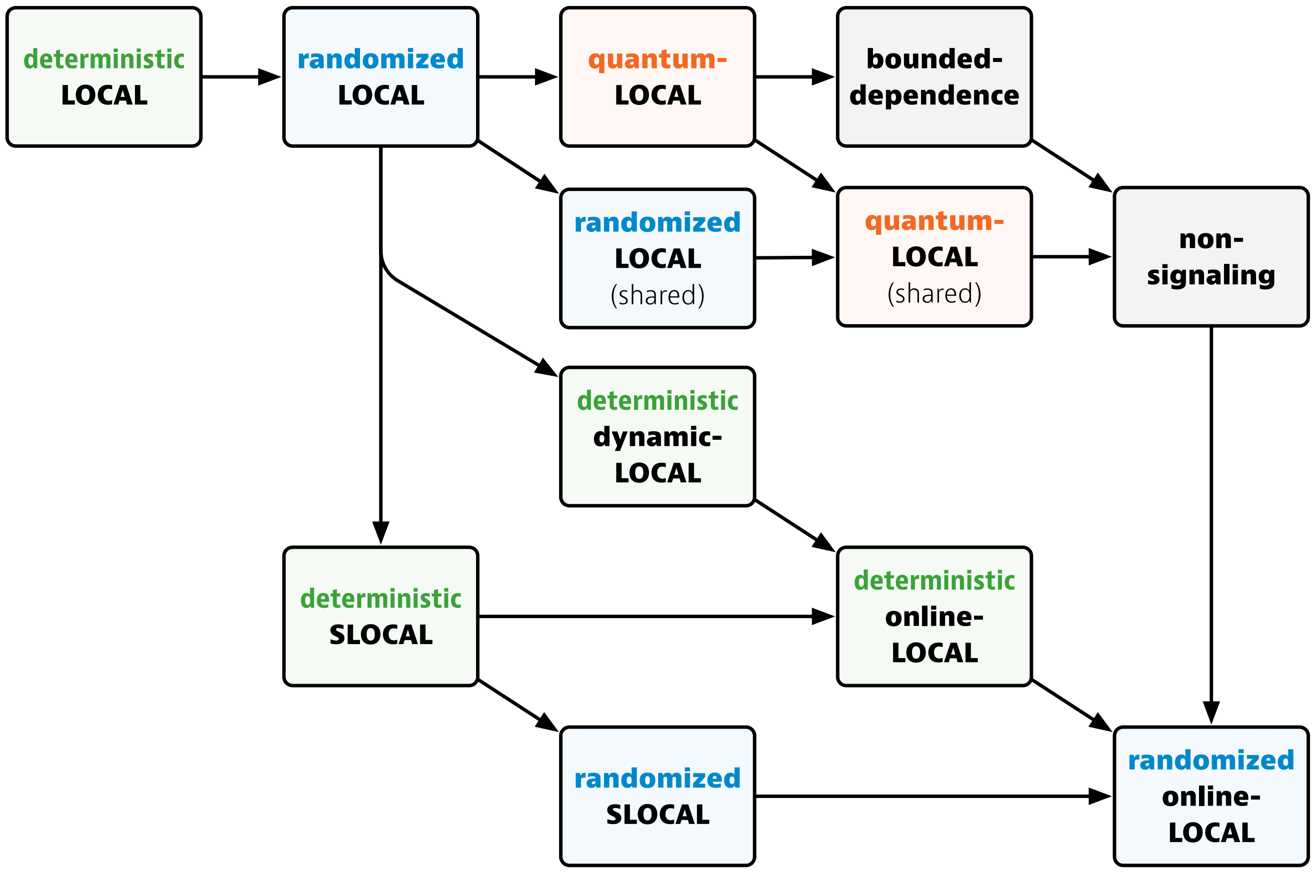
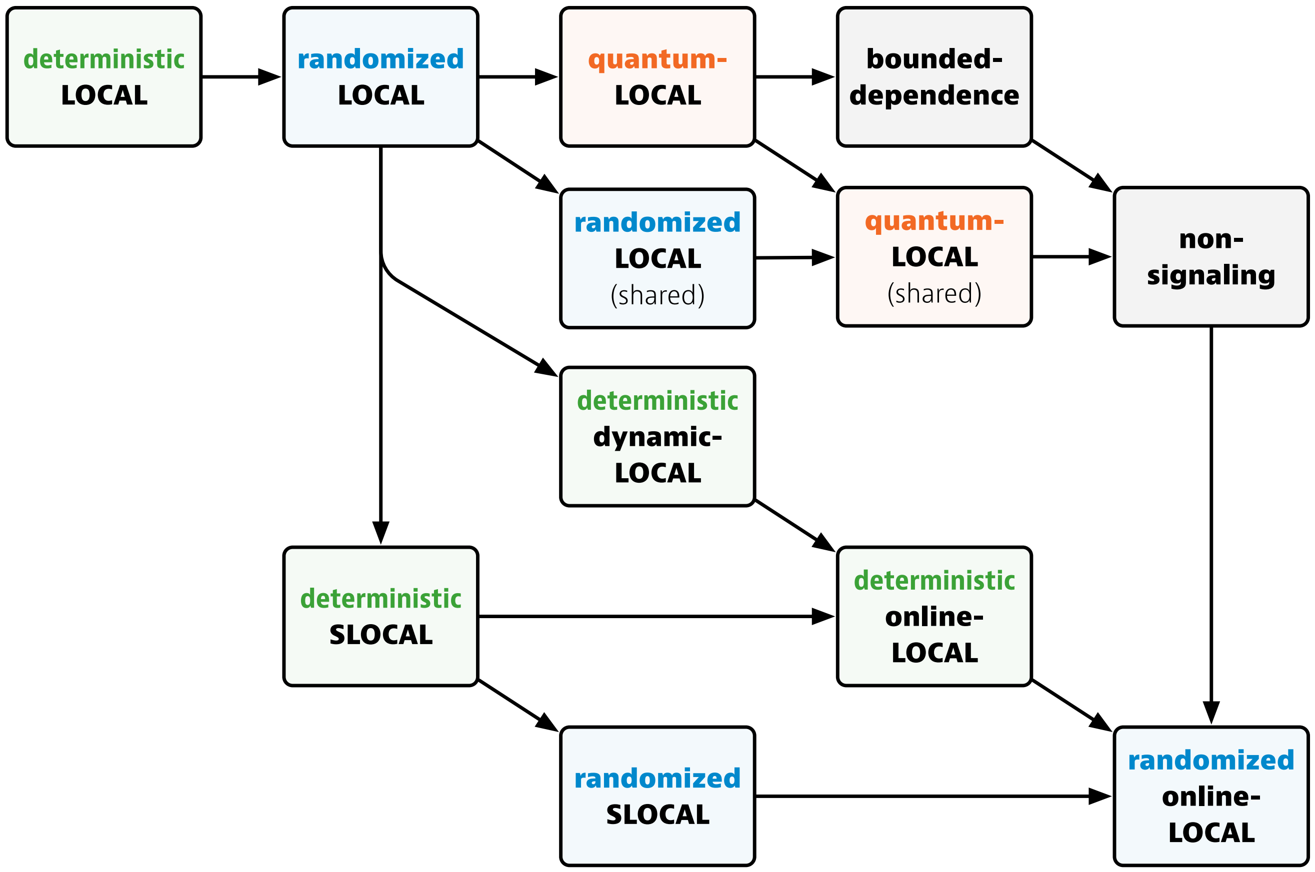
Fun stuff & latest news: **Locality** *in all kinds of weird settings*



Jukka Suomela
Aalto University

Joint work with e.g.

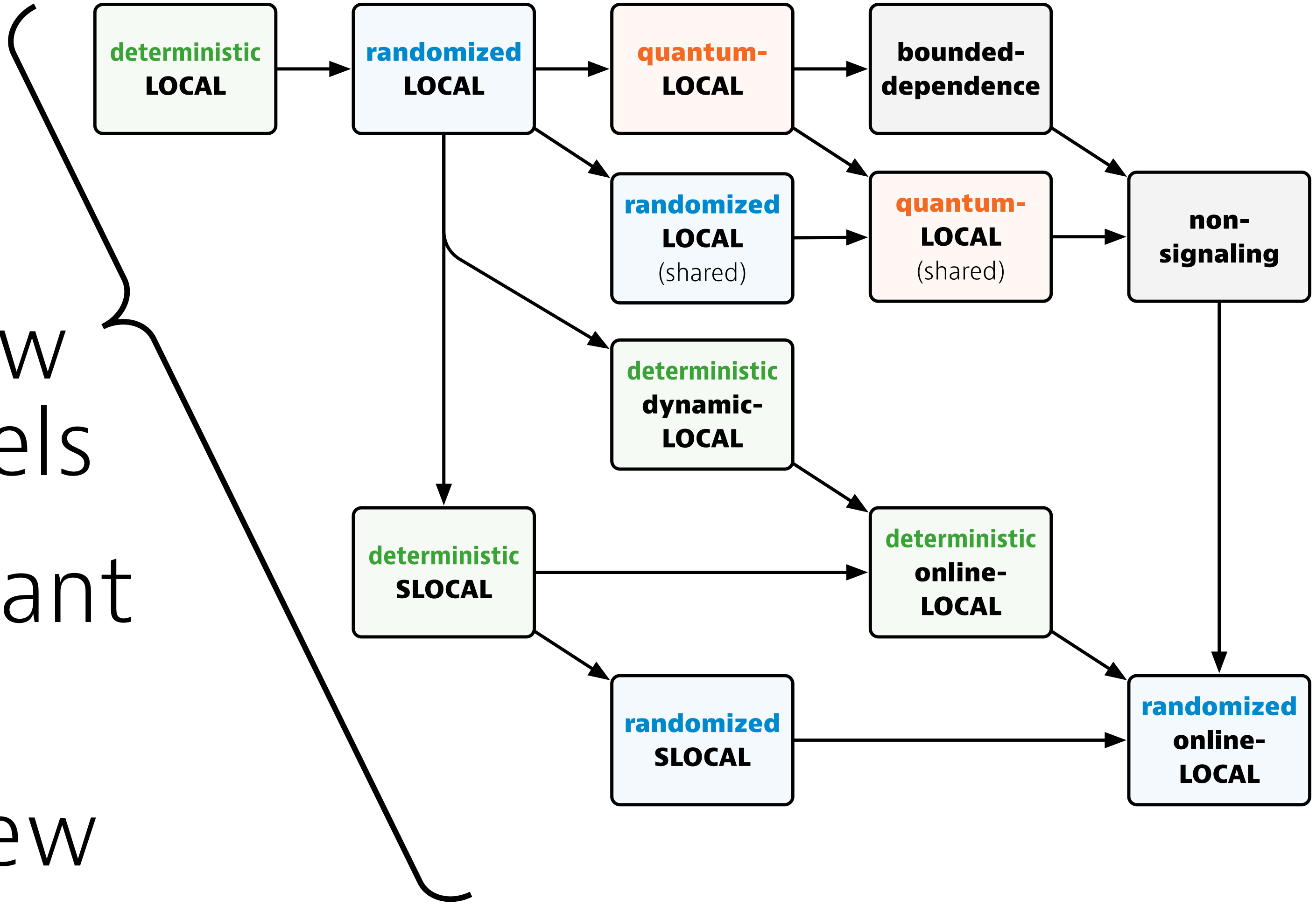
- Amirreza Akbari
- Francesco d'Amore
- Xavier Coiteux-Roy
- Navid Eslami
- Rishikesh Gajjala
- Fabian Kuhn
- François Le Gall
- Henrik Lievonen
- Darya Melnyk
- Augusto Modanese
- Shreyas Pai
- Marc-Olivier Renou
- Václav Rozhoň
- Gustav Schmid
- Joona Särkijärvi

