Local algorithms

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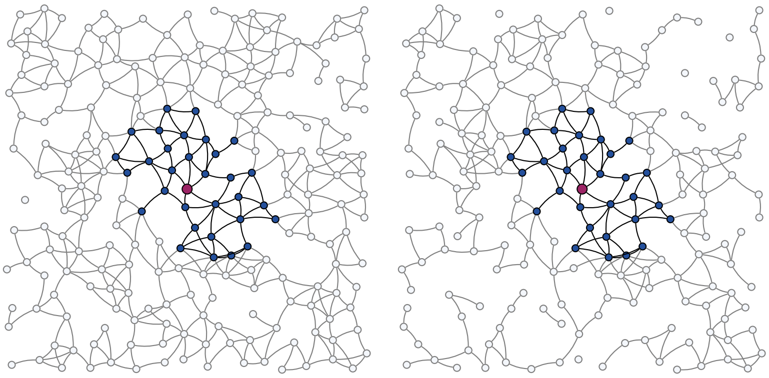
1 Local algorithms

Local algorithms are *constant-time distributed algorithms*. The output of a node is a function of the input available within its *constant-radius neighbourhood*.



Changes outside the *local horizon* of a node do not affect its output.

2 Max-min LPs

Max-min LPs are linear programs of the form

$$\begin{aligned} \text{maximise} \quad & \min_{k \in K} \textstyle\sum_{v \in V_k} c_{kv} x_v \\ \text{subject to} \quad & \textstyle\sum_{v \in V_i} a_{iv} x_v \leq 1 \quad \forall i \in I, \\ & x_v \geq 0 \quad \forall v \in V. \end{aligned}$$

Here

- $a_{iv} \geq 0$ and $c_{kv} \geq 0$
- $V_i \subseteq V$ and $V_k \subseteq V$
- $|V_i| \leq \Delta_I$ and $|V_k| \leq \Delta_K$
- $\Delta_I \geq 2$ and $\Delta_K \geq 2$ are constants.

Bipartite version: each $v \in V$ is in exactly one set V_i and exactly one set V_k.

Local algorithms

The underlying *communication graph* $\mathcal{G}$:

- the vertex set is $V \cup I \cup K$
- an edge $\{i, v\}$ for each $i \in I$, $v \in V_i$
- an edge $\{k, v\}$ for each $k \in K$, $v \in V_k$.

Each *agent* $v \in V$ must choose the value of x_v.

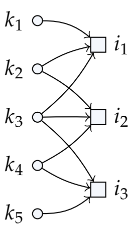
Theorem 1 (Papadimitriou & Yannakakis 1993). *There is a local algorithm for max-min LPs with the approximation ratio Δ_I.*

Theorem 2. *For any $\epsilon > 0$, there is a local algorithm for bipartite max-min LPs with the approximation ratio $\Delta_I(1 - 1/\Delta_K) + \epsilon$.*

Theorem 3. *No local algorithm achieves the approximation ratio $\Delta_I(1 - 1/\Delta_K)$ for bipartite max-min LPs.*

There is a local approximation algorithm that achieves a better approximation ratio if $\mathcal{G}$ has *bounded relative growth*.

Application: fair bandwidth allocation



$$\begin{aligned} \max \quad & \min \{x_{11},\ x_{21} + x_{22},\ x_{31} + x_{32} + x_{33},\ x_{42} + x_{43},\ x_{53}\} \\ \text{s.t.} \quad & x_{11} + x_{21} + x_{31} \leq 1, \\ & x_{22} + x_{32} + x_{42} \leq 1, \\ & x_{33} + x_{43} + x_{53} \leq 1, \\ & x_{11}, x_{21}, \dots, x_{53} \geq 0. \end{aligned}$$

3 Sleep scheduling

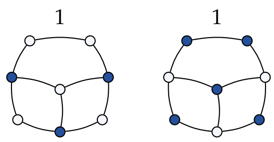
Input is a *redundancy graph* $\mathcal{R}$:

- If nodes u and v are adjacent in $\mathcal{R}$, then u and v are *pairwise redundant*: if v is awake then u can be asleep and vice versa.
- D is a valid set of nodes that are awake iff D is a *dominating set* of $\mathcal{R}$.

