Causal limits of distributed quantum computation
Joint work with

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arxiv.org/abs/2307.09444
Quick recap: Classical distributed algorithms
Distributed algorithms

- Graph = communication network
  - node = computer
  - edge = communication link
Distributed algorithms

• **Initially:** each node only aware of itself
• **Goal:** each announces its local output
  - e.g. graph coloring: “my color is 5”
Distributed algorithms

- **Running time** = **number of rounds** until all nodes have stopped
Knowledge before round 1
Knowledge after round 1
Knowledge after round 2
Knowledge after round 3
Knowledge after round 4
number of communication rounds

= 

how far do you need to see
What about quantum?
Distributed algorithms

**Classical**
- node = classical computer
- edge = classical communication channel

**Quantum**
- node = quantum computer
- edge = quantum communication channel
0 rounds
1 round
2 rounds
Does it help?
Quantum advantage

• Is there any graph problem for which we can show distributed quantum advantage?
  • Yes!
    • $O(1)$-round quantum algorithm
    • no $o(n)$-round classical algorithm

Le Gall, Nishimura, Rosmani 2019
Quantum advantage

• Is there any graph problem that someone actually cares about for which we can show distributed quantum advantage?

• Nobody knows!
Quantum advantage

• It is hard to characterize the exact limits of distributed quantum computation
  • and you need to understand quantum things...

• But for many graph problems we can use causality to show that there is no quantum advantage!
  • without knowing anything about quantum things!
Causality
Not faster than light

• No physical thing can allow faster-than-light communication

• Not even your quantum distributed algorithm
2 rounds
2 rounds
2 rounds

light cone

measure

measure

measure
Non-signaling model

• Key idea: **define** a model so that it can do **anything** except violating causality
Non-signaling model

Definition (*non-signaling distribution*):
- fix any *set of nodes* $X$ ...
Non-signaling model

**Definition** *(non-signaling distribution)*:
- fix any set of nodes $X$
- changes in the input more than $T$ hops away from $X$ do not influence the output distribution of $X$

Gavoille, Kosowski, Markiewicz 2009
Arfaoui, Fraigniaud 2014
Three models

Classical (randomized) distributed algorithms
\[\downarrow\]
Quantum distributed algorithms
\[\downarrow\]
Non-signaling “algorithms”
Example: graph coloring

• Any quantum advantage?
• No! (up to polylog factors)
Example: graph coloring

• Any quantum advantage?
• No! (up to polylog factors)
• Example: 3-coloring bipartite graphs
  • $\tilde{O}(n^{1/2})$ — classical distributed algorithm
  • $o(n^{1/2})$ — impossible for non-signaling
Example: graph coloring

• Any quantum advantage?
  • **No!** (up to polylog factors)

• Example: 4-coloring bipartite graphs
  • $\tilde{O}(n^{1/3})$ — classical distributed algorithm
  • $o(n^{1/3})$ — impossible for non-signaling
Example: graph coloring

• Any quantum advantage?
• **No!** (up to polylog factors)

• Example: **25-coloring 7-colorable** graphs
  • $\tilde{O}(n^{1/4})$ — classical distributed algorithm
  • $o(n^{1/4})$ — impossible for non-signaling
Summary

- Causality
  - no faster-than-light communication
  - “non-signaling model”

- Tight lower bounds for distributed quantum algorithms
  - without touching weird quantum things
An open question

- **3-coloring cycles:**
  - classical: $O(\log^* n)$
  - quantum: ???
  - non-signaling: $O(1)$

Linial 1992
Holroyd, Liggett 2016
Holroyd, Hutchcroft, Levy 2018