Local 3-approximation algorithms for weighted dominating set and vertex cover in quasi unit-disk graphs

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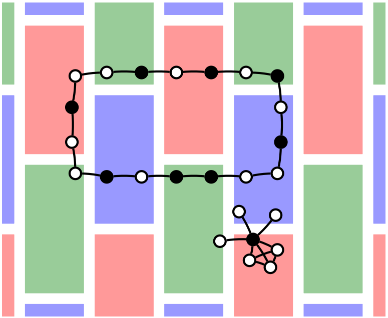


LOCALGOS
14 June 2008

Introduction

Local algorithms: output at each node depends only on the constant-radius neighbourhood of the node

(Linial 1992, Naor and Stockmeyer 1995)

Assumptions:

- Unit-disk graph
- Each node knows its coordinates

Problems:

- Dominating set
- Vertex cover

Prior work

Dominating set:

- 15-approximation (Urrutia 2007)
- 5-approximation (Czyzowicz et al. 2008)
- $(1 + \epsilon)$-approximation (Wiese and Kranakis 2007)

Vertex cover:

- 12-approximation trivial
- $(1 + \epsilon)$-approximation (Wiese and Kranakis 2008)

Our contributions

Simple local algorithm

3-approximation

Small local horizon (locality distance):

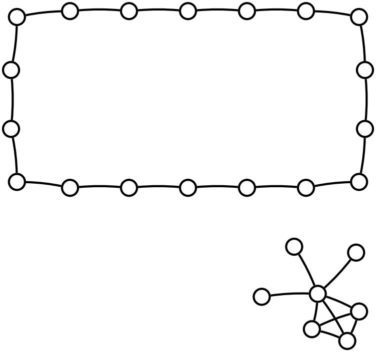
- Present algorithm: $r = 83$
- Wiese and Kranakis (2007):
  $r = 46814$ for 3-approximation