Each node v can be awake for 1 time unit:

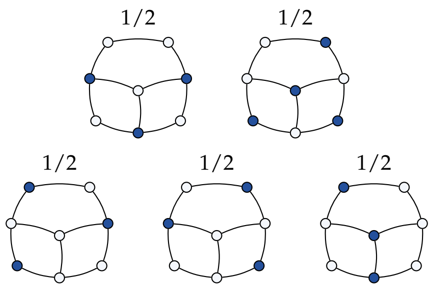
$$\begin{aligned} \text{maximise} \quad & \textstyle\sum_D x(D) \\ \text{subject to} \quad & \textstyle\sum_{D:\, v \in D} x(D) \leq 1 \quad \forall v, \\ & x(D) \geq 0 \quad \forall D. \end{aligned}$$

Integral solutions are *domatic partitions*:



○ $\notin D$ (asleep) ● $\in D$ (awake)

Our focus is on *fractional domatic partitions*:



○ $\notin D$ (asleep) ● $\in D$ (awake)

Local algorithms

A graph $\mathcal{G}$ is a *$(\Delta, \ell_1, \ell_\mu, \mu)$-marked graph* if

- the maximum degree of $\mathcal{G}$ is Δ
- some nodes are designated as *markers* such that for any node of $\mathcal{G}$ there is at least one marker within distance ℓ_1 and at most μ markers within distance ℓ_μ.

Theorem 4. *Assume that $\mathcal{R}$ is a $(\Delta, \ell_1, \ell_\mu, \mu)$-marked graph. Then there is a local algorithm for sleep scheduling with the approximation ratio $(1 + \epsilon)$ for any $\epsilon > 4\Delta / \lfloor (\ell_\mu - \ell_1)/\mu \rfloor$.*

4 Activity scheduling

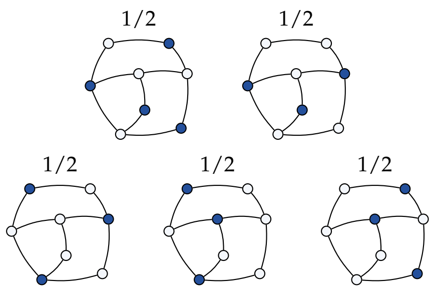
Input is a *conflict graph* $\mathcal{C}$:

- If nodes u and v are adjacent in $\mathcal{C}$, then u and v are *mutually conflicting*: if v is active then u must be inactive and vice versa.
- I is a valid set of active nodes iff I is an *independent set* of $\mathcal{C}$.

Each node v must be active for 1 time unit:

$$\begin{aligned} \text{minimise} \quad & \textstyle\sum_I x(I) \\ \text{subject to} \quad & \textstyle\sum_{I:\, v \in I} x(I) \geq 1 \quad \forall v, \\ & x(I) \geq 0 \quad \forall I. \end{aligned}$$

Integral solutions are *graph (vertex) colourings*. Our focus is on *fractional graph colourings*:



○ $\notin I$ (inactive) ● $\in I$ (active)

Theorem 5. *Assume that $\mathcal{C}$ is a $(\Delta, \ell_1, \ell_\mu, \mu)$-marked graph. Then there is a local algorithm for activity scheduling with the approximation ratio $1/(1 - \epsilon)$ for any $\epsilon > 4/\lfloor (\ell_\mu - \ell_1)/\mu \rfloor$.*

References

[1] P. Floréen, P. Kaski, T. Musto, and J. Suomela. Approximating max-min linear programs with local algorithms. *IPDPS 2008.*

[2] P. Floréen, P. Kaski, T. Musto, and J. Suomela. Local approximation algorithms for scheduling problems in sensor networks. *Algosensors 2007.*

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