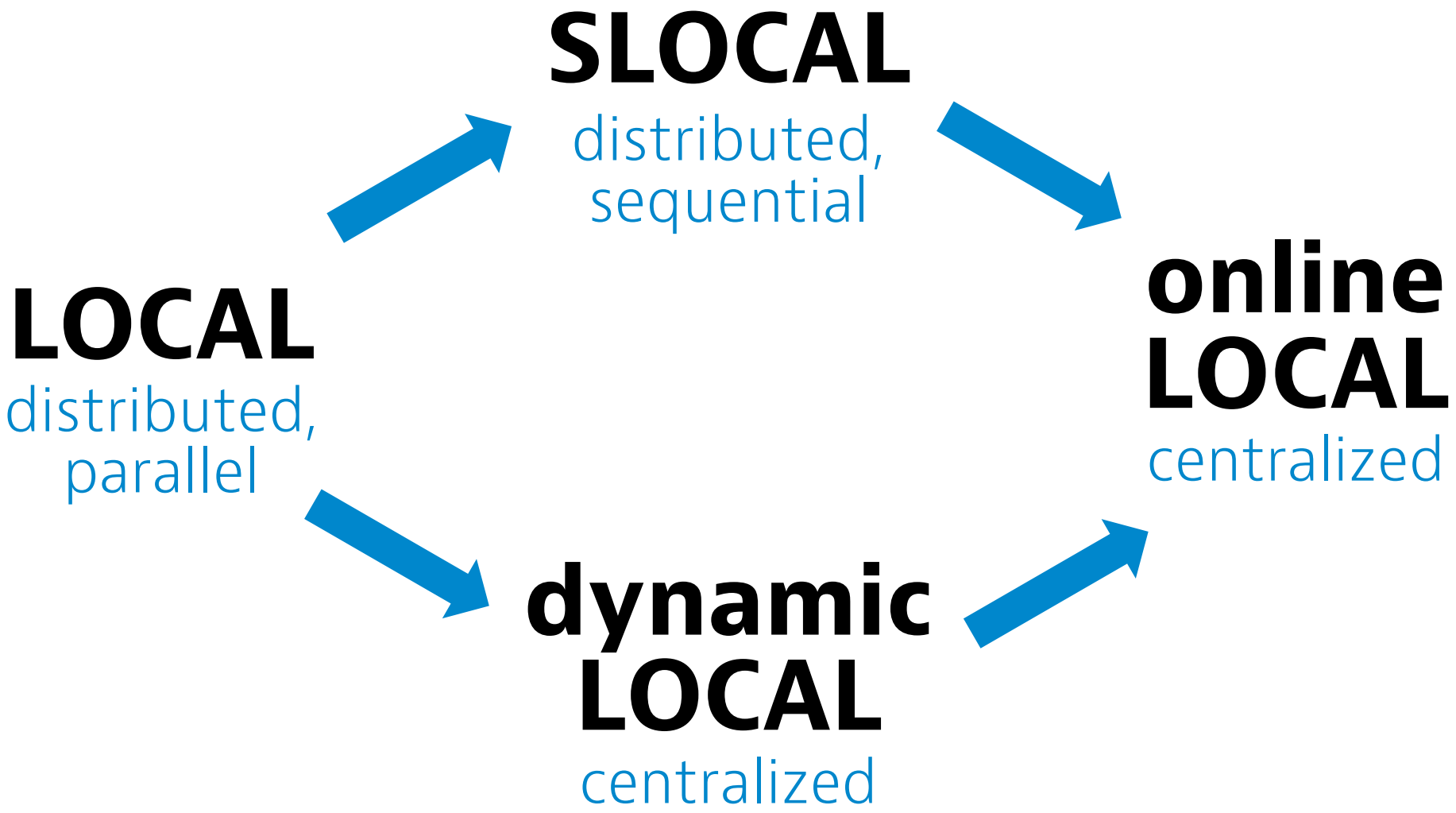

SLOCAL
distributed,
sequential
online
LOCAL
centralized
LOCAL
distributed,
parallel
dynamic
LOCAL
centralized
just recompute
everything after
each change!