Quasi unit-disk graphs

Weighted versions

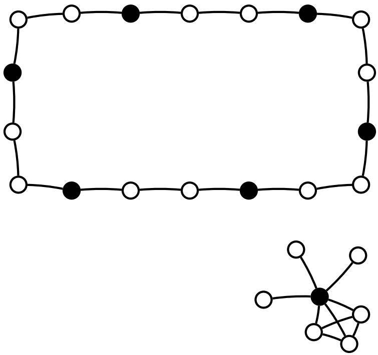
Dominating set

Input — assumed to be a unit-disk graph

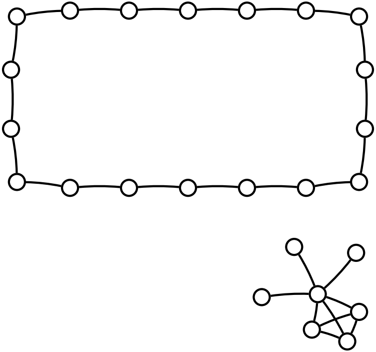


Dominating set

An optimal solution

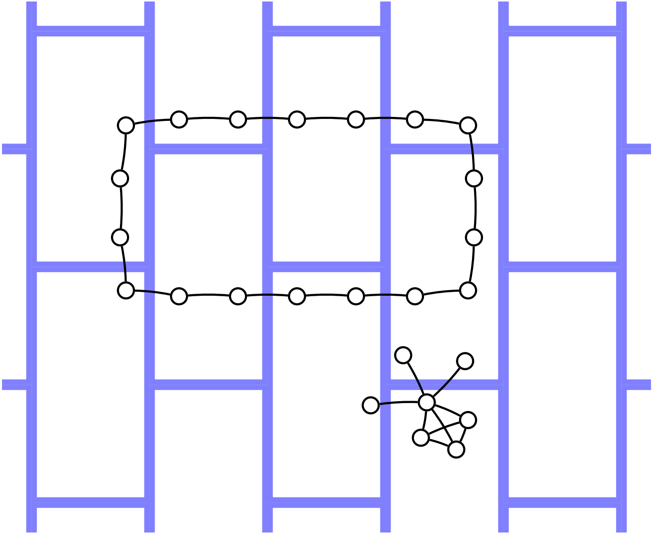


Dominating set: local algorithm



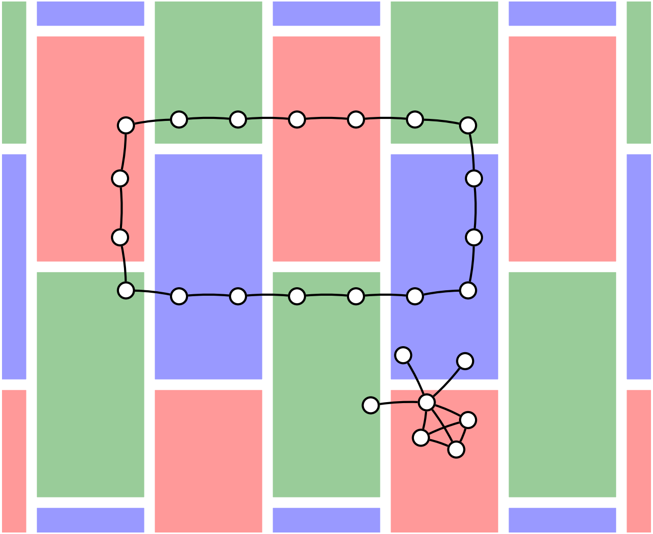
Dominating set: local algorithm

Tile the plane with 2×4 rectangles



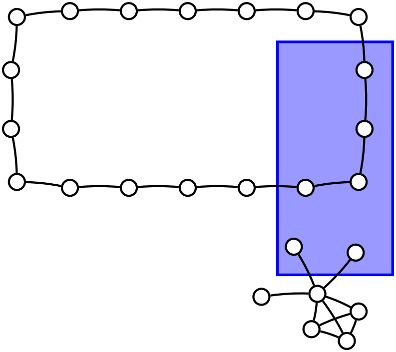
Dominating set: local algorithm

3-colour the rectangles



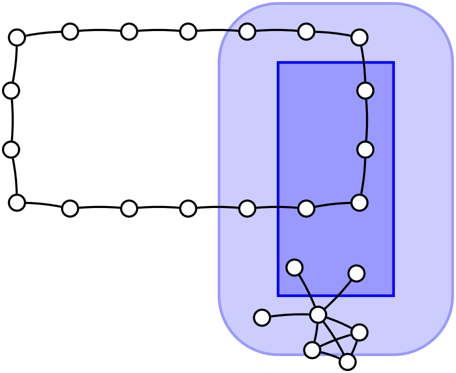
Dominating set: local algorithm

For each rectangle. . .