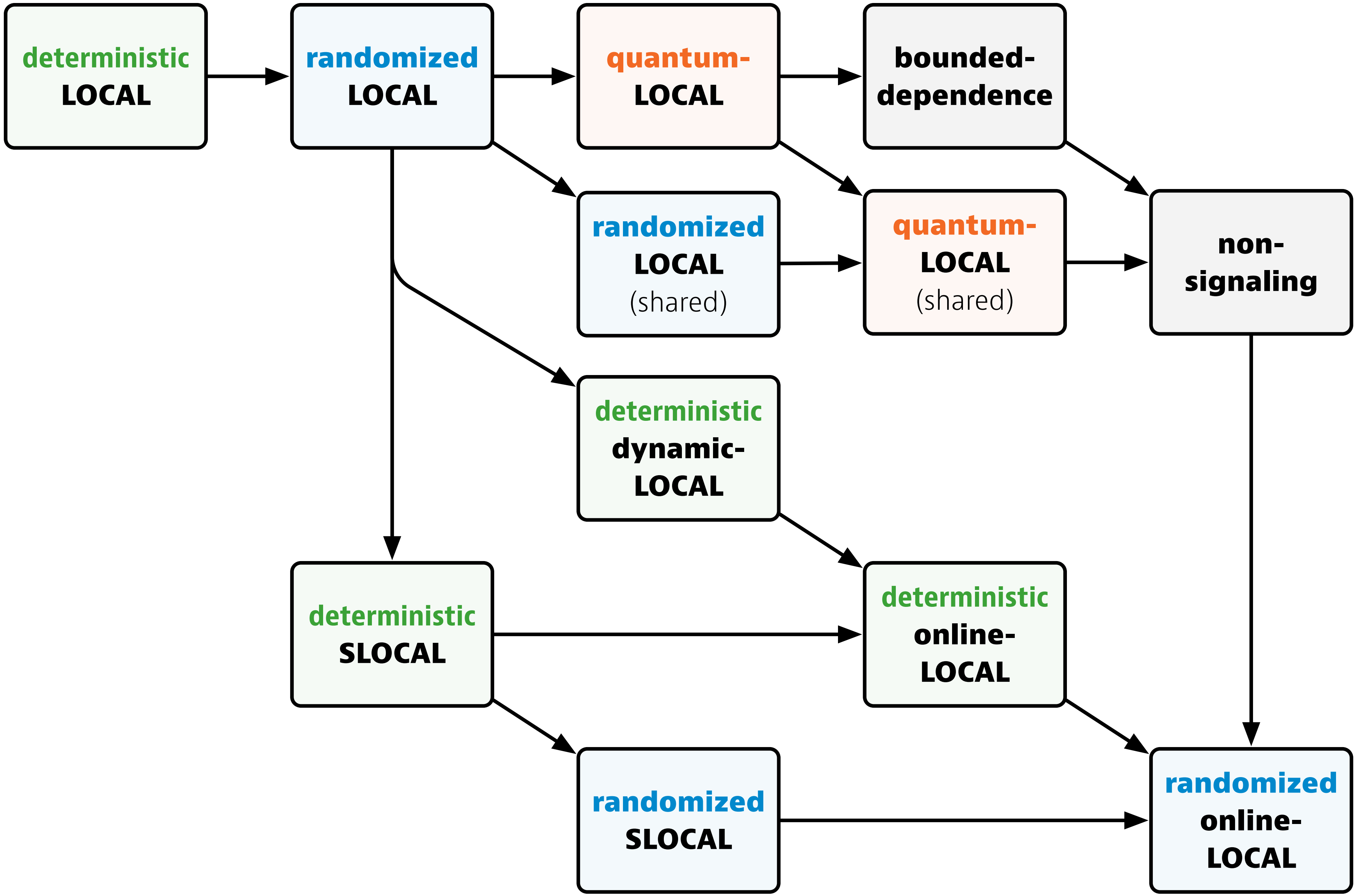


+ many results & concepts by others

Goals

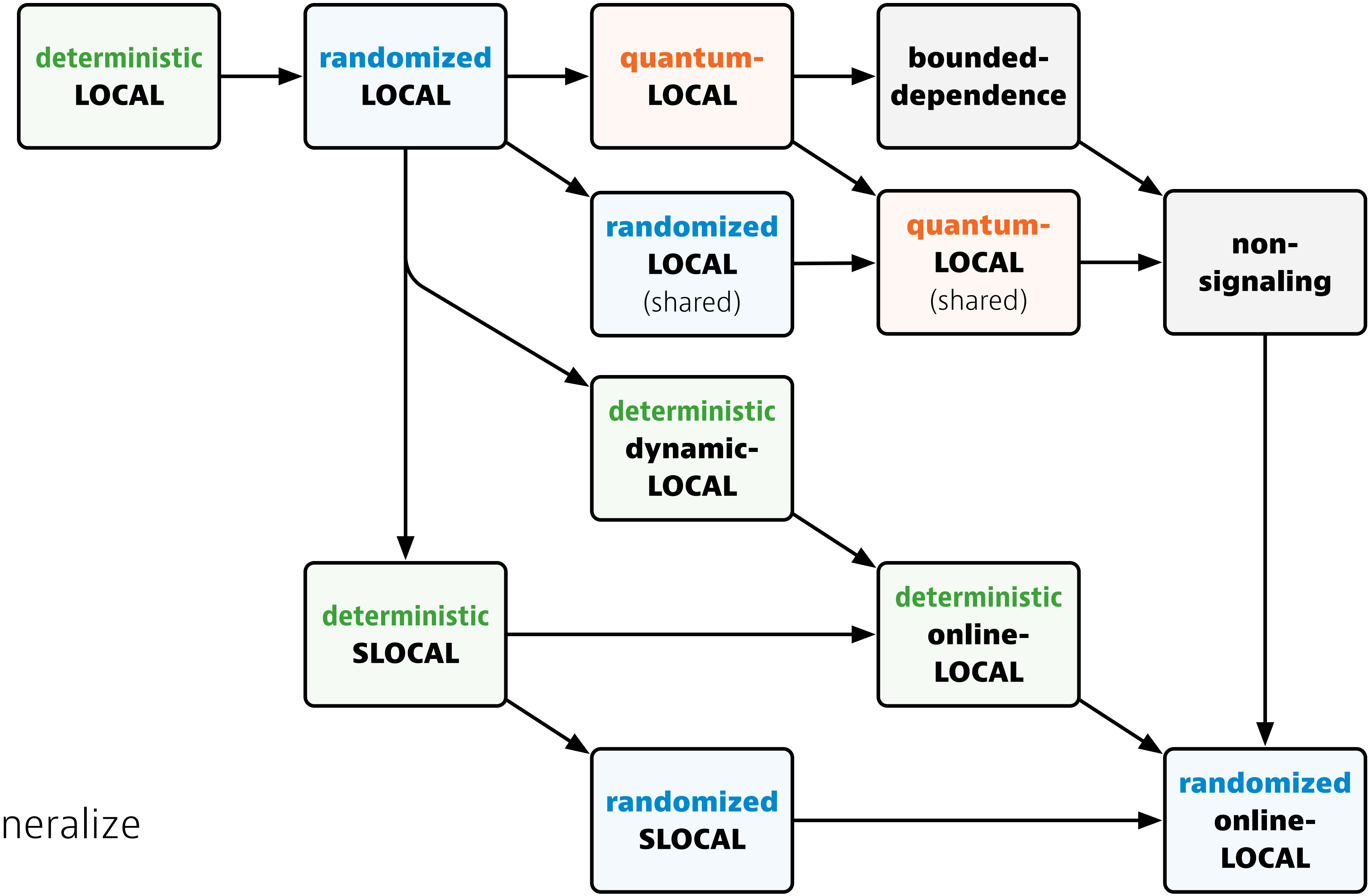
1. Informal overview of all these models
2. Why is this relevant and interesting?
3. Inspiration for new connections with e.g. measurable combinatorics?



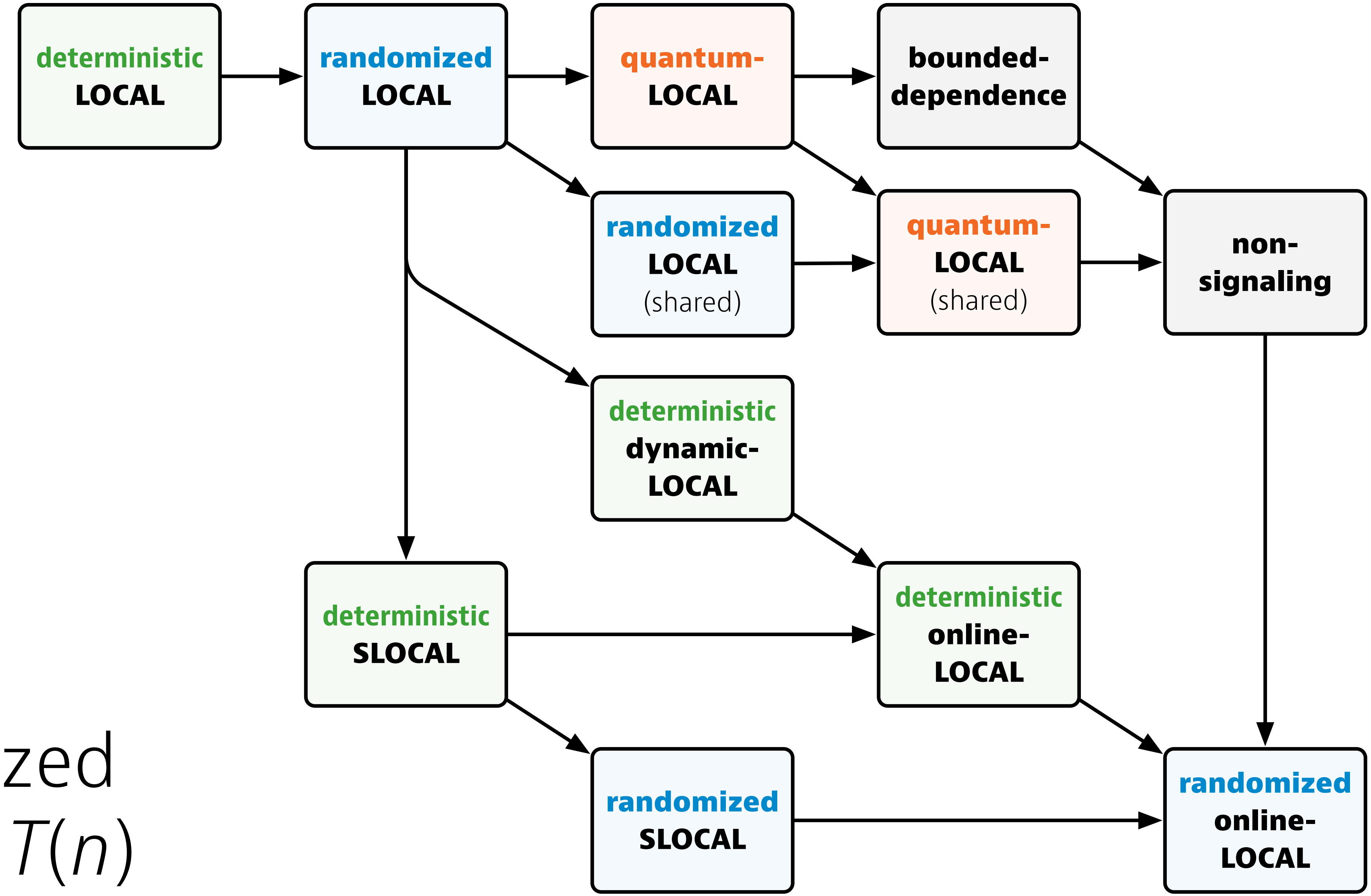

deterministic LOCAL
randomized LOCAL
quantum-LOCAL
bounded-dependence
randomized LOCAL (shared)
quantum-LOCAL (shared)
non-signaling
deterministic dynamic-LOCAL
deterministic SLOCAL
deterministic online-LOCAL
randomized SLOCAL
randomized online-LOCAL

Context:
locally checkable labelings (LCL)

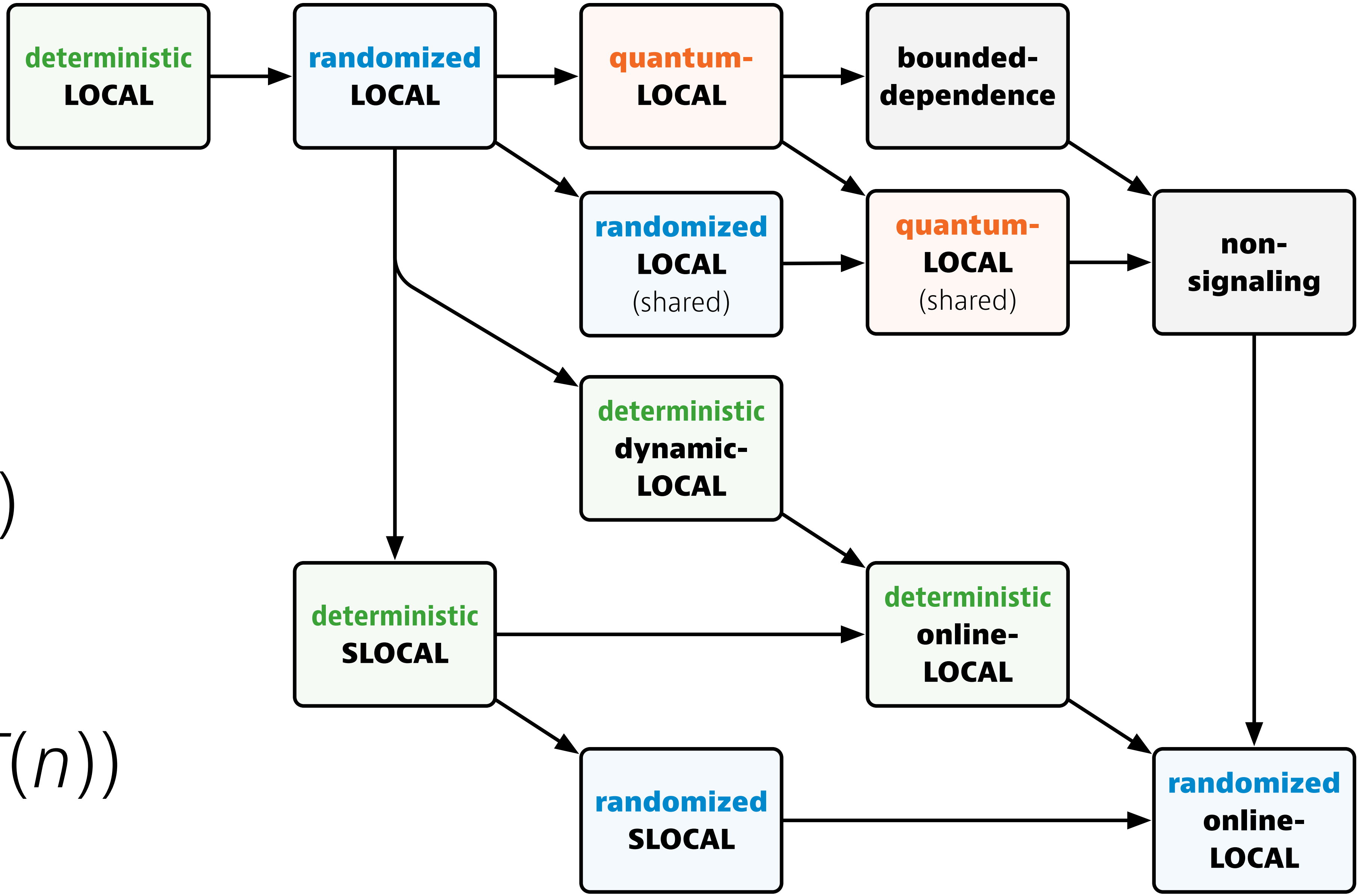
but many things generalize far beyond LCLs

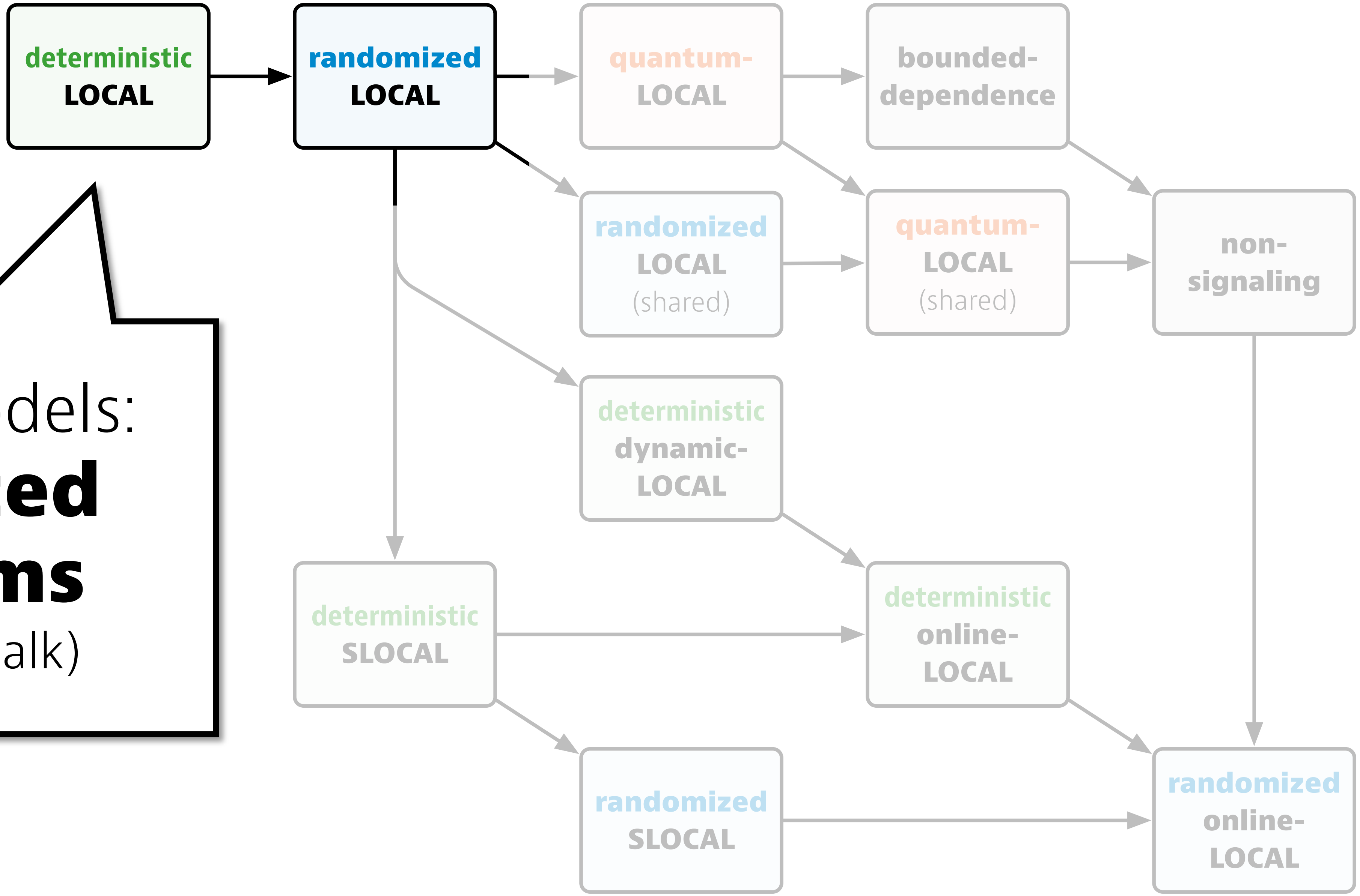


Each box:
model of computing parameterized by locality $T(n)$



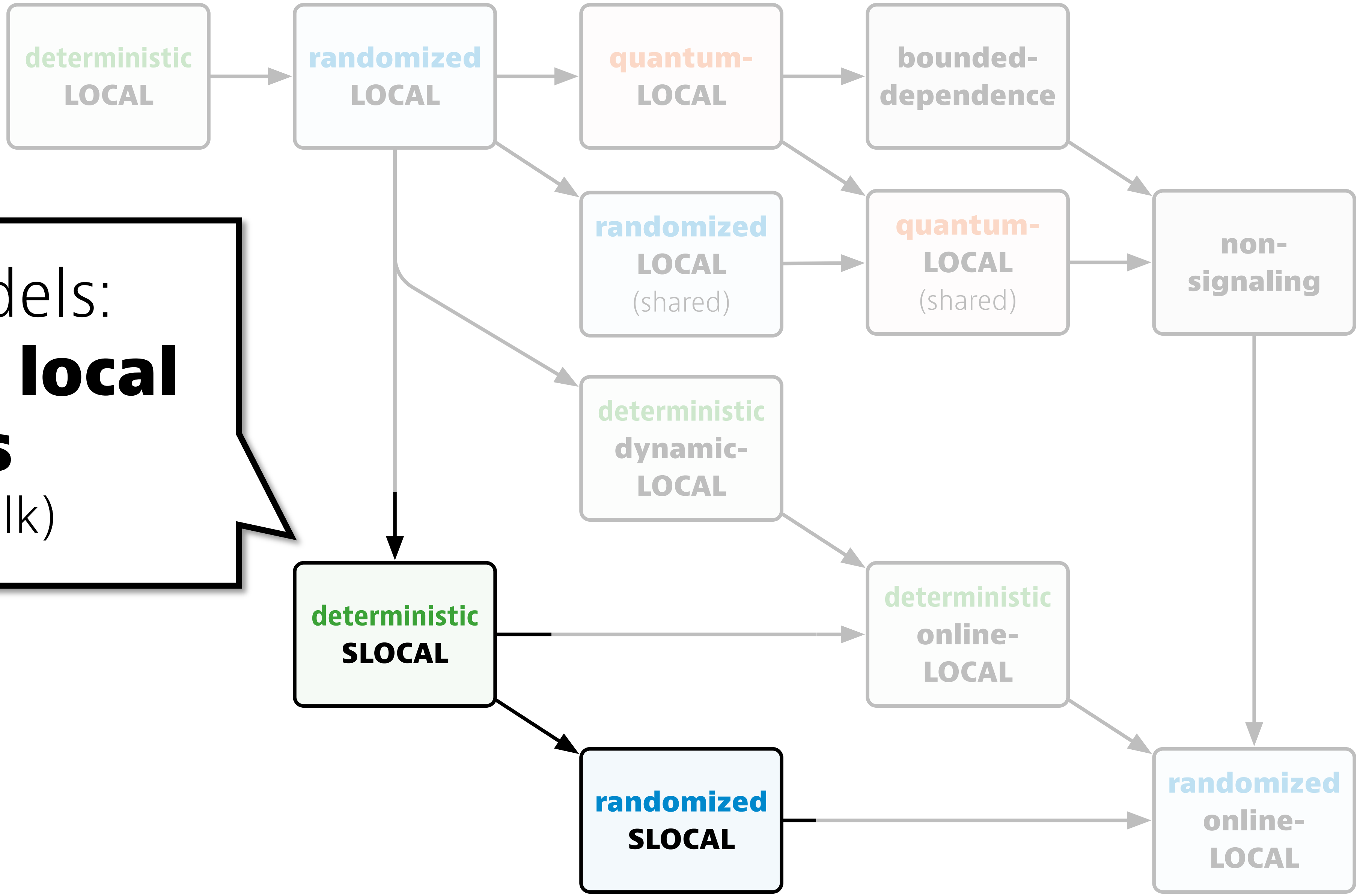
A* → *B:
locality $T(n)$
in model A
implies
locality $O(T(n))$
in model B

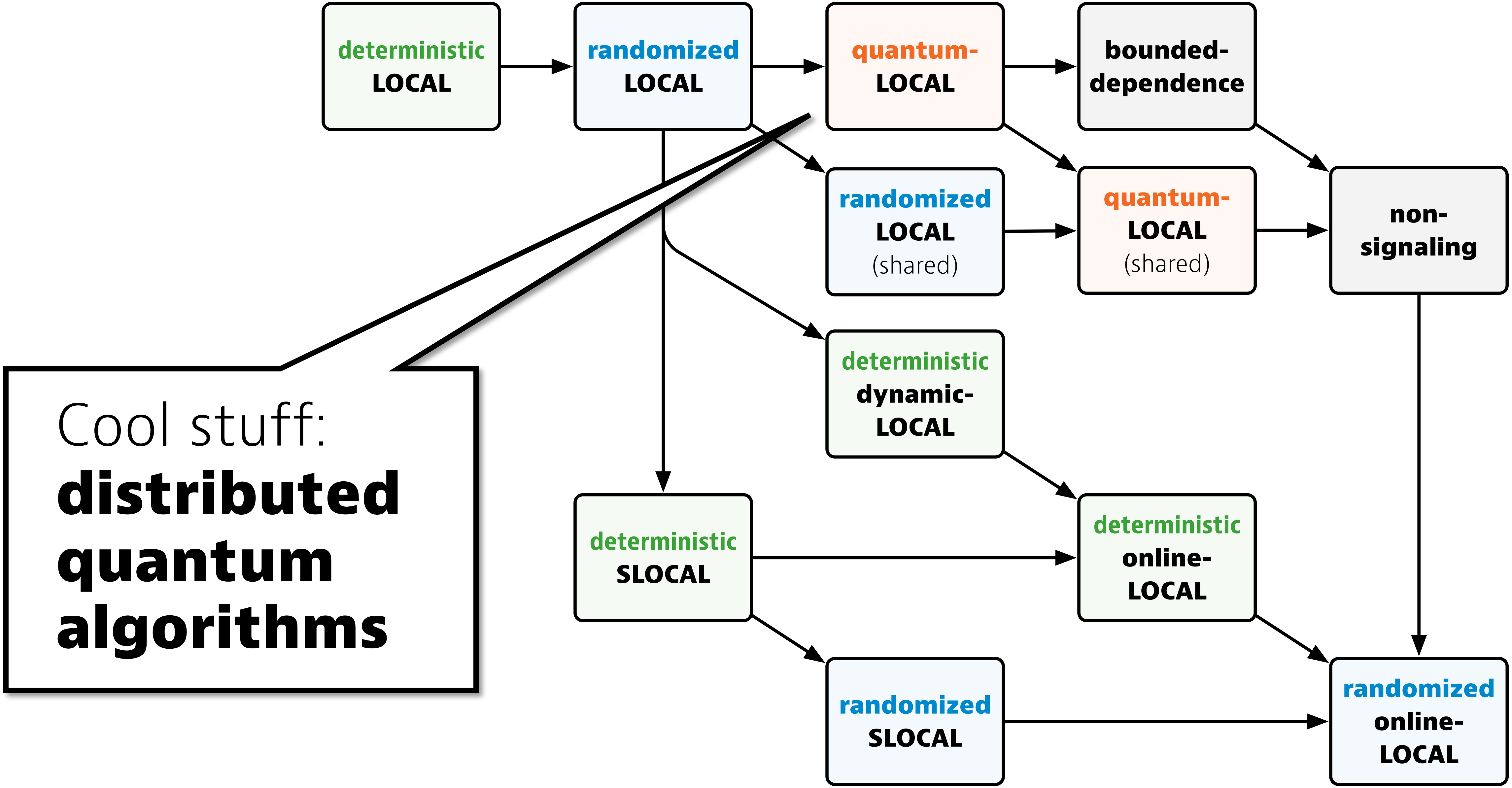




Familiar models:
distributed algorithms
(recall Vašek's talk)

Familiar models:
sequential local algorithms
(recall Vašek’s talk)

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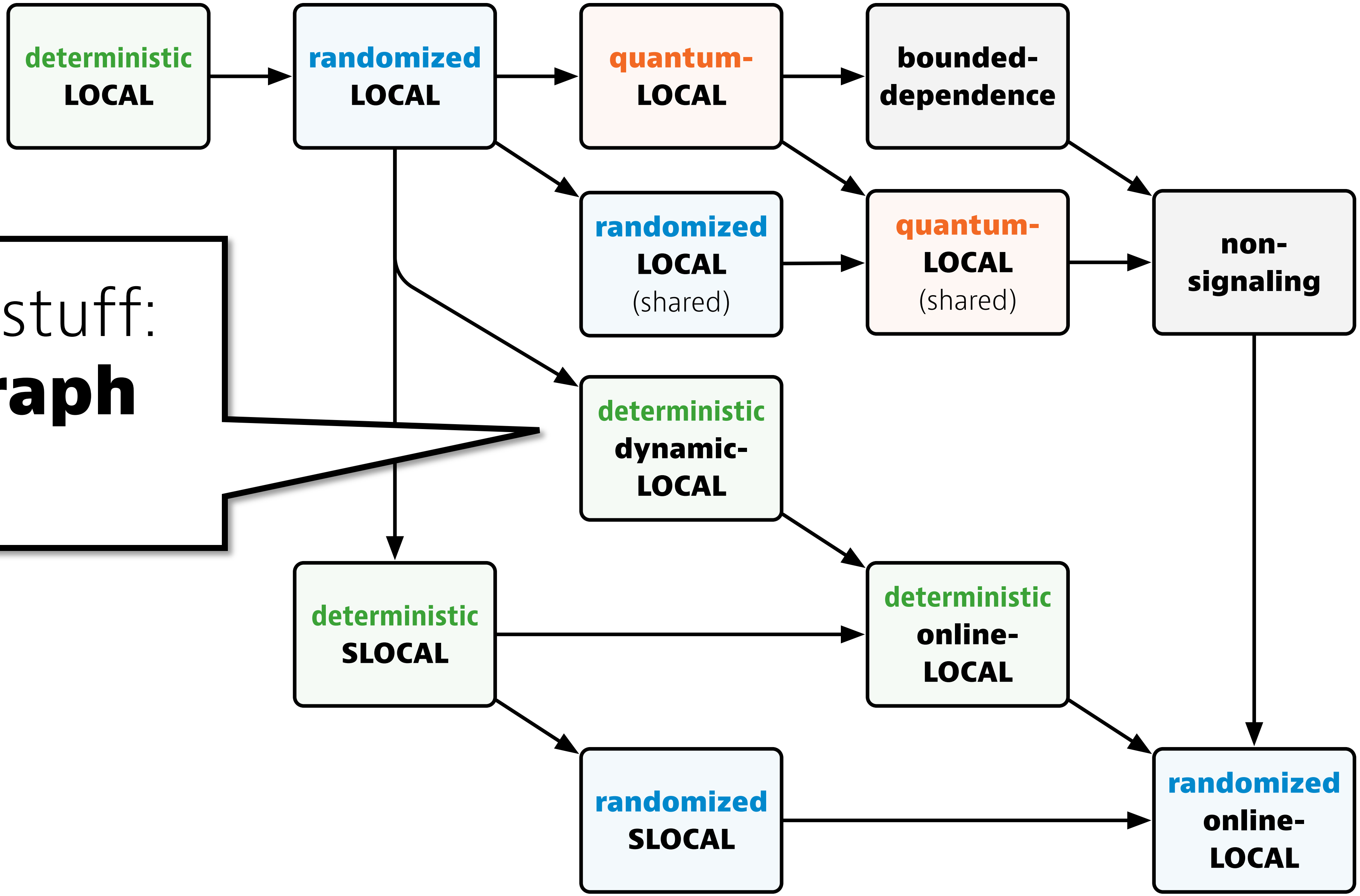
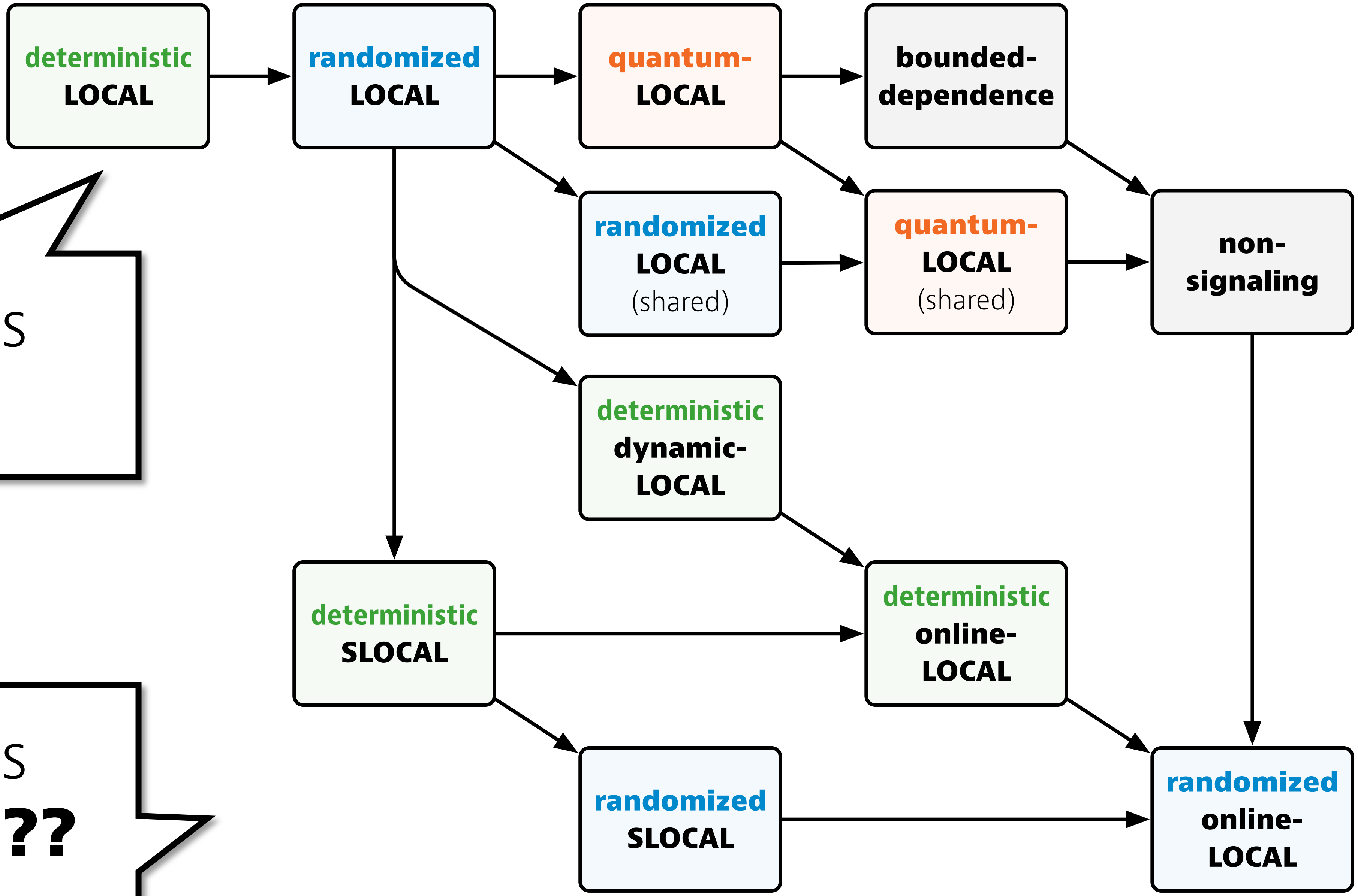
Cool stuff:
distributed quantum algorithms

Unexpected stuff:

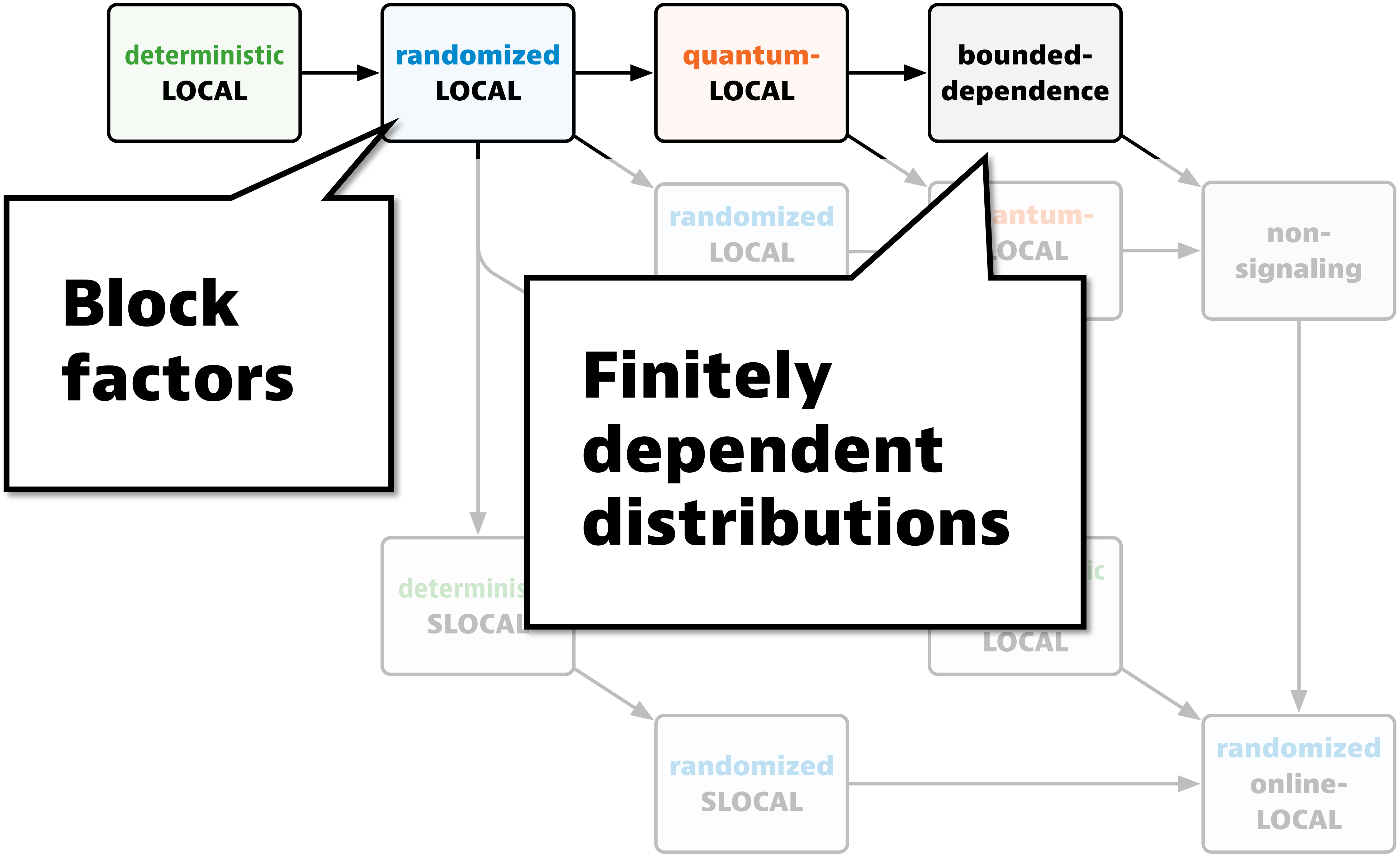
dynamic graph algorithms



Connections with **Borel**



Connections with **Borel??**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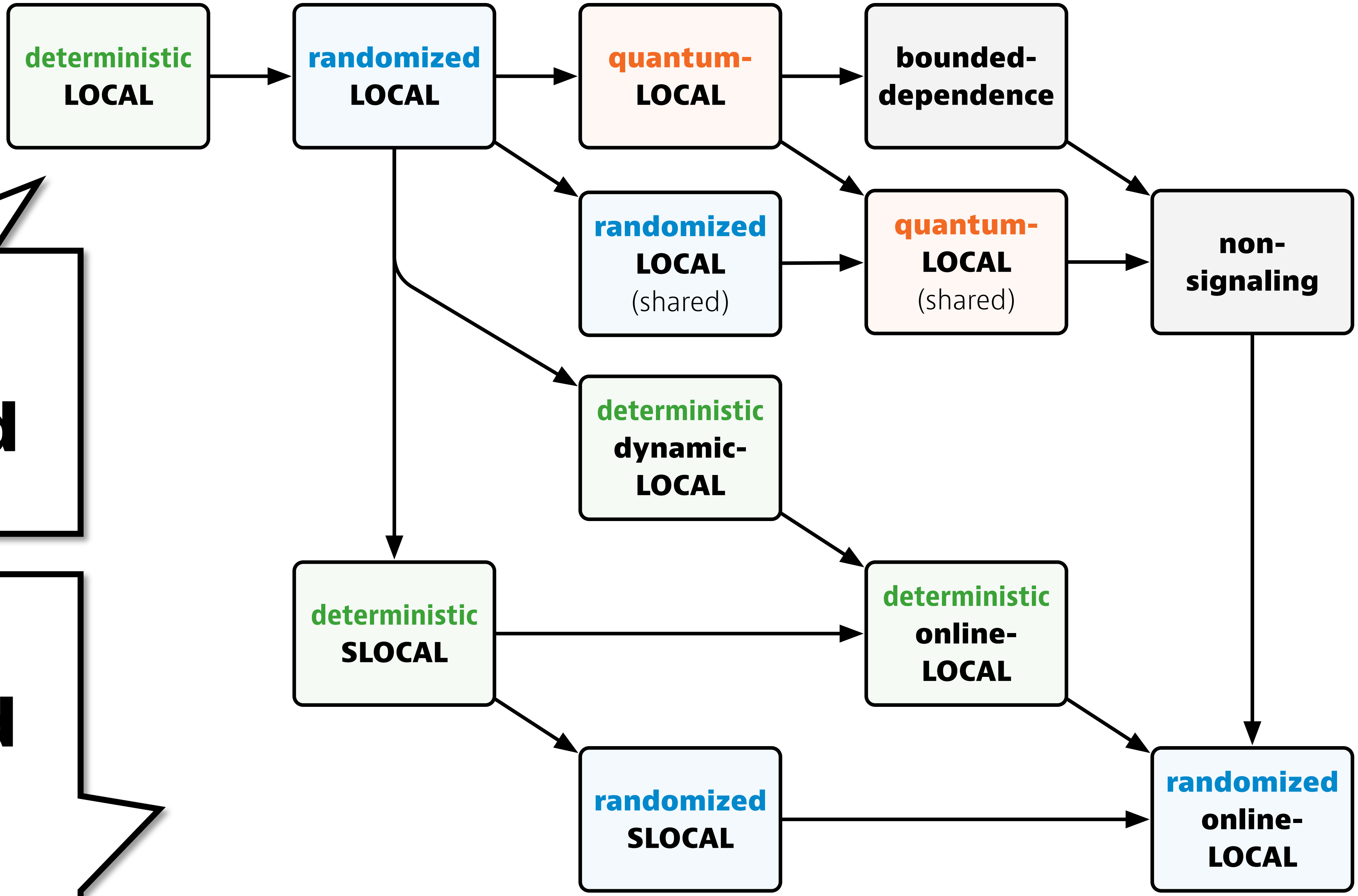
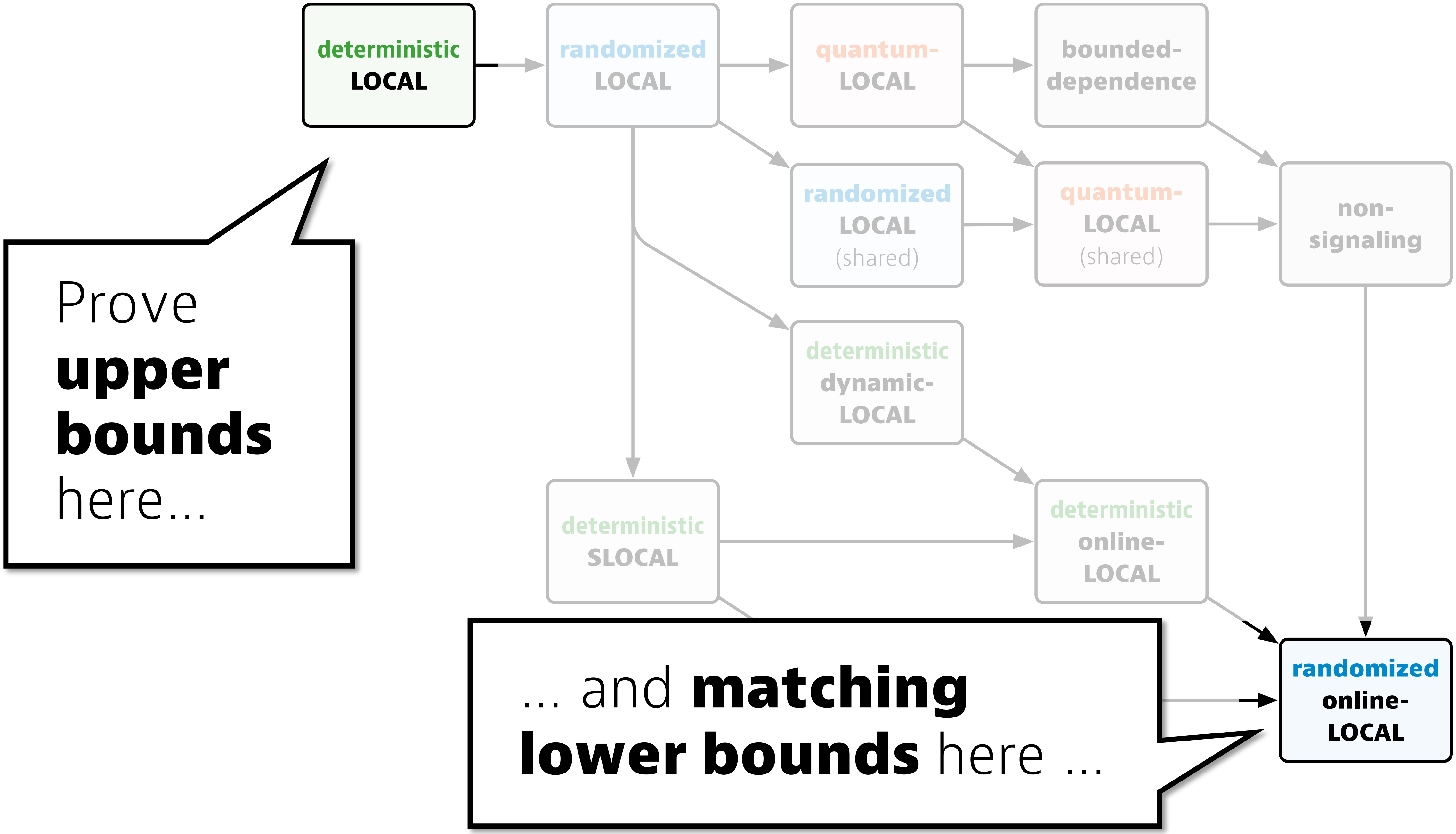
Block factors
Finitely dependent distributions

This is **well understood**

Could we **understand** also all the rest??

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deterministic SLOCAL
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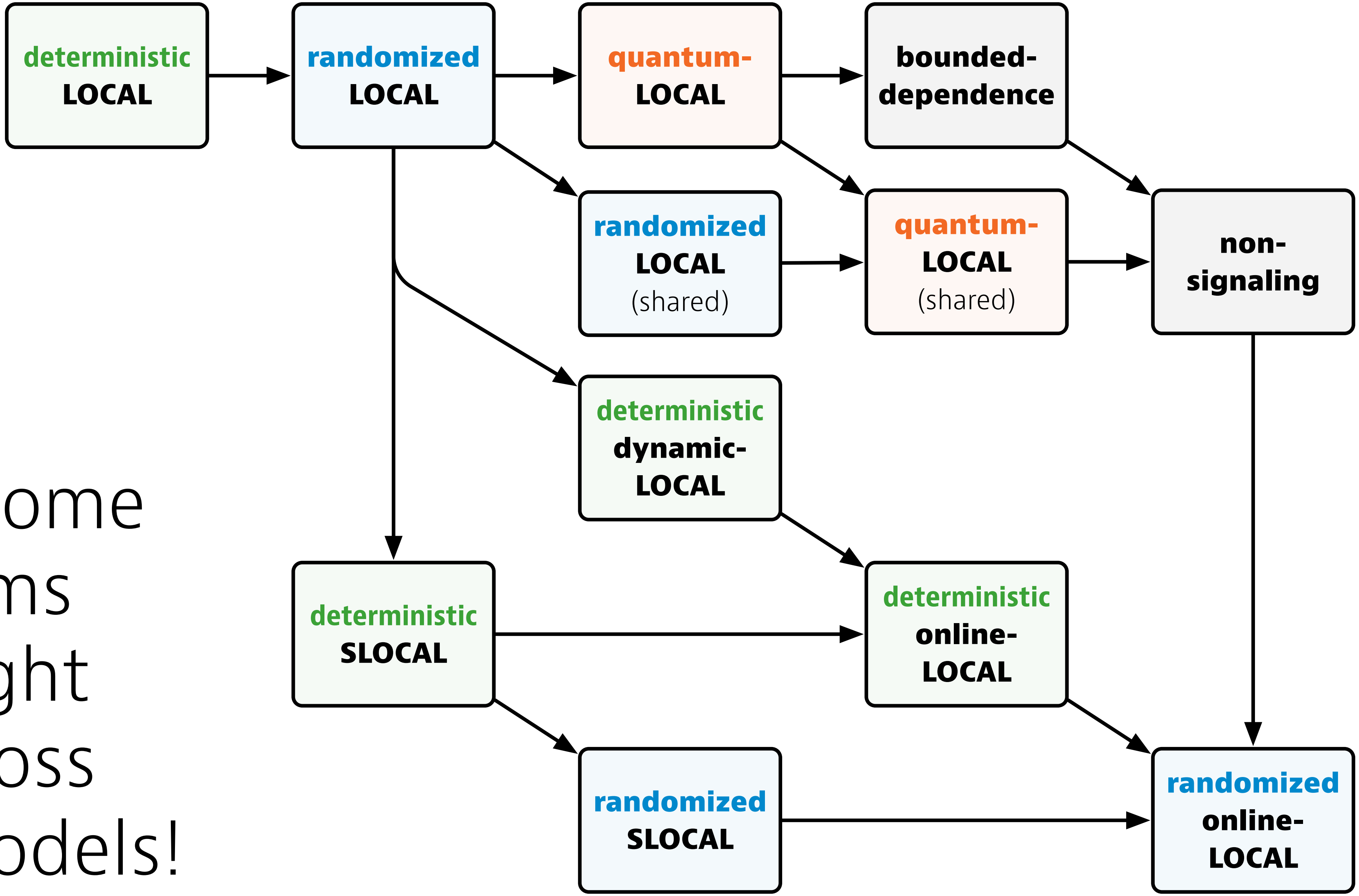
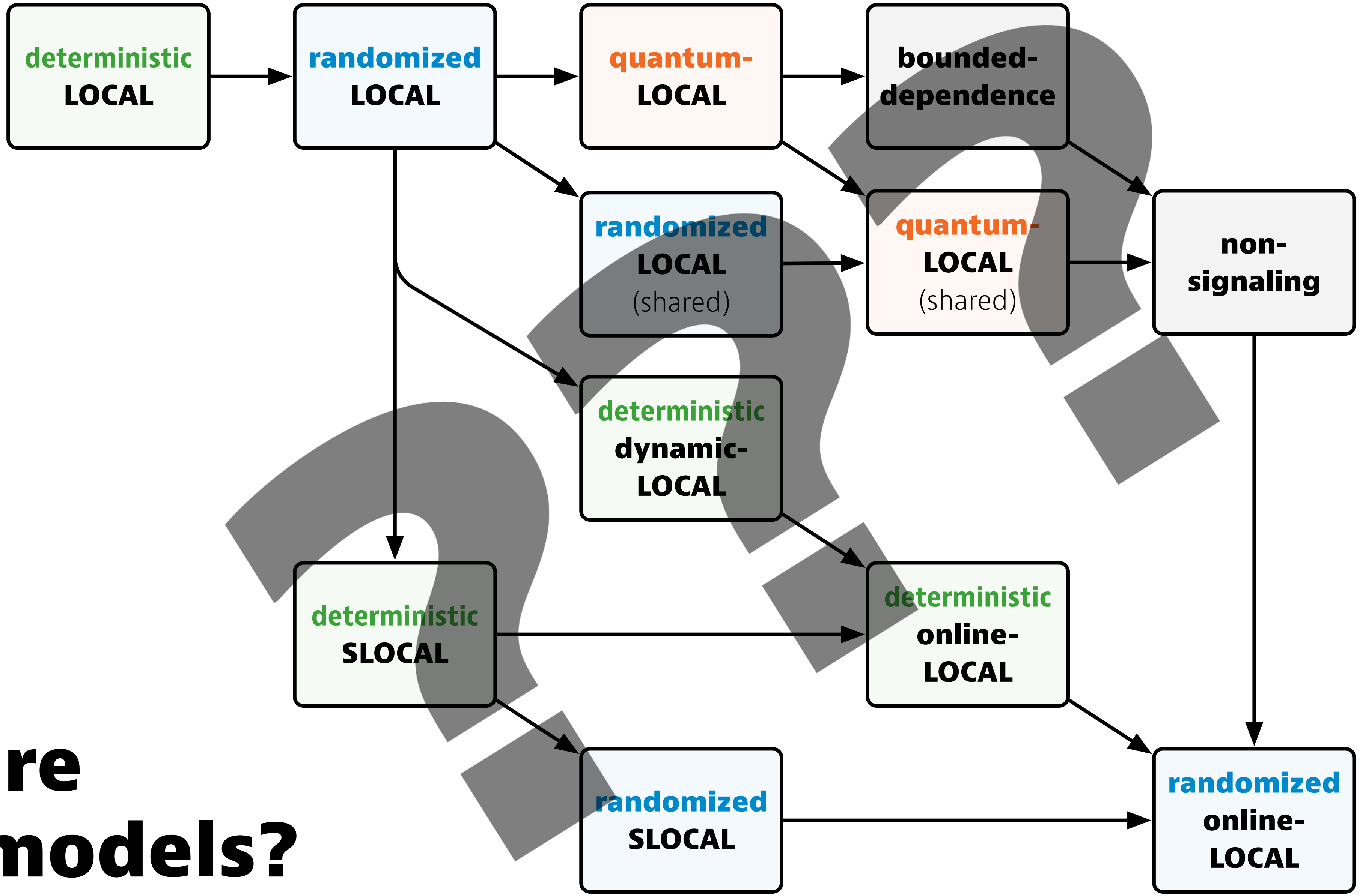
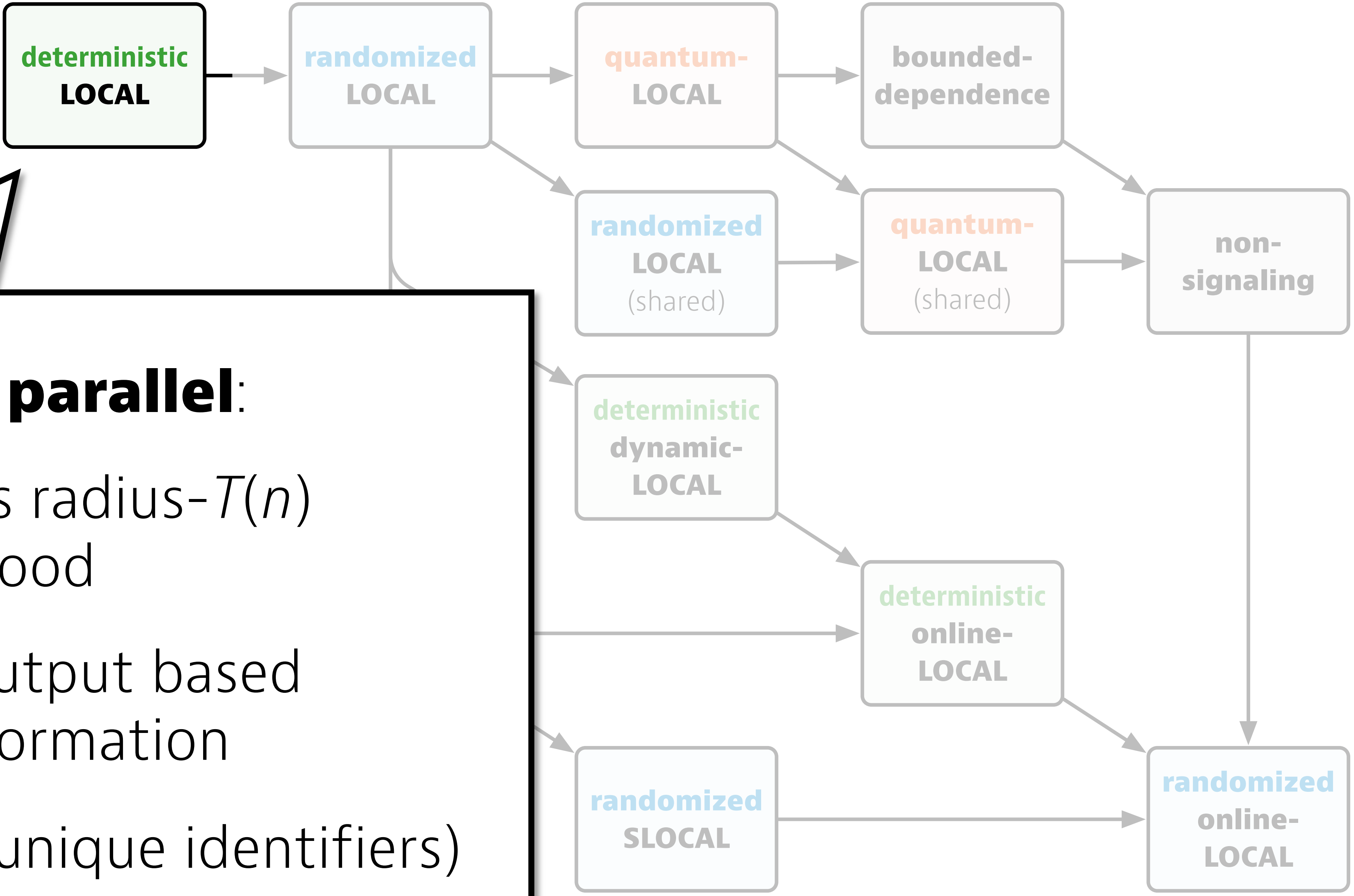
randomized online-LOCAL
Prove **upper bounds** here...
... and **matching lower bounds** here ...



… and for some LCL problems we have tight bounds across all these models!



So what are all these models?



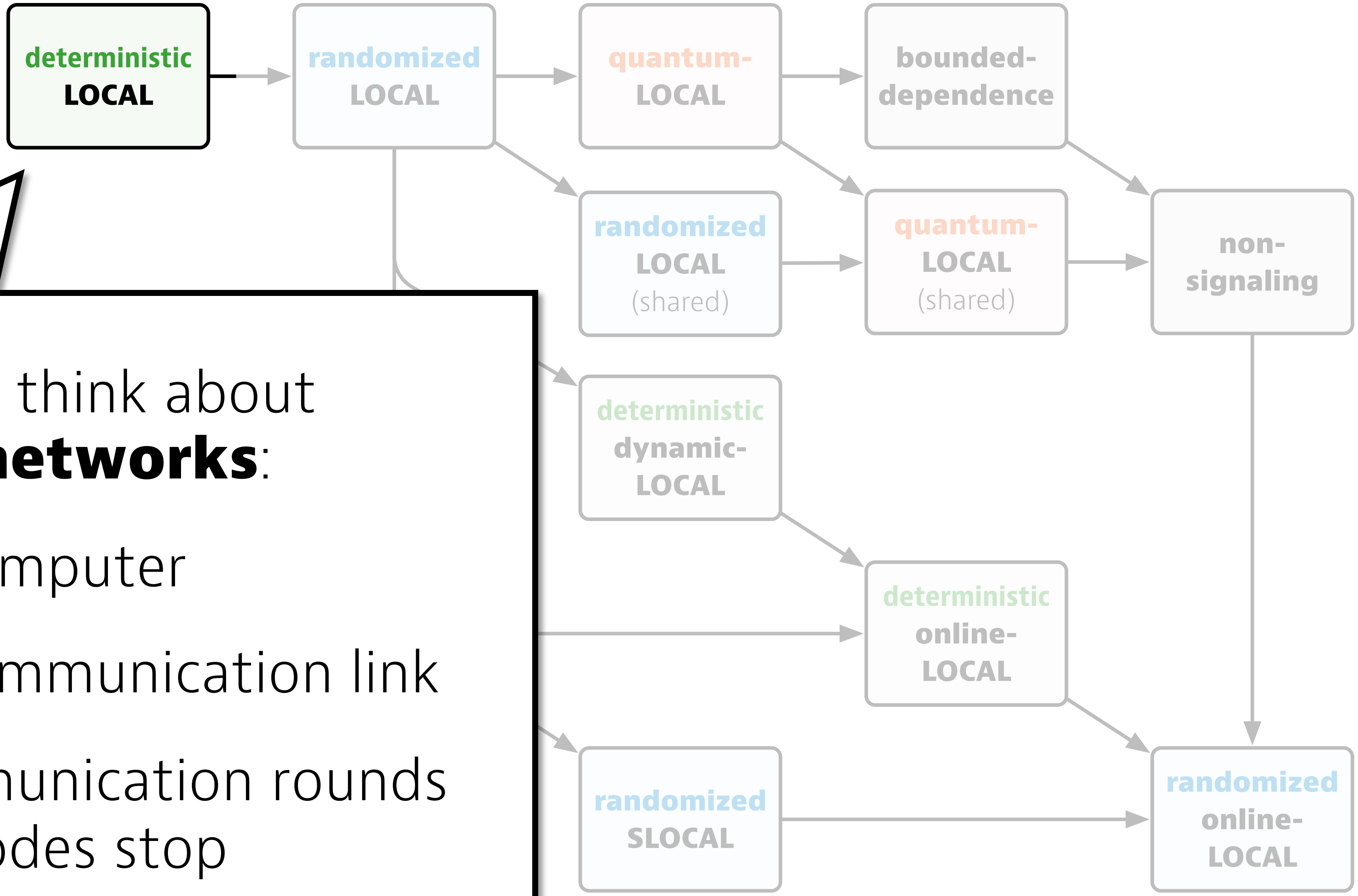
Each node in **parallel**:

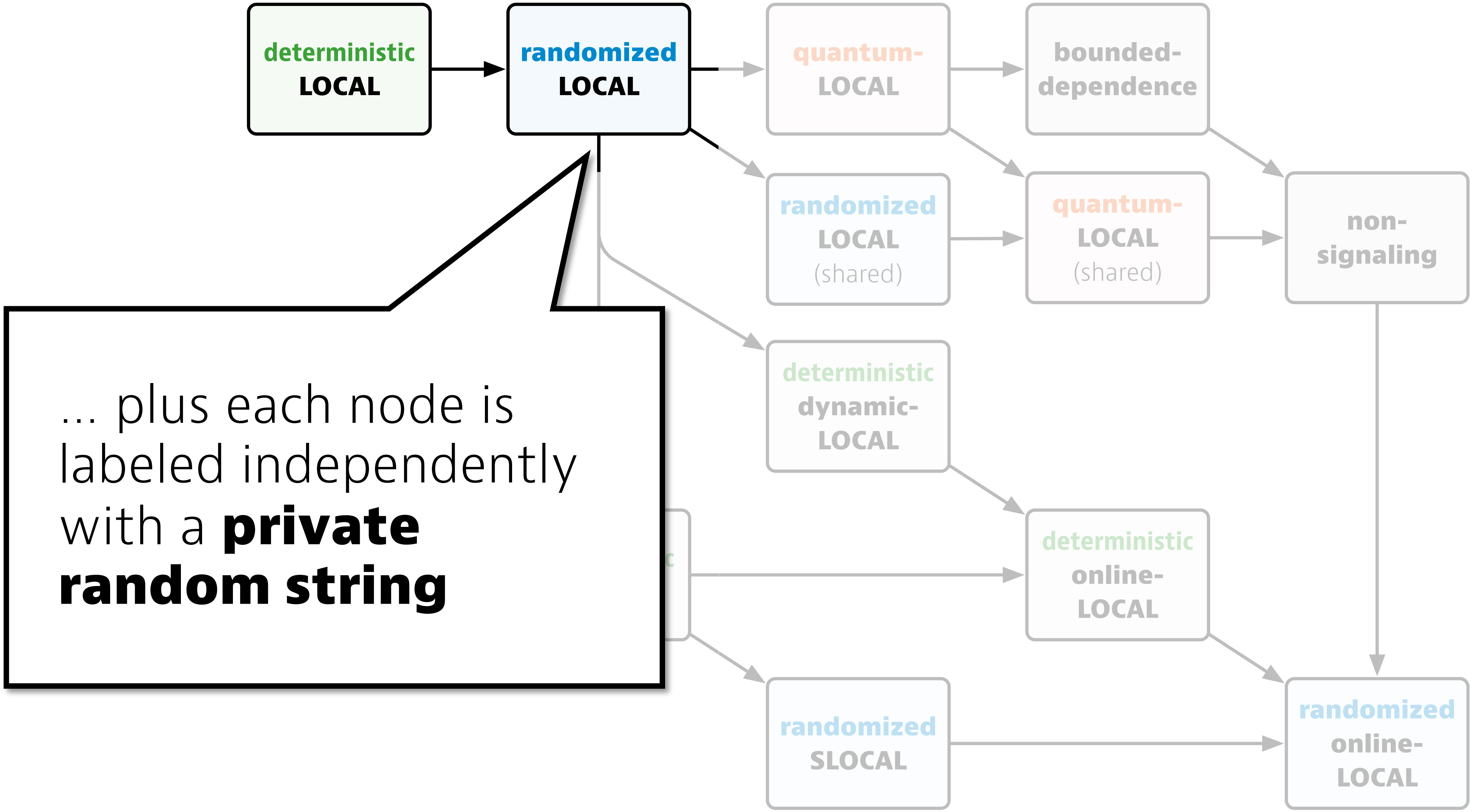
- looks at its radius-$T(n)$ neighborhood
- picks its output based on this information

(nodes have unique identifiers)

… or you can think about **computer networks**:

- node = computer
- edge = communication link
- $T(n)$ communication rounds until all nodes stop

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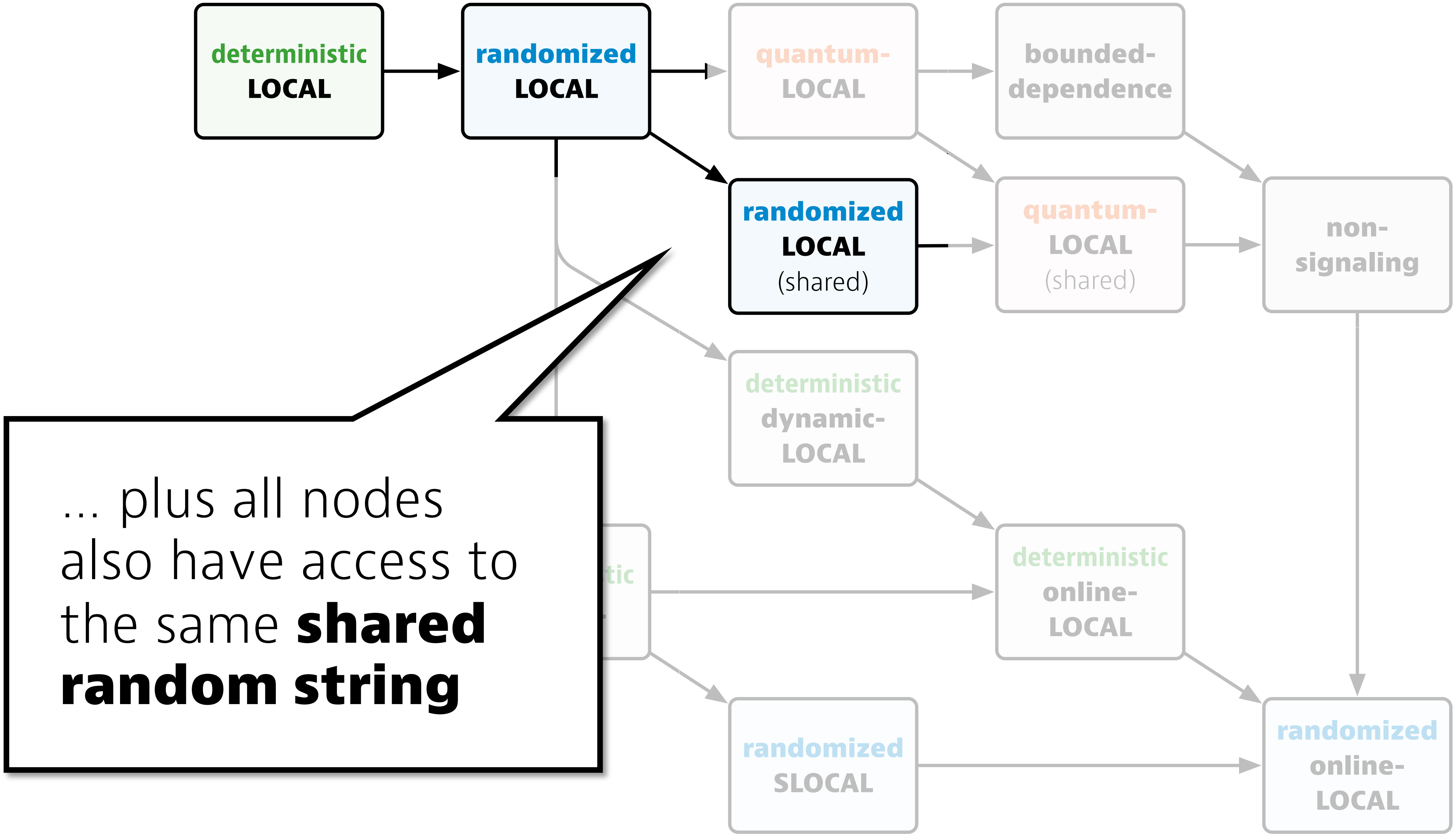
deterministic online-LOCAL
randomized SLOCAL
randomized online-LOCAL
… plus each node is labeled independently with a **private random string**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non-signaling
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deterministic online-LOCAL
randomized SLOCAL
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