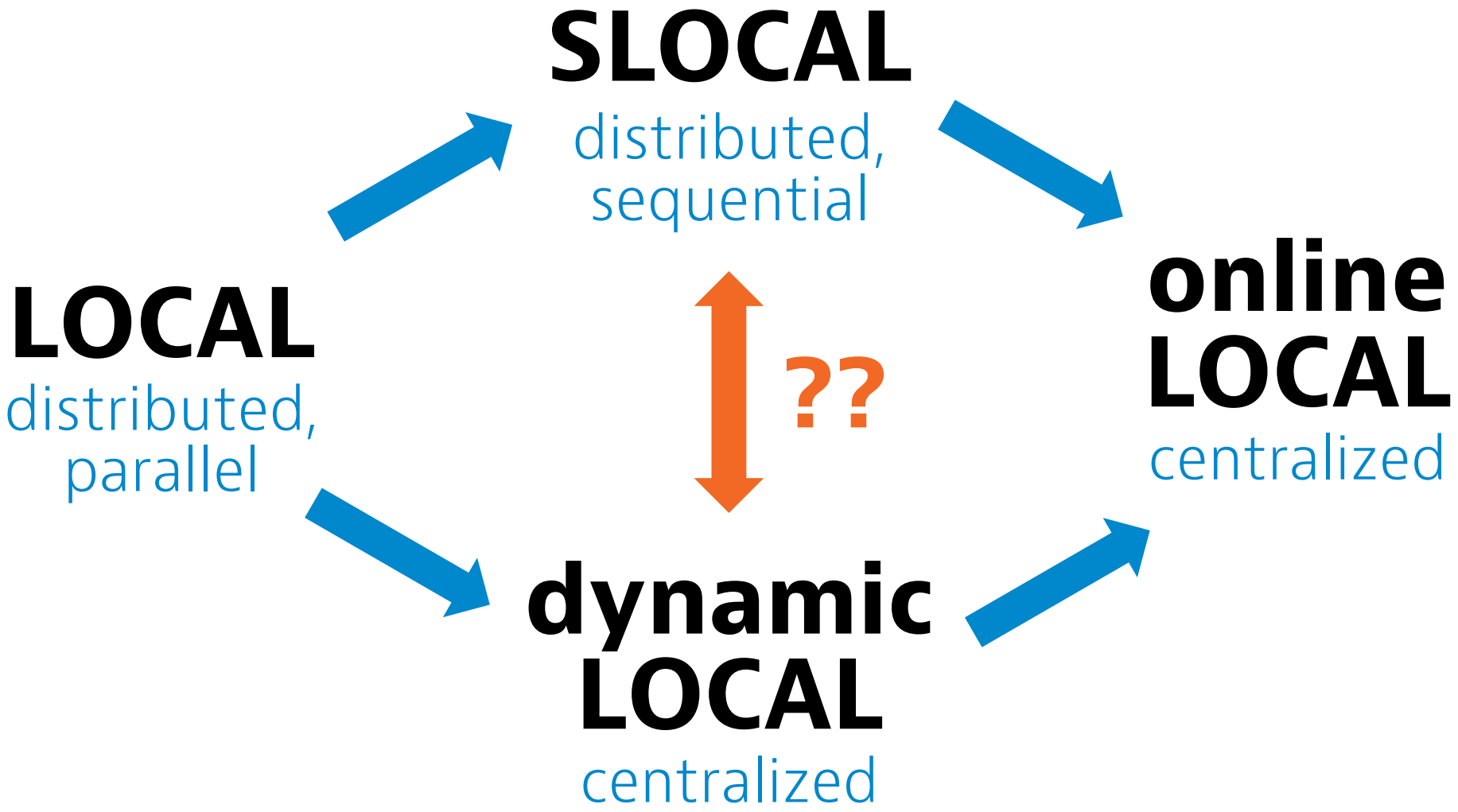
Equivalent perspectives

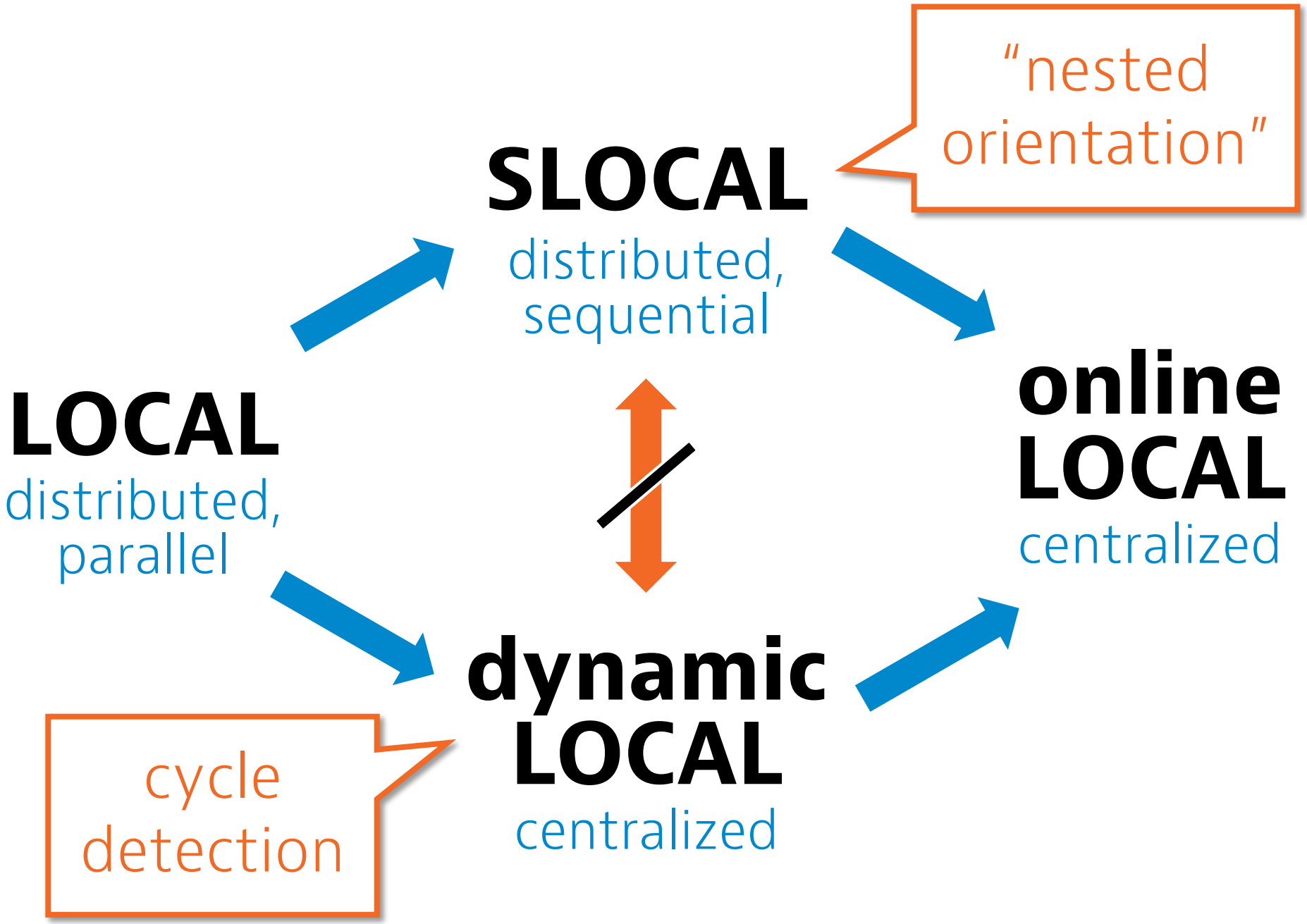
- **Output of node *v* only depends on inputs of nodes *u* with dist(*u*, *v*) ≤ *T***
  - this is what we have by definition in LOCAL algorithms
- **Changes at node *u* can only influence outputs of nodes *v* with dist(*u*, *v*) ≤ *T***
  - this is enough to have a dynamic LOCAL algorithm