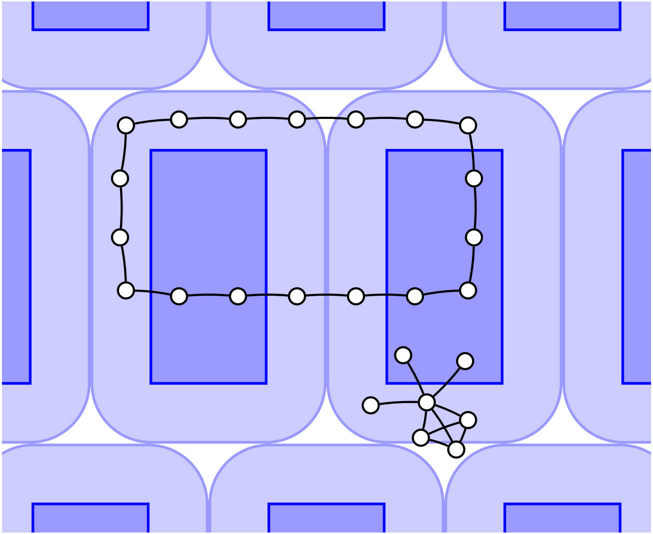
Dominating set: local algorithm

For each rectangle construct an extended rectangle



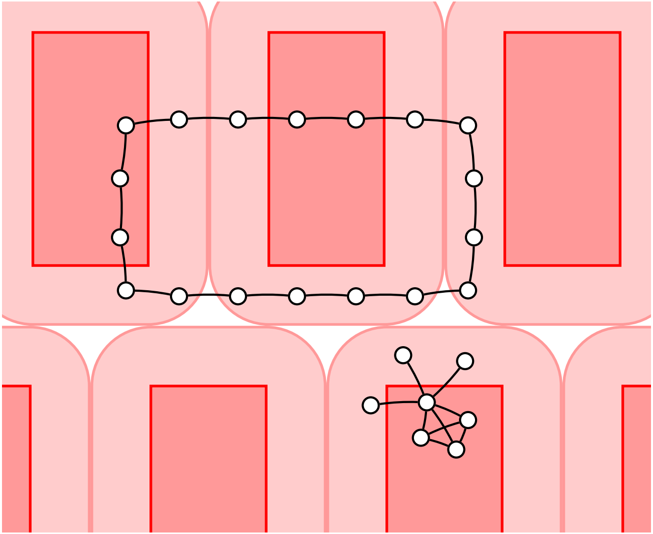
Dominating set: local algorithm

Extended rectangles are non-intersecting for each colour



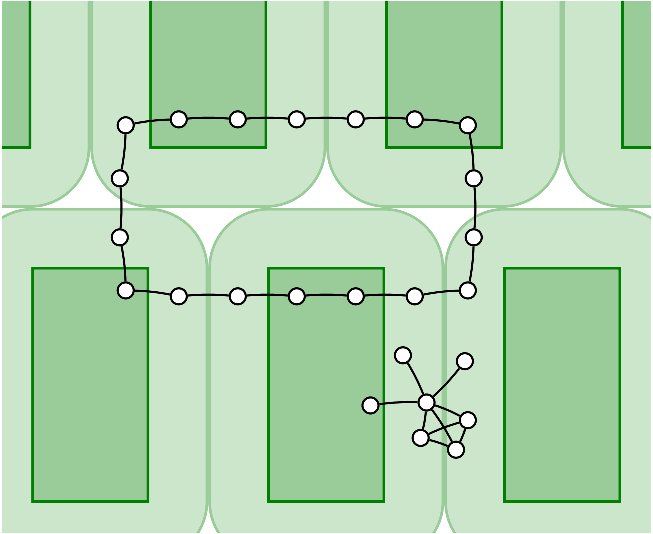
Dominating set: local algorithm

Extended rectangles are non-intersecting for each colour



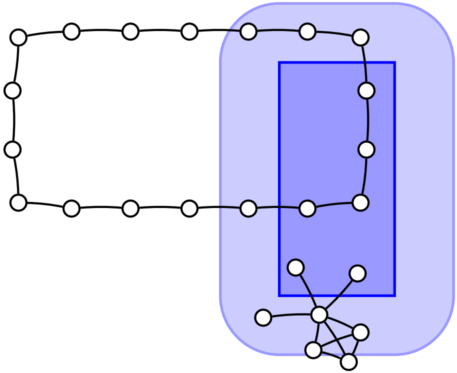
Dominating set: local algorithm

Extended rectangles are non-intersecting for each colour



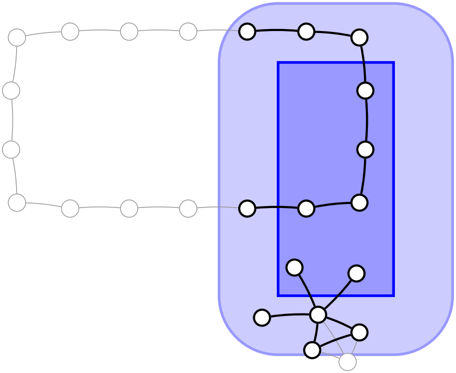
Dominating set: local algorithm

For each extended rectangle. . .



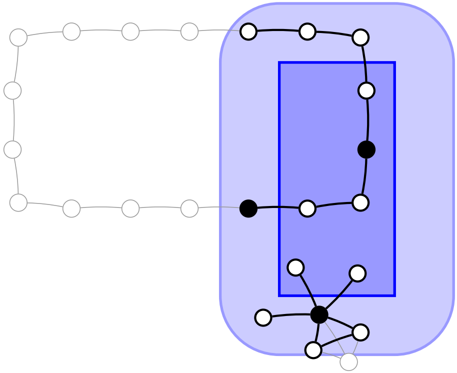
Dominating set: local algorithm

For each extended rectangle, form a subproblem. . .



