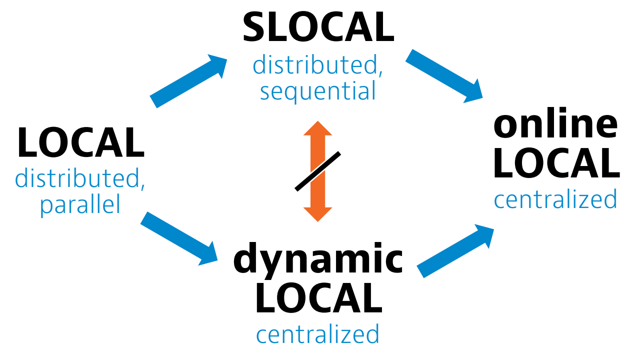
Jukka Suomela
Aalto University

Online Locality Meets Distributed Quantum Computing

Foundations Friday in December 2022

Locality

in online, dynamic, sequential, and distributed graph algorithms



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Foundations Friday in December 2023

Causal limits

of distributed quantum computation

Three models

Classical probability theory

Classical (randomized) distributed algorithms

↓

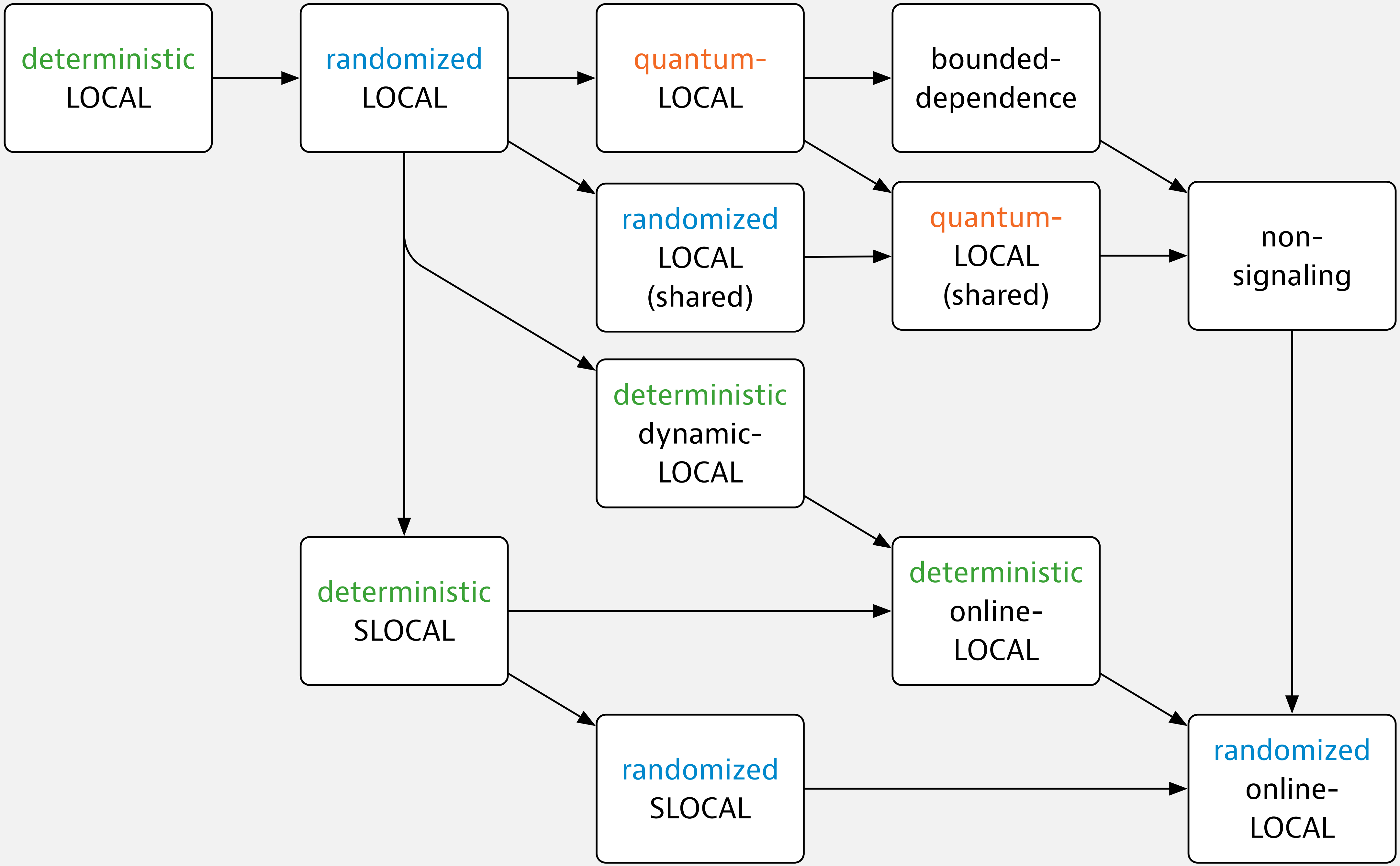
Quantum distributed algorithms