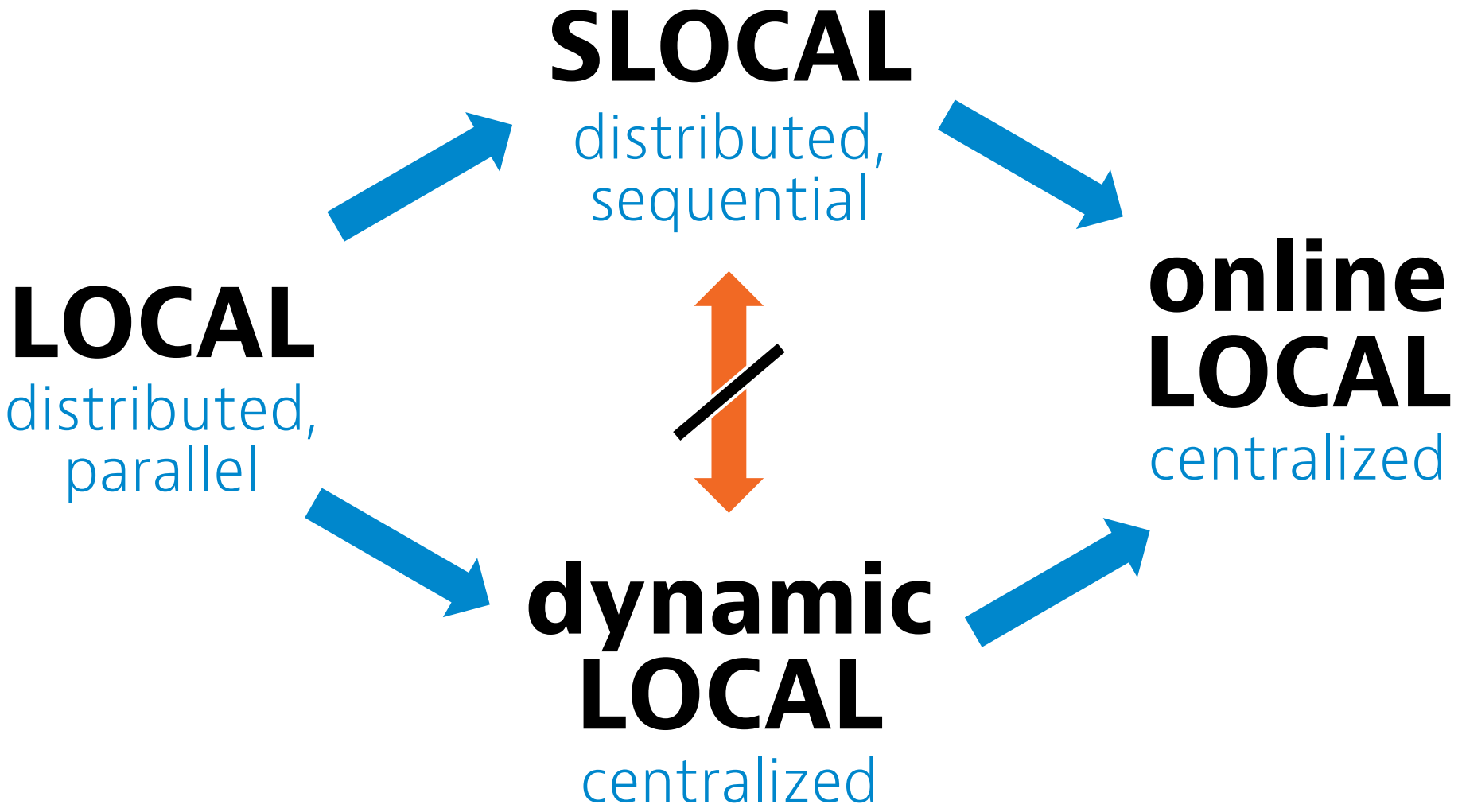

SLOCAL
distributed,
sequential
online
LOCAL
centralized
LOCAL
distributed,
parallel
dynamic
LOCAL
centralized
just recompute
everything after
each change!


SLOCAL
distributed,
sequential
LOCAL
distributed,
parallel
online
LOCAL
centralized
dynamic
LOCAL
centralized
some
thinking
required


SLOCAL
distributed,
sequential
LOCAL
distributed,
parallel
online
LOCAL
centralized
dynamic
LOCAL
centralized


SLOCAL
distributed,
sequential
LOCAL
distributed,
parallel
??
online
LOCAL
centralized
dynamic
LOCAL
centralized


SLOCAL
distributed,
sequential
"nested
orientation"
LOCAL
distributed,
parallel
online
LOCAL
centralized
dynamic
LOCAL
centralized
cycle
detection


SLOCAL
distributed,
sequential
LOCAL
distributed,
parallel
online
LOCAL
centralized
dynamic
LOCAL
centralized