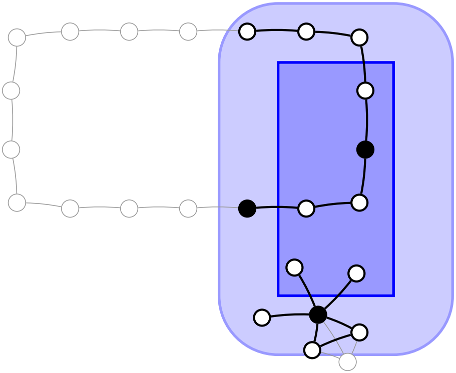
Dominating set: local algorithm

. . . and solve the subproblem optimally



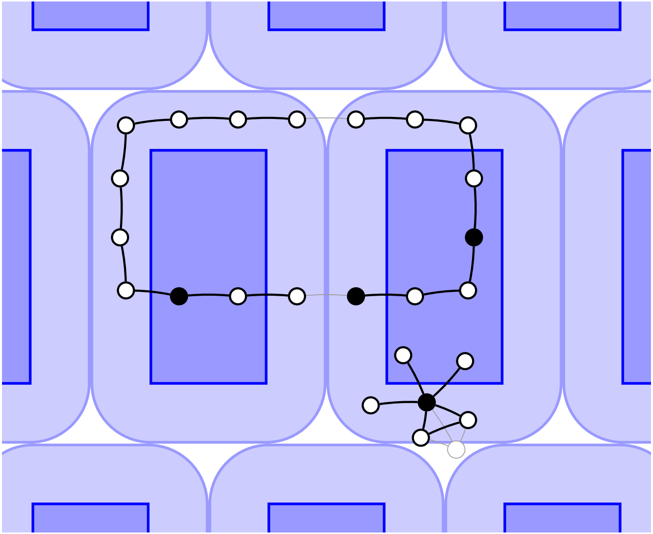
Dominating set: local algorithm

Only inside needs to be dominated



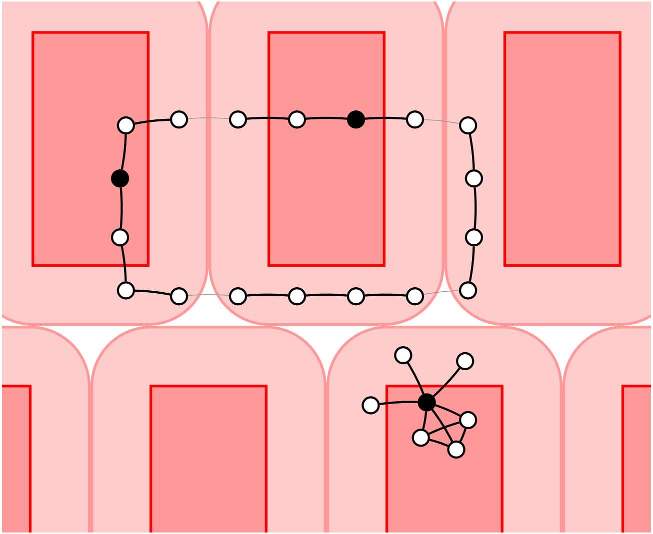
Dominating set: local algorithm

Repeat for each rectangle



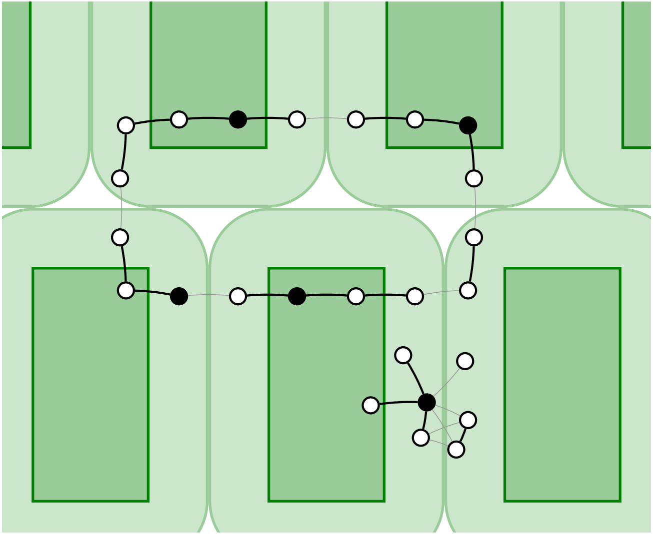
Dominating set: local algorithm

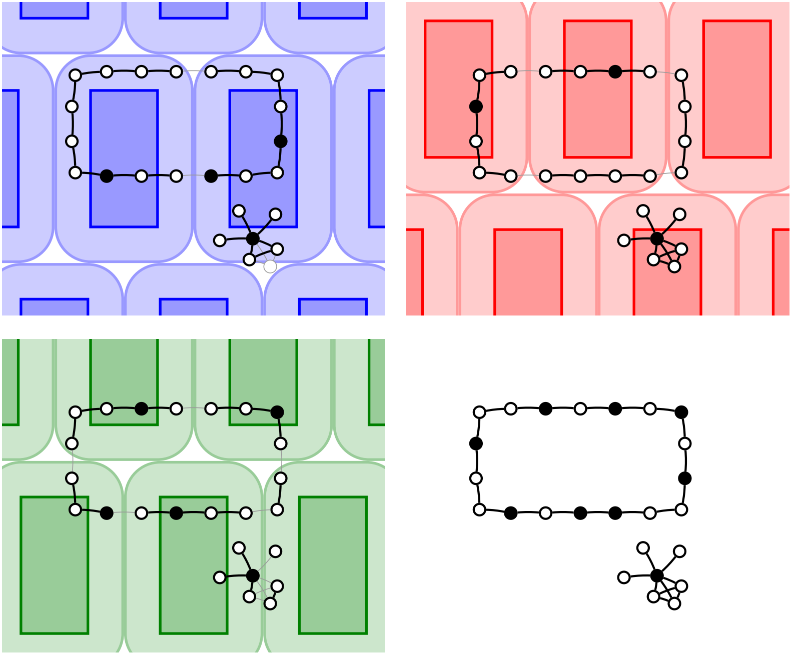
Repeat for each rectangle



Dominating set: local algorithm

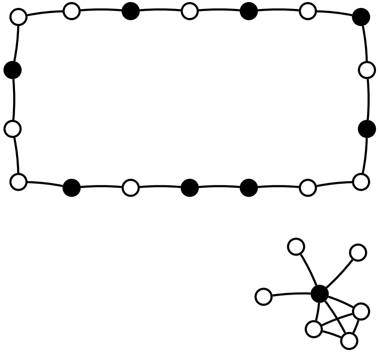
Repeat for each rectangle





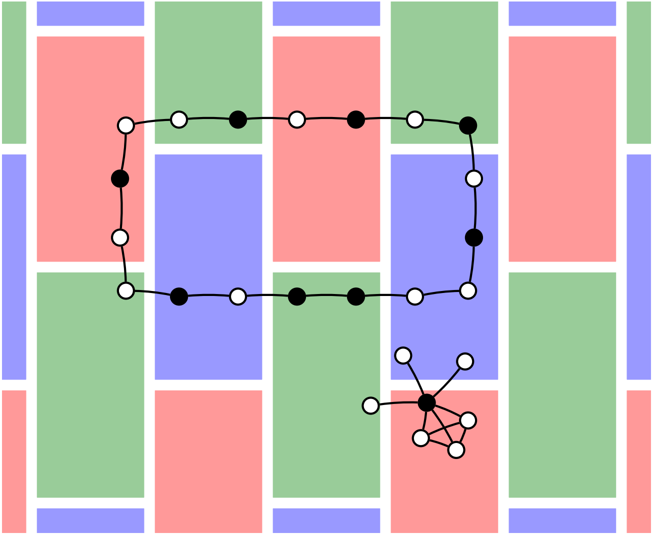
Dominating set: local algorithm

Union of local solutions



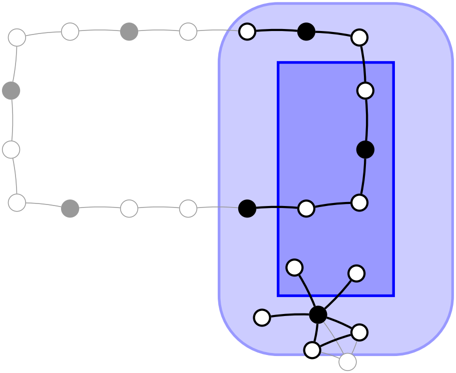
Dominating set: feasibility

Each node is dominated in at least one subproblem



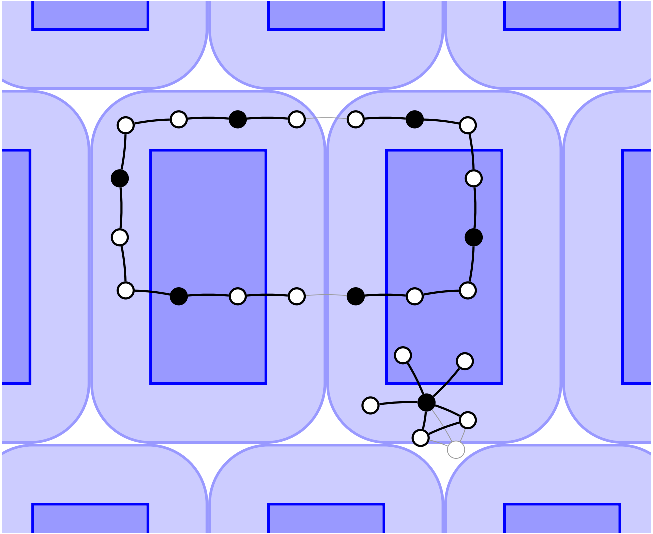
Dominating set: approximation ratio

OPT is a feasible solution to each subproblem



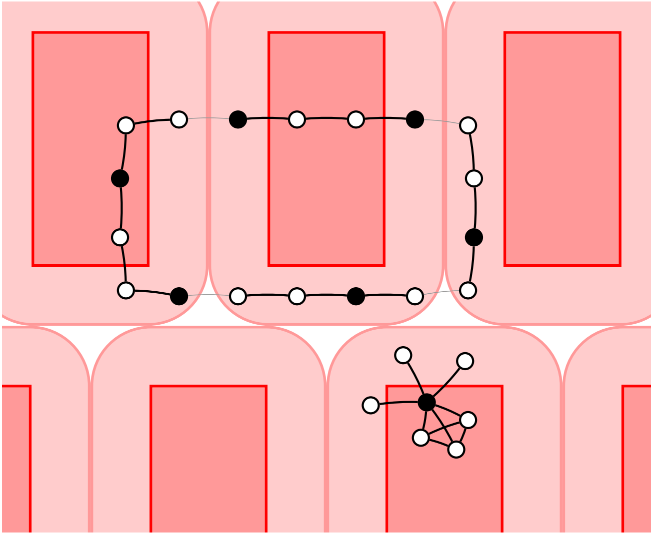
Dominating set: approximation ratio

OPT is a feasible solution to each subproblem



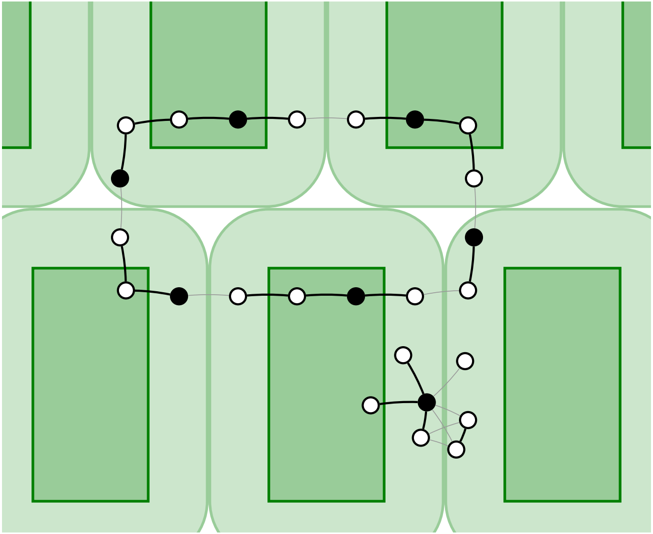
Dominating set: approximation ratio

OPT is a feasible solution to each subproblem



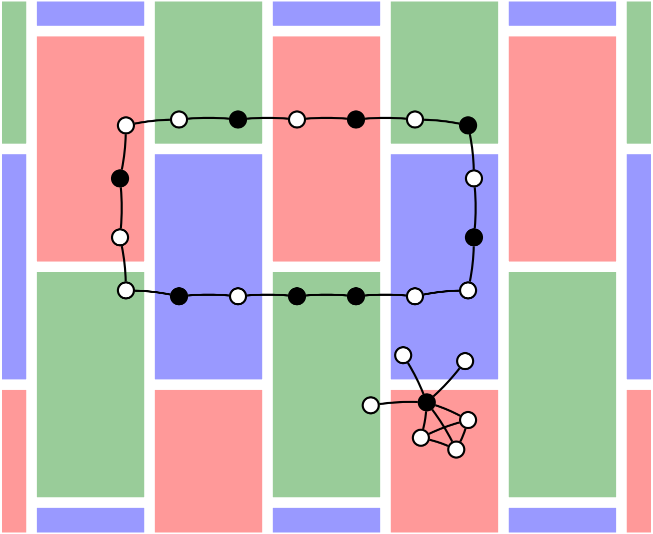