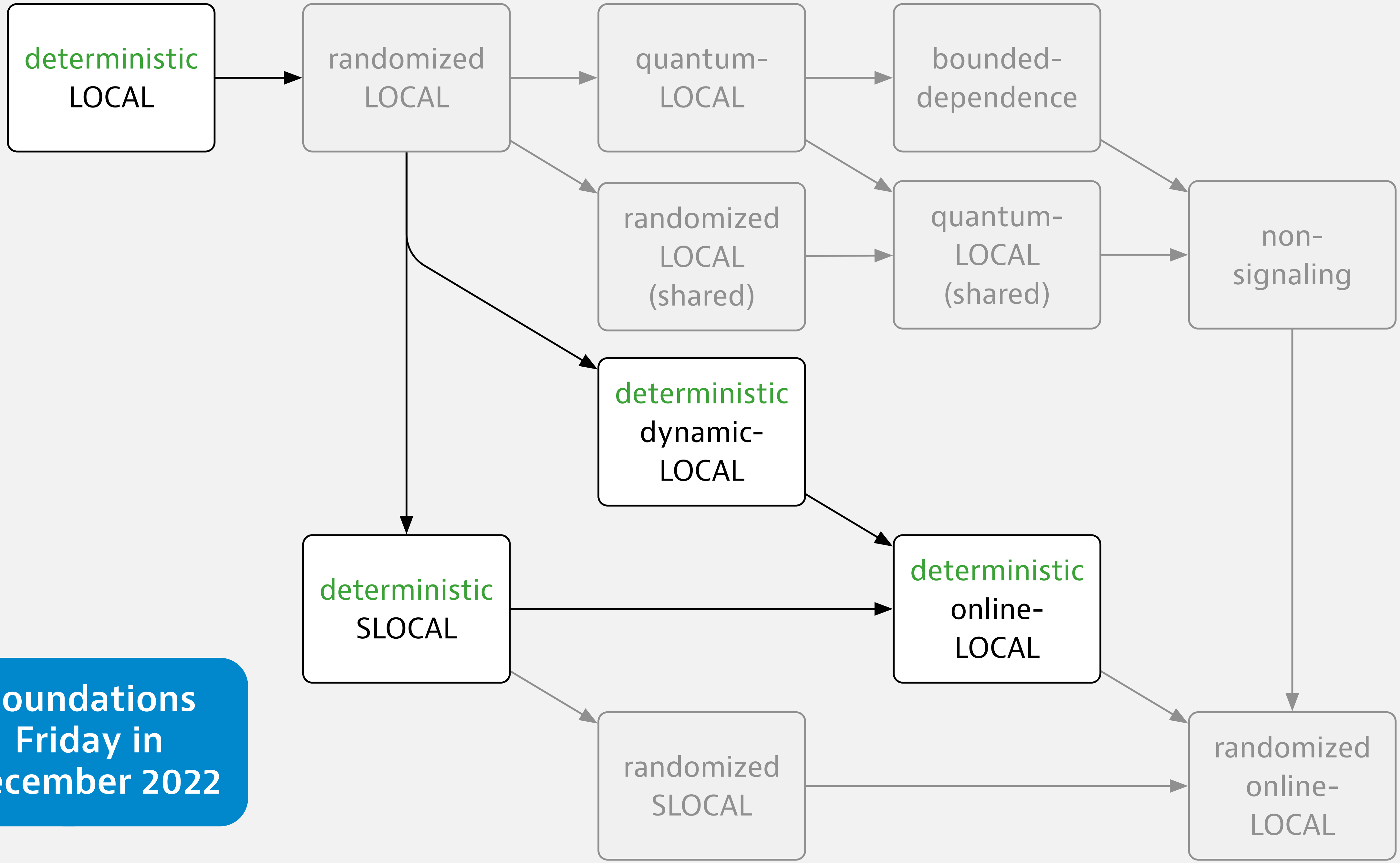
Weird quantum things

↓

Non-signaling "algorithms"

Classical probability theory


deterministic LOCAL
randomized LOCAL
quantum-LOCAL
bounded-dependence
randomized LOCAL (shared)
quantum-LOCAL (shared)
non-signaling
deterministic dynamic-LOCAL
deterministic SLOCAL
deterministic online-LOCAL
randomized SLOCAL
randomized online-LOCAL



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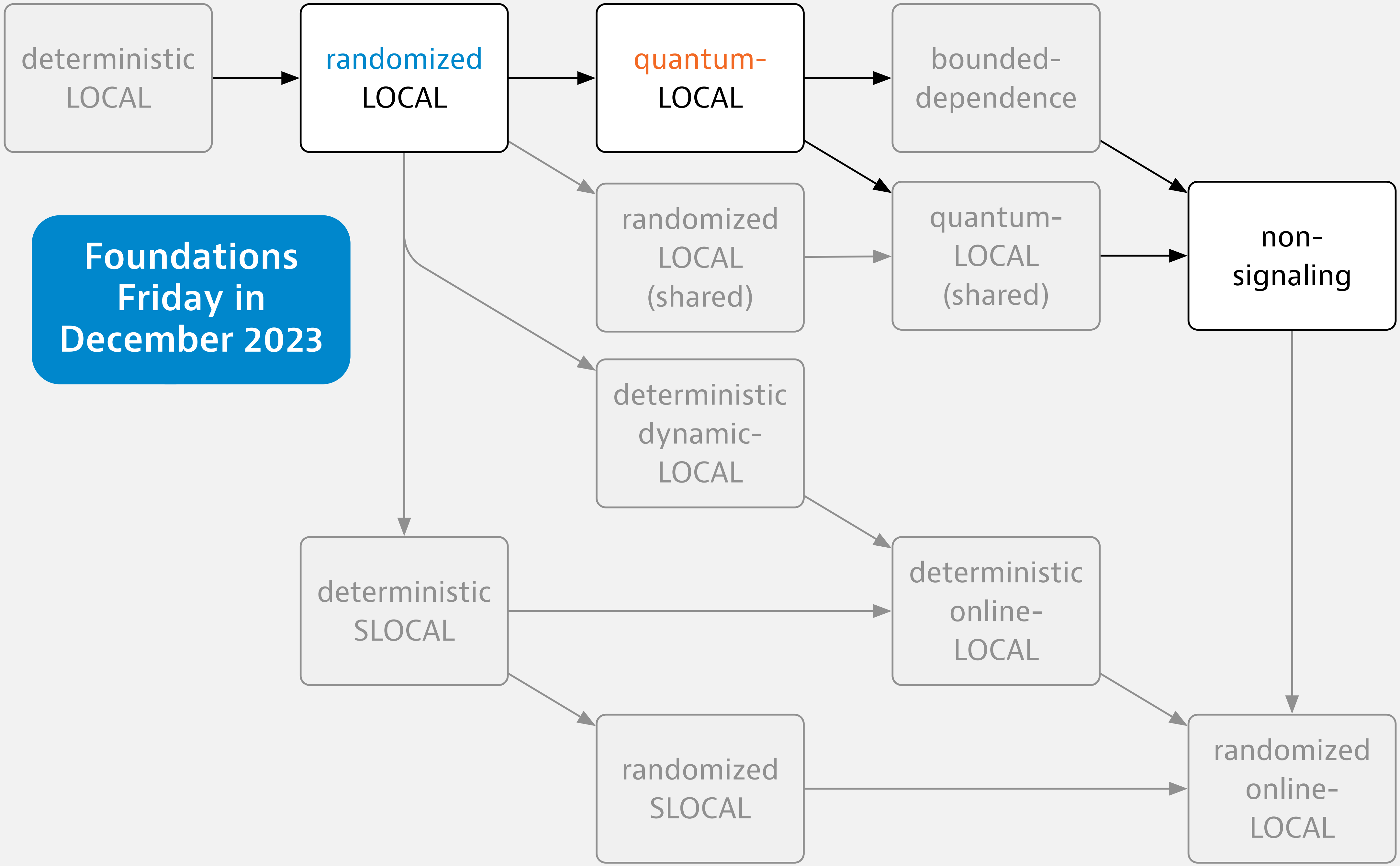
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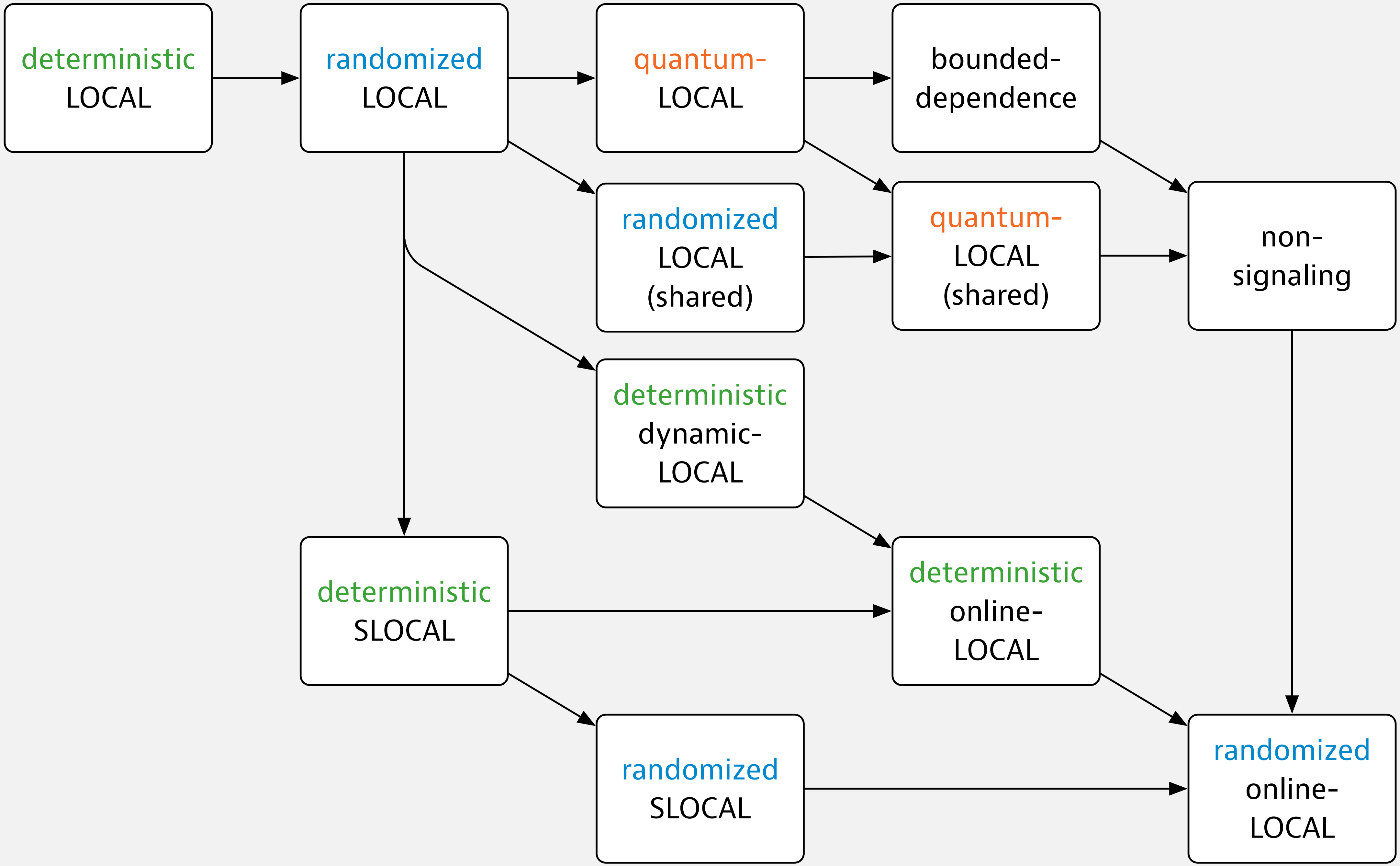