Collapse in rooted trees

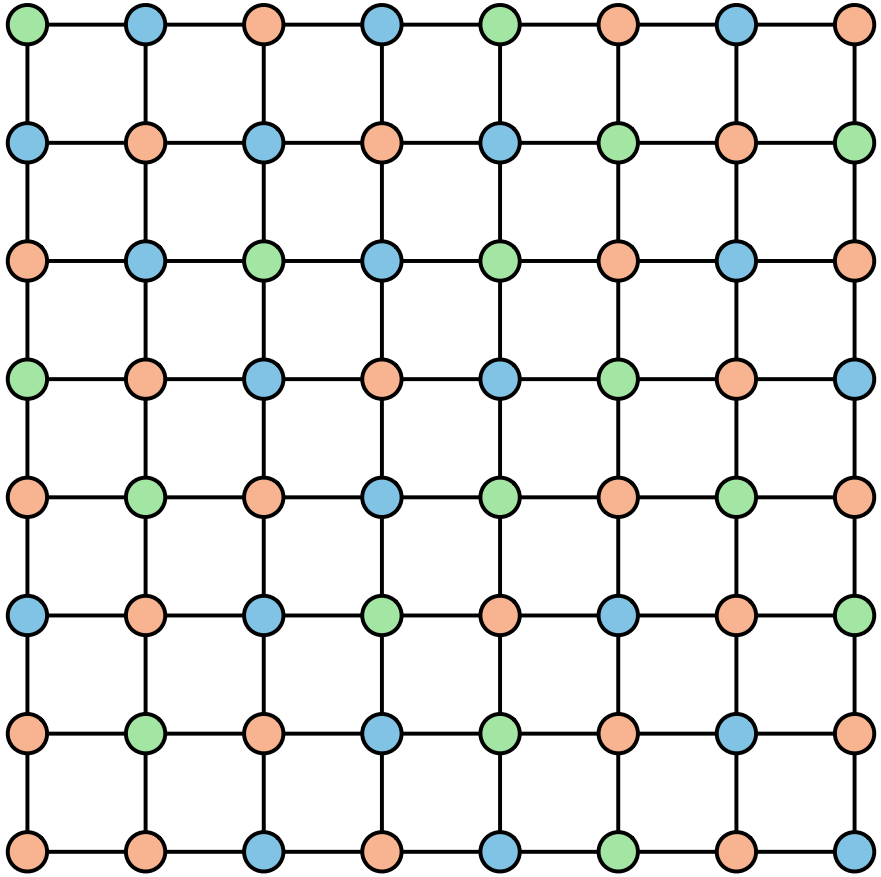
LCLs in rooted trees

- **Rooted regular trees**
- **Locally checkable labelings** (LCLs)
  - solution valid if it "looks good everywhere"
  - example: 3-coloring
- ***In this setting all models equally strong!***

LCLs in rooted trees

Case study: grids

Coloring grids



Coloring grids

- **5-coloring:** local in all models (easy to see)

Coloring grids

- **5-coloring:** local in all models (easy to see)
- **4-coloring:** local in all models (hard to see)

Coloring grids

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- **4-coloring:** local in all models (hard to see)
- **3-coloring:**
  - LOCAL, SLOCAL: global

Coloring grids

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- **4-coloring:** local in all models (hard to see)
- **3-coloring:**
  - LOCAL, SLOCAL: global
  - **online-LOCAL: $O(\log n)$**

Coloring grids

- **5-coloring:** local in all models (easy to see)
- **4-coloring:** local in all models (hard to see)
- **3-coloring:**
  - LOCAL, SLOCAL: global
  - online-LOCAL: $O(\log n)$ — **is this tight?**

Coloring grids

- **5-coloring:** local in all models (easy to see)
- **4-coloring:** local in all models (hard to see)
- **3-coloring:**
  - LOCAL, SLOCAL: global
  - online-LOCAL: $O(\log n)$ — **is this tight?**
  - dynamic-LOCAL: **open**

Follow-up questions

- **Distance** (how far do you see/touch) vs. **volume** (how many nodes do you see/touch)
- **Randomized** versions (oblivious, adaptive …)
- **Geometric** versions
- Which **other models** are sandwiched between LOCAL and online-LOCAL?

Distinct in general, equivalent for LCLs in trees