Dominating set: approximation ratio

OPT is a feasible solution to each subproblem



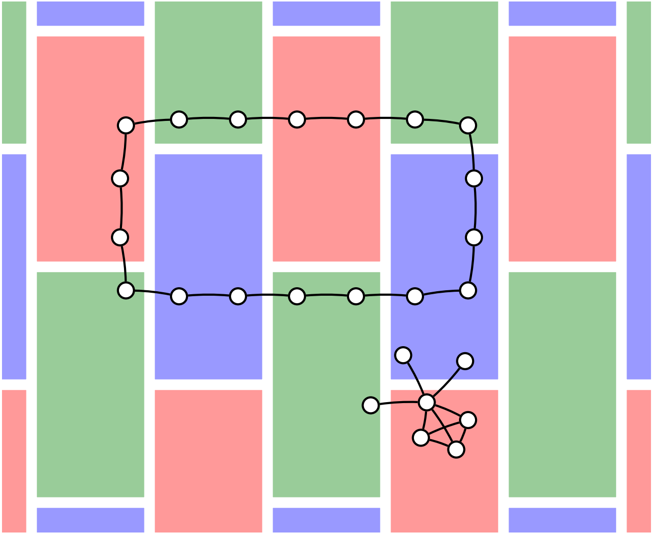
Dominating set: approximation ratio

Factor 3 approximation from 3-colouring



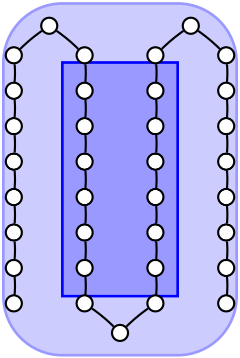
Vertex cover

The same basic approach applies here as well



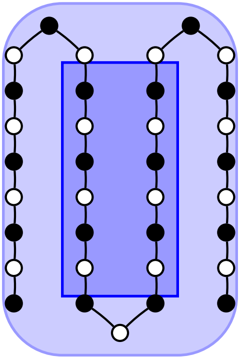
Local horizon: worst case

Consider a shortest path within an extended rectangle



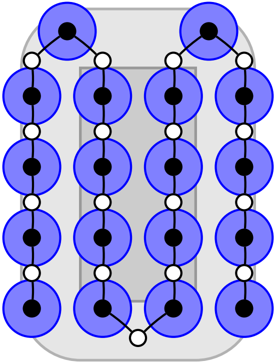
Local horizon: worst case

Pick even nodes — distance between any pair > 1



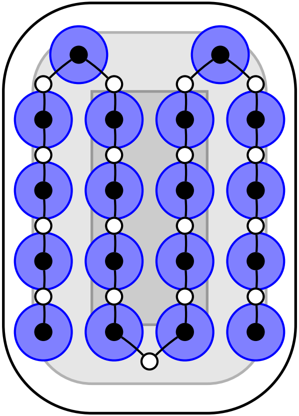
Local horizon: worst case

Place disks of radius $1/2$ on even nodes — non-intersecting

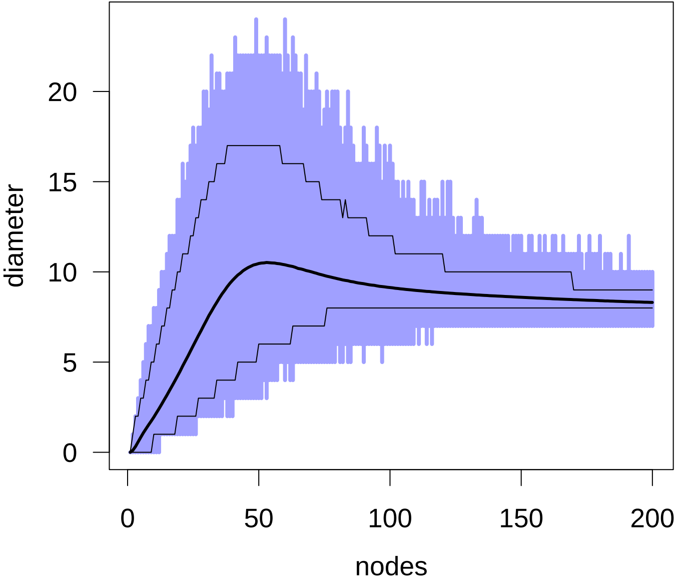


Local horizon: worst case

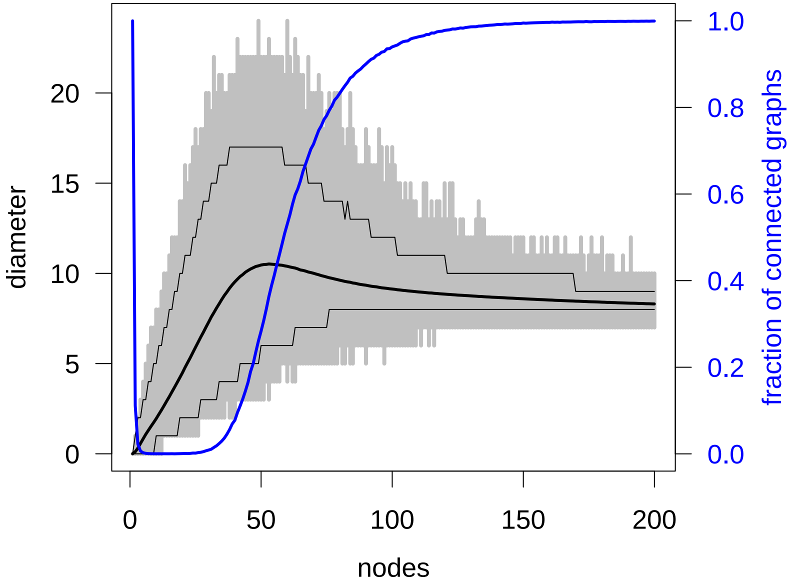
Area bound: at most 42 such disks $\implies$ at most 83 edges



Local horizon: average case



Local horizon: average case



Conclusions

Local 3-approximation algorithm for dominating set and vertex cover

Assumptions: (quasi) unit-disk graphs, coordinates known

Unweighted case: local and poly-time

Weighted case: local — but not necessarily poly-time!

- Other complexity measures for local algorithms besides the local horizon?

Challenge: apply the same idea to other problems!

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J. Czyzowicz, S. Dobrev, T. Fevens, H. González-Aguilar, E. Kranakis, J. Opatrny, and J. Urrutia. Local algorithms for dominating and connected dominating sets of unit disk graphs with location aware nodes. In *Proc. 8th Latin American Theoretical Informatics Symposium (LATIN, Búzios, Brazil, April 2008)*, volume 4957 of *Lecture Notes in Computer Science*, pages 158–169, Berlin, Germany, 2008. Springer-Verlag. [DOI]

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