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non-signaling
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randomized online-LOCAL

Recap: locality in distributed, dynamic and online settings

SLOCAL
distributed,
sequential

LOCAL
distributed,
parallel

online LOCAL
centralized

dynamic LOCAL
centralized

LOCAL
distributed,
parallel

SLOCAL
distributed,
sequential

online LOCAL
centralized

dynamic LOCAL
centralized

LOCAL

distributed,
parallel

Each node **in parallel**:

- looks at its radius-T neighborhood
- picks its output based on this information

(nodes have unique identifiers)

SLOCAL
distributed,
sequential

LOCAL
distributed,
parallel

online LOCAL
centralized

dynamic LOCAL
centralized

SLOCAL

distributed,
sequential

LOCAL

distributed,
parallel

online LOCAL

centralized

dynamic LOCAL

centralized

SLOCAL

distributed,
sequential

Each node in a **sequential, adversarial order**:

- looks at its radius-T neighborhood
- picks its output & state based on this information

SLOCAL
distributed,
sequential

LOCAL
distributed,
parallel

online
LOCAL
centralized

dynamic
LOCAL
centralized

SLOCAL

distributed,
sequential

LOCAL

distributed,
parallel

online LOCAL

centralized

dynamic LOCAL

centralized

Graph **constructed** by an adversary that adds nodes and edges one by one

We can **see everything**

We can **change** our output only within distance *T* from a point of change

dynamic

LOCAL

centralized

SLOCAL
distributed,
sequential

LOCAL
distributed,
parallel

online LOCAL
centralized

dynamic LOCAL
centralized

SLOCAL
distributed,
sequential

LOCAL
distributed,
parallel

online LOCAL
centralized

dynamic LOCAL
centralized

Some unknown input graph is **revealed** piece by piece:

- adversary points at a node v
- we can see the radius-T neighborhood of v
- we have to choose the label for v

We can **remember** everything

online LOCAL

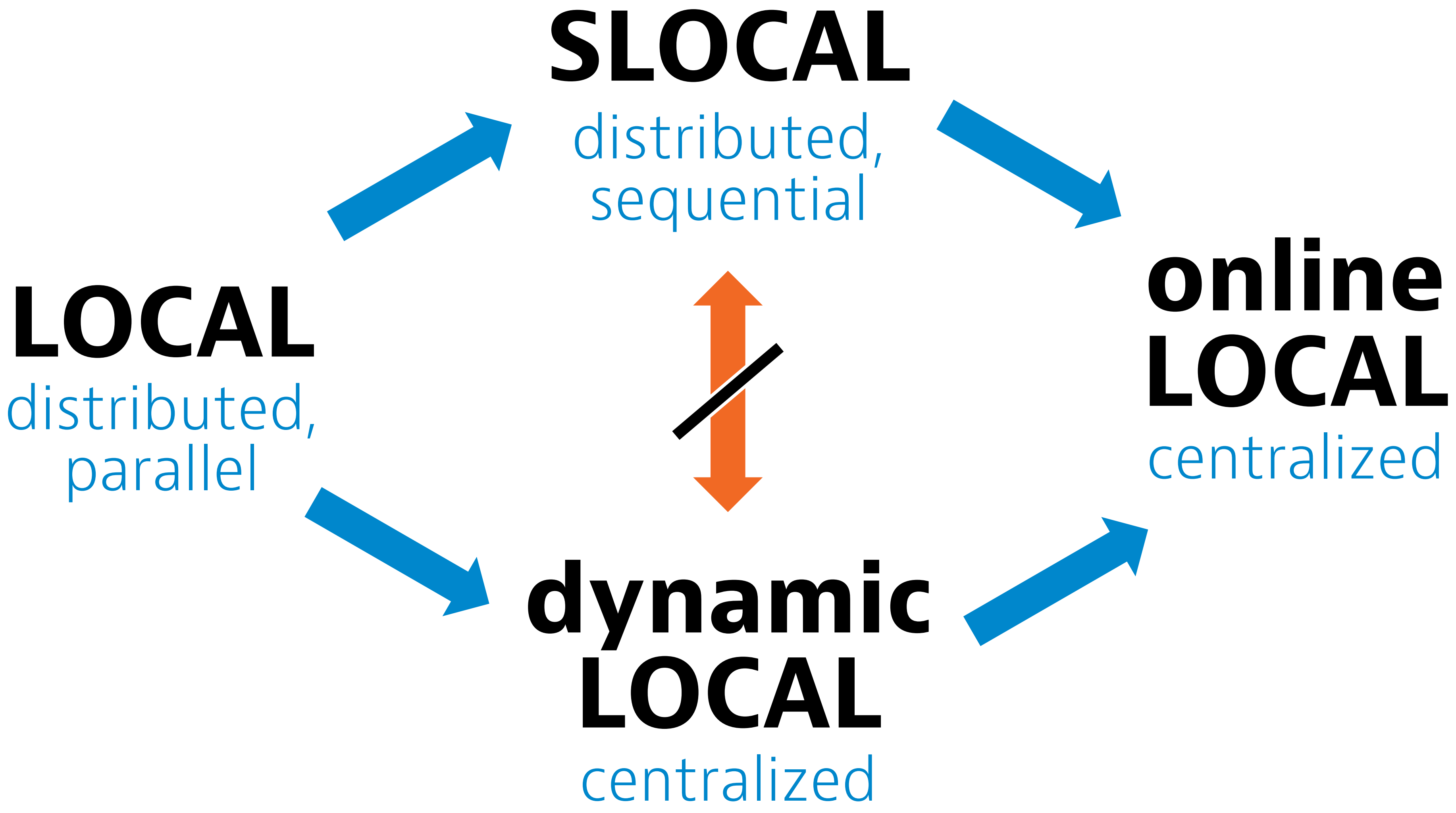
centralized

SLOCAL
distributed,
sequential

LOCAL
distributed,
parallel

online LOCAL
centralized

dynamic LOCAL
centralized


SLOCAL
distributed,
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LOCAL
distributed,
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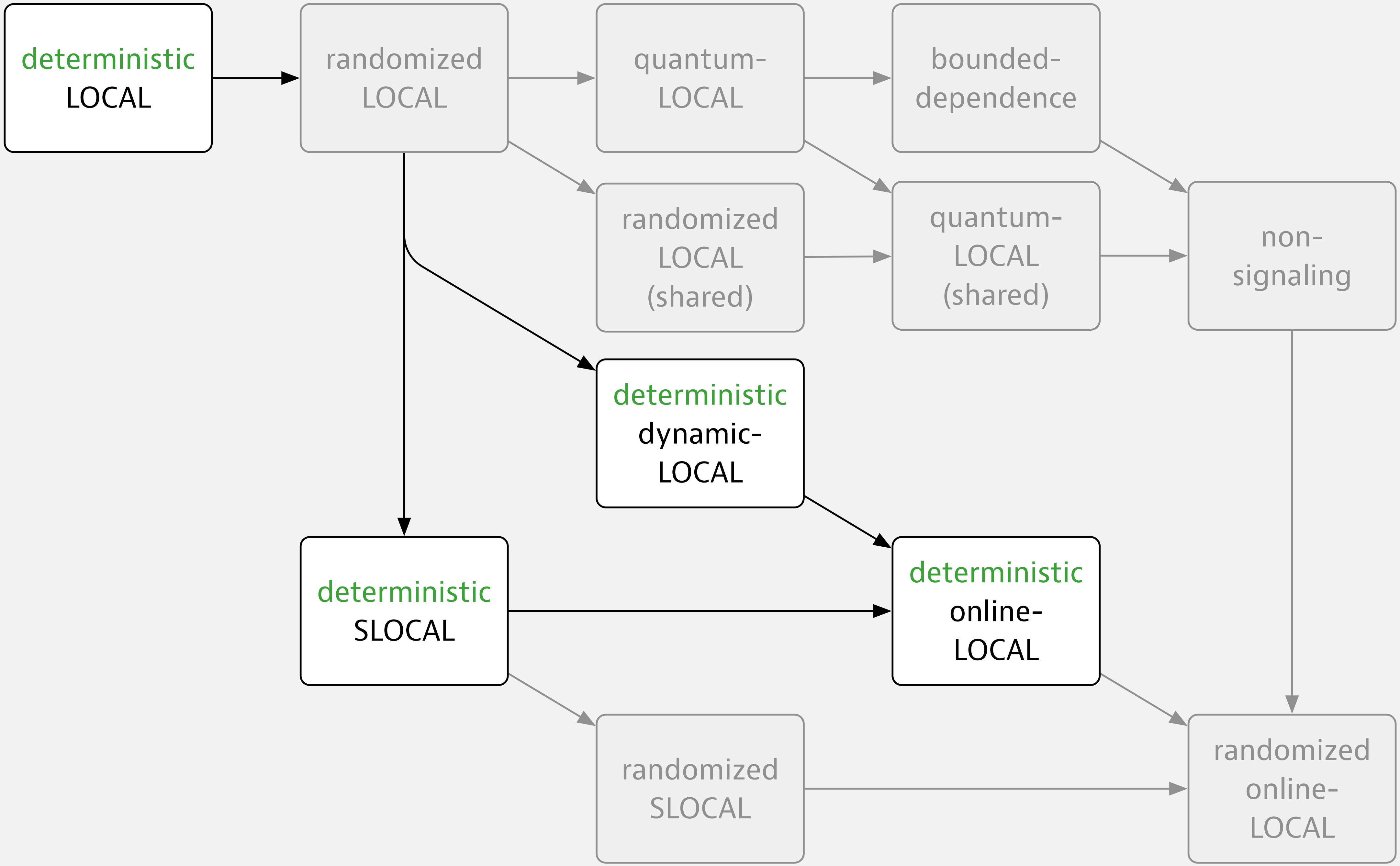
online
LOCAL
centralized
dynamic
LOCAL
centralized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Recap: distributed quantum computing & causality

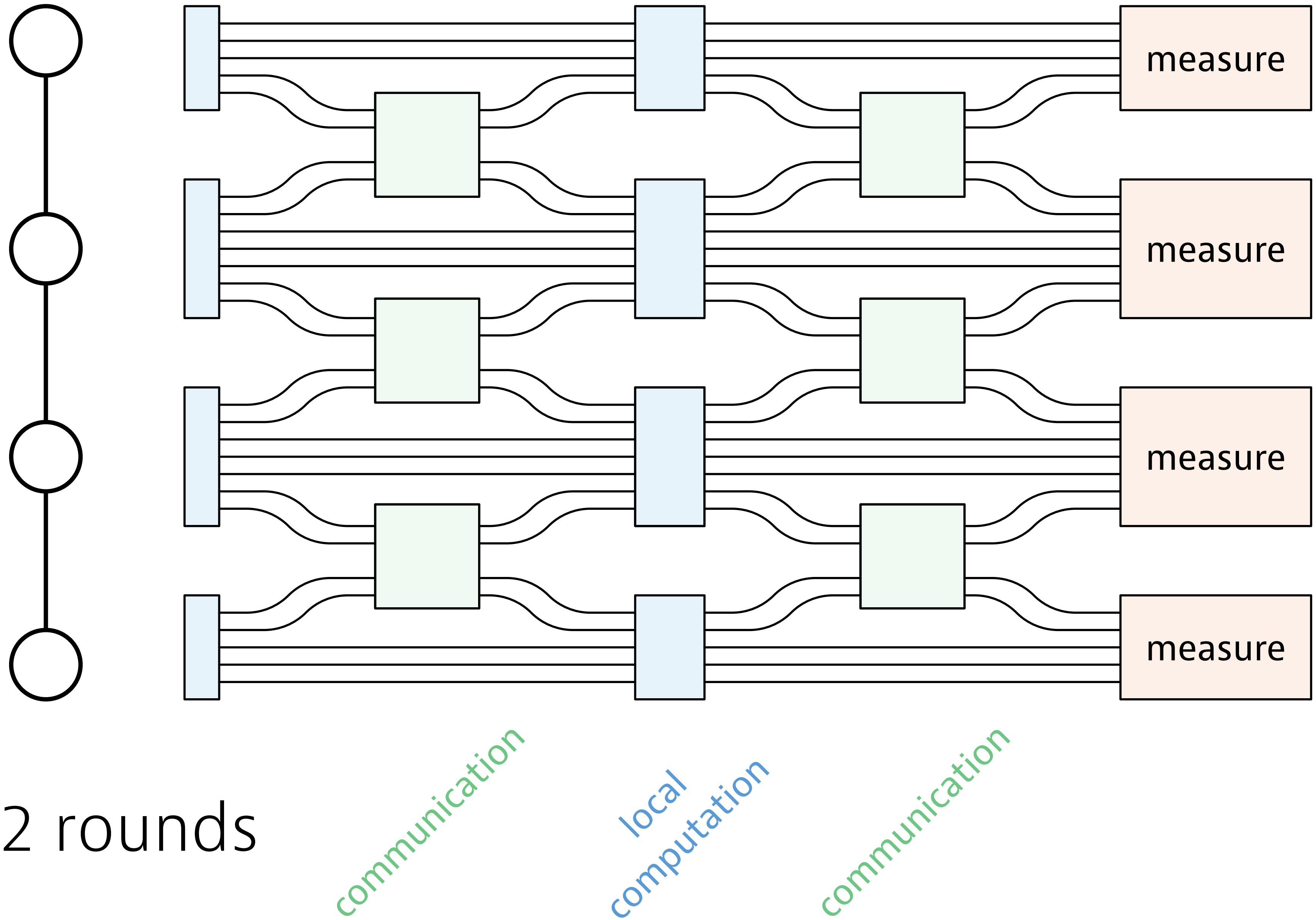
Distributed algorithms

Classical

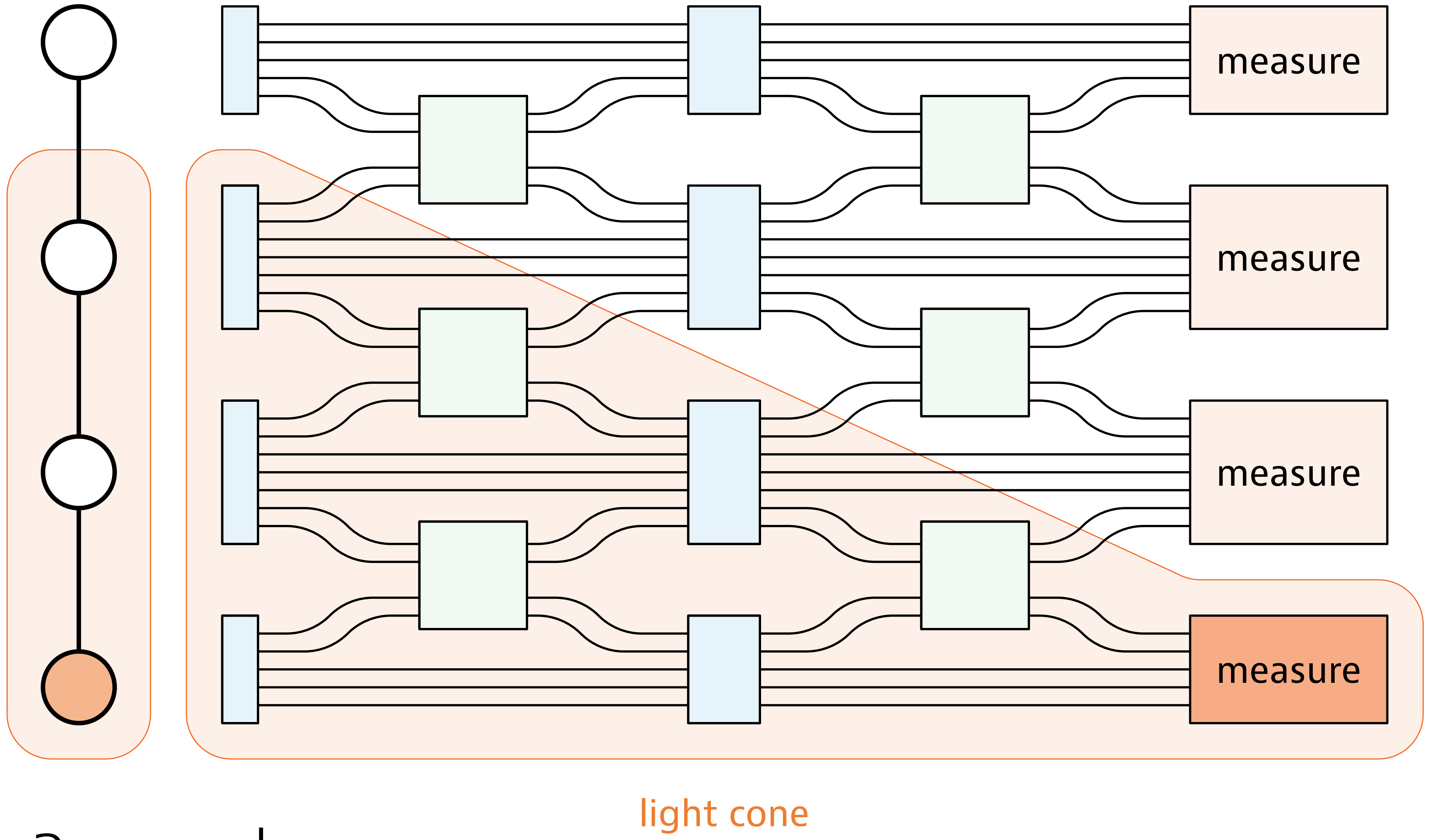
- node = classical computer
- edge = classical communication channel

Quantum

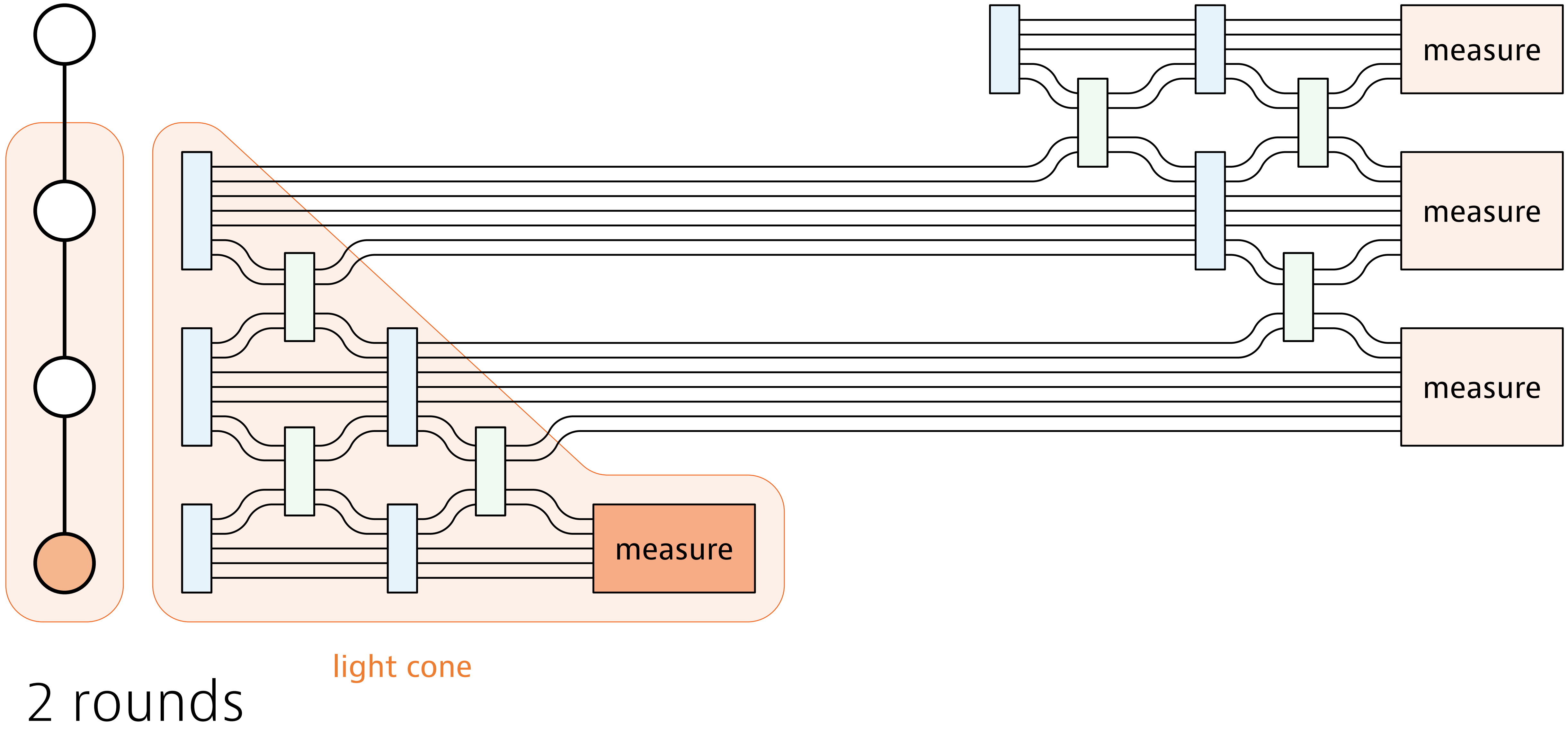
- node = quantum computer
- edge = quantum communication channel


measure
measure
measure
measure
communication
local
computation
communication

2 rounds



2 rounds


measure
measure
measure
measure
light cone
2 rounds

Non-signaling model

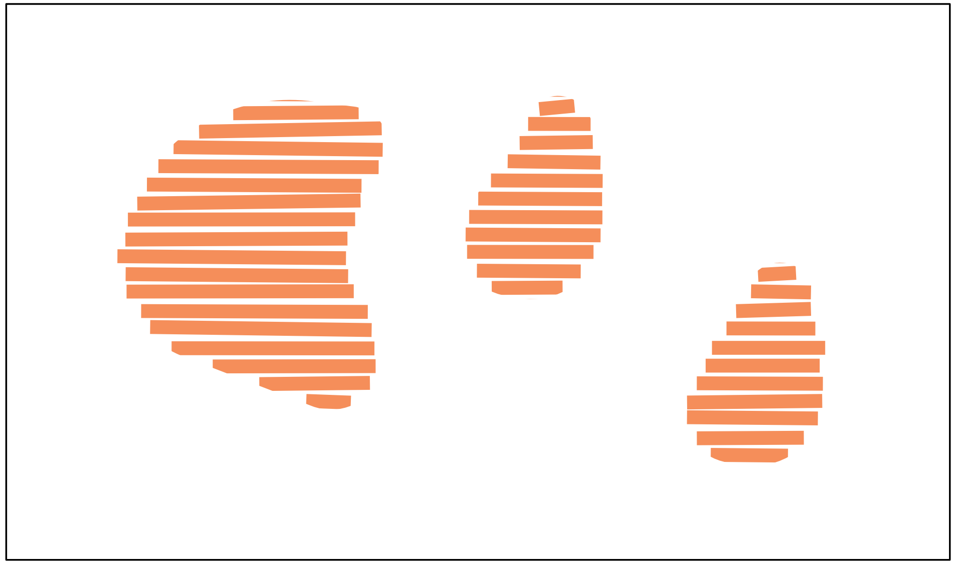
- Key idea: **define** a model so that it can do **anything** except violating causality

Non-signaling model

Definition (*non-signaling distribution*):

- fix any **set of nodes *X*** …

Gavoille, Kosowski, Markiewicz 2009
Arfaoui, Fraigniaud 2014



Non-signaling model

Definition (*non-signaling distribution*):

- fix any **set of nodes *X***
- changes in the input **more than *T* hops away** from ***X*** do not influence the output distribution of ***X***

Gavoille, Kosowski, Markiewicz 2009
Arfaoui, Fraigniaud 2014

Three models

Classical (randomized) distributed algorithms

Classical probability theory

↓

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Weird quantum things

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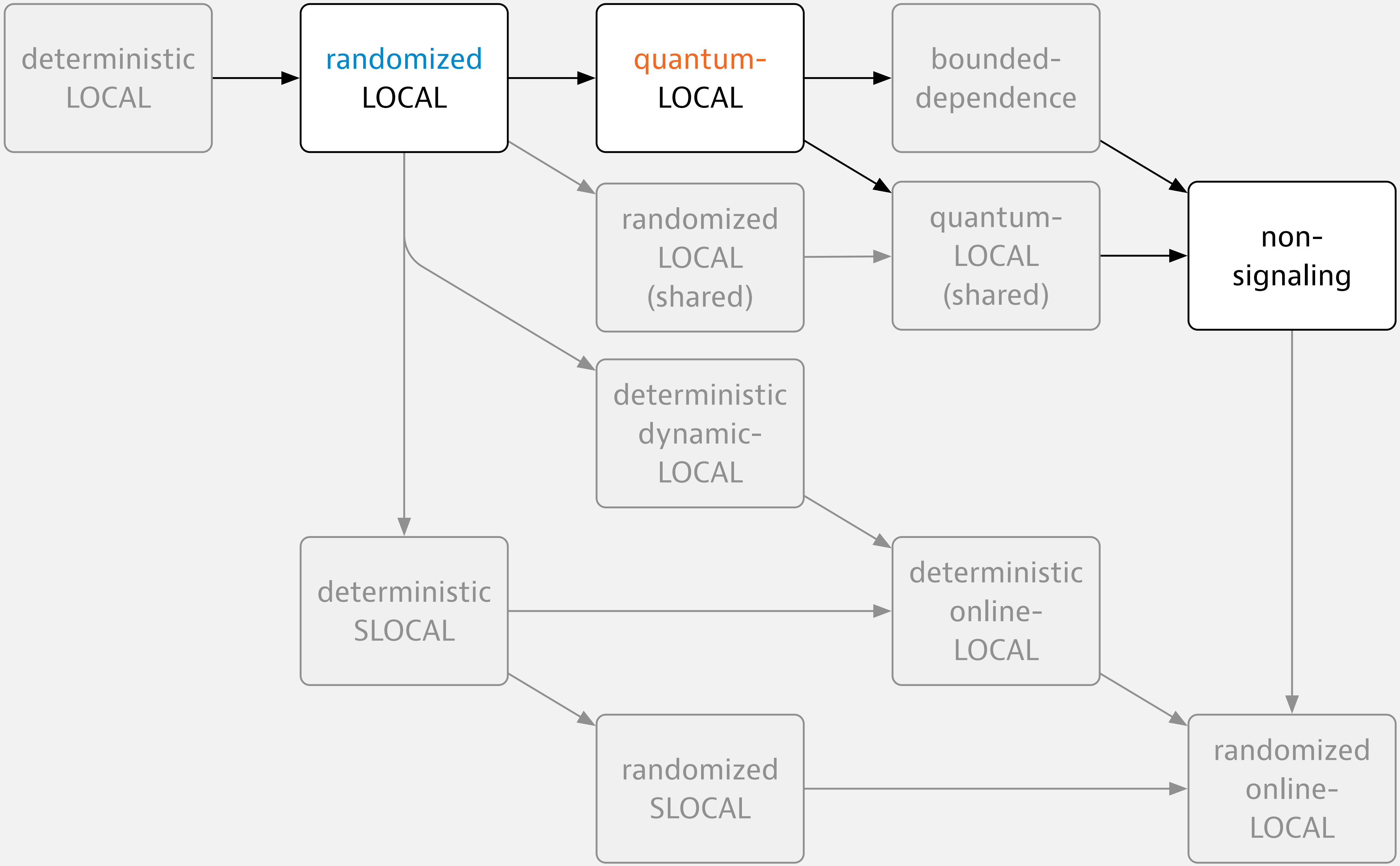
Non-signaling “algorithms”

Classical probability theory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Unifying model: randomized online LOCAL

Rand. online-LOCAL

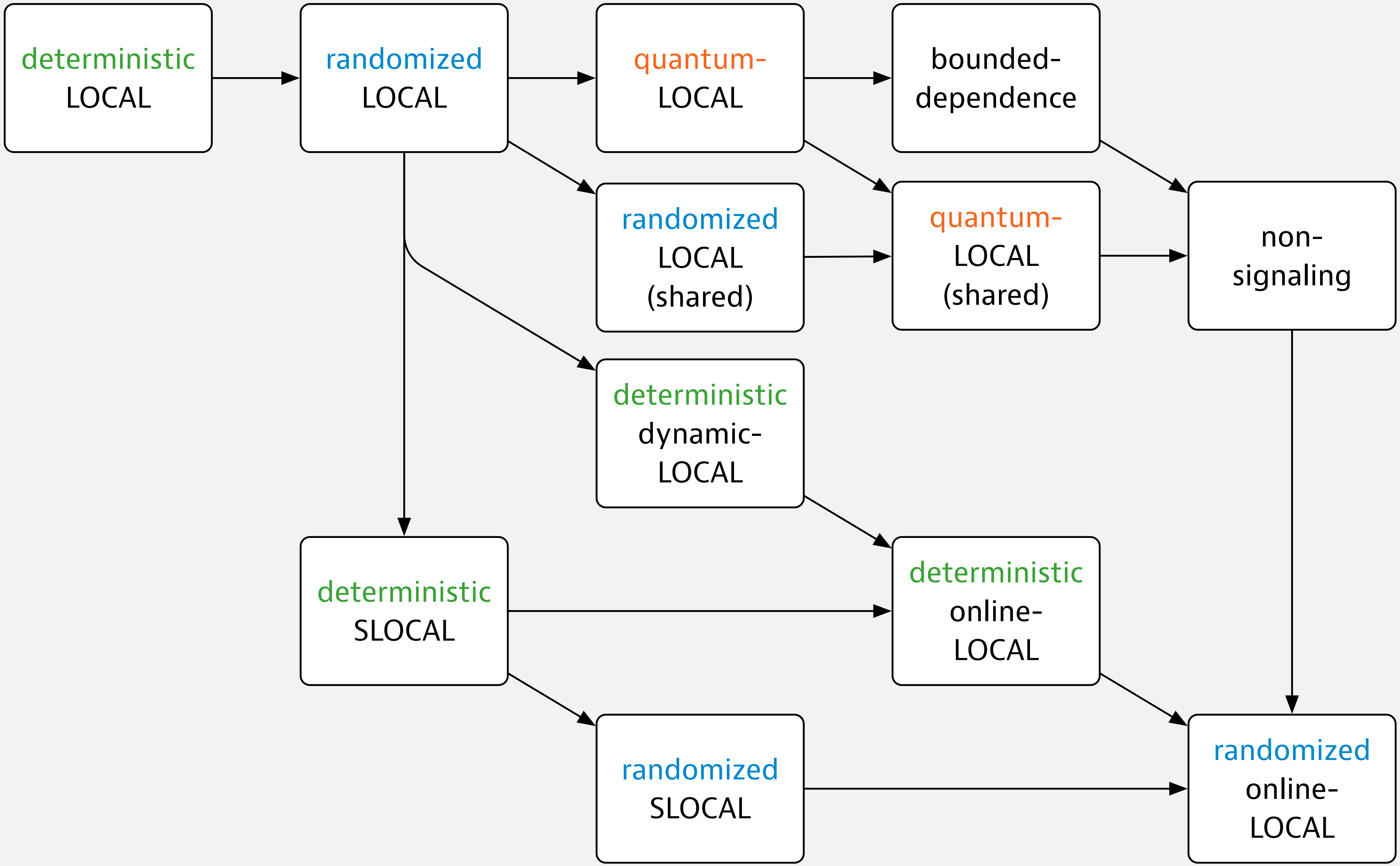
- Adversary fixes a **graph** + **order** in which nodes are revealed

Oblivious adversary

- For each node v
  - algorithm sees radius-T neighborhood of v
  - algorithm must choose the label of v
- Algorithm can **remember** everything, algorithm can use **randomness**

Rand. online-LOCAL

- **Trivial:**
  - randomize online-LOCAL can simulate deterministic online-LOCAL
- **Suprise:**
  - randomized online-LOCAL ***can simulate any non-signaling distribution*** (with the same asymptotic locality)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Why does it matter?

- We can prove for *some problem families* that **det. LOCAL ≈ rand. online-LOCAL**

Why does it matter?

- We can prove for ***some problem families*** that **det. LOCAL ≈ rand. online-LOCAL**
- Implies: **det. LOCAL ≈ rand. LOCAL ≈ quantum-LOCAL ≈ bounded-dependence ≈ non-signaling ≈ det. SLOCAL ≈ rand. SLOCAL ≈ det. dynamic-LOCAL ≈ det. online-LOCAL ≈ rand. online-LOCAL** for the same problem families

Why does it matter?

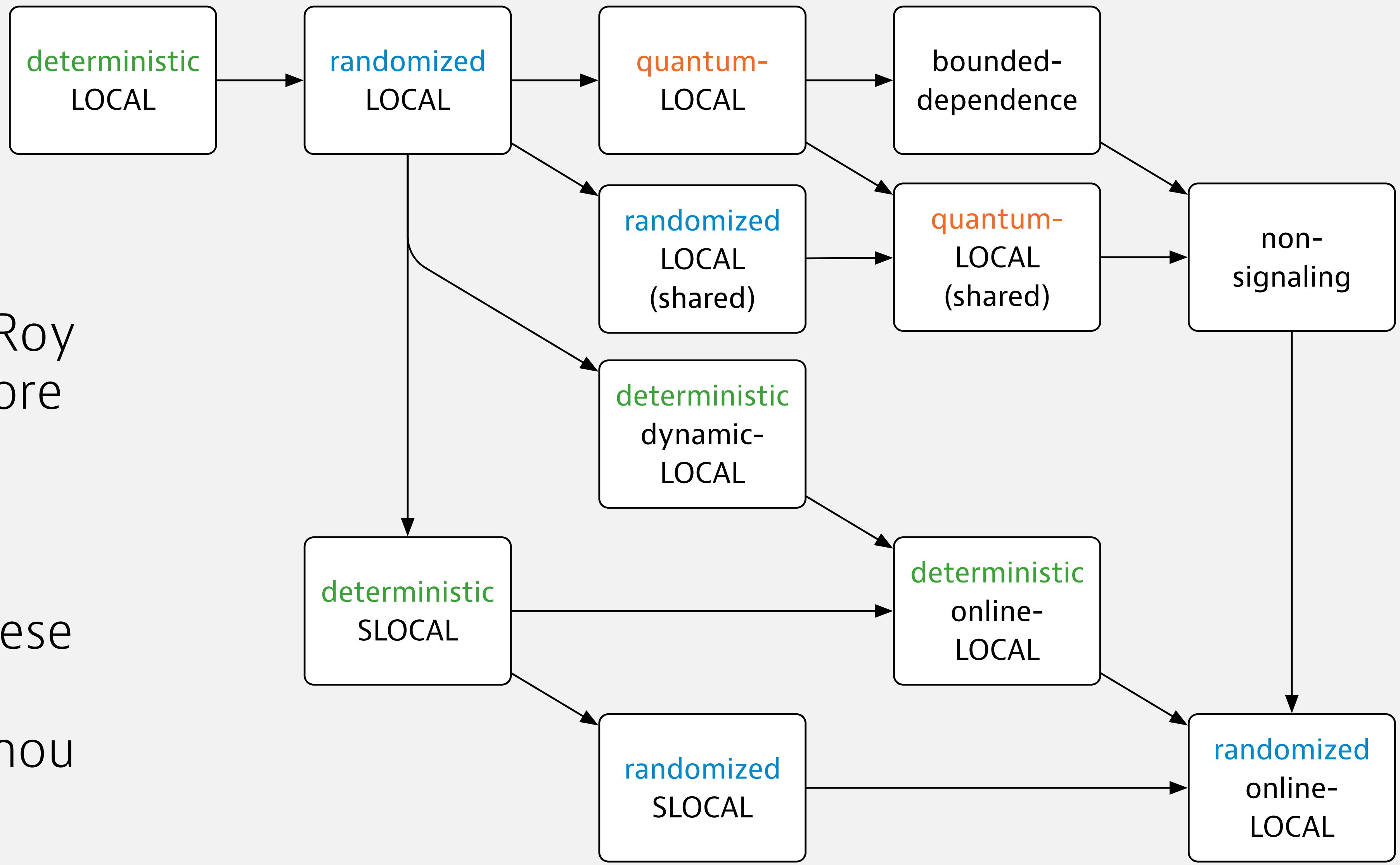
- We can prove for ***some problem families*** that **det. LOCAL ≈ rand. online-LOCAL**
- Holds for:
  - locally checkable labeling problems (LCLs)
  - in rooted trees
  - in $o(\log \log n)$ region
- Puts limits on distributed quantum advantage

One implication

- $O(\log^* n)$**-round** distributed **quantum** algorithms are not any stronger than $O(\log^* n)$**-round** classical algorithms for LCLs in rooted trees

Still wide open

- Are $O(1)$**-round** distributed **quantum** algorithms any stronger than $O(1)$**-round** classical algorithms for LCLs in rooted trees?
- In directed cycles?
- Anywhere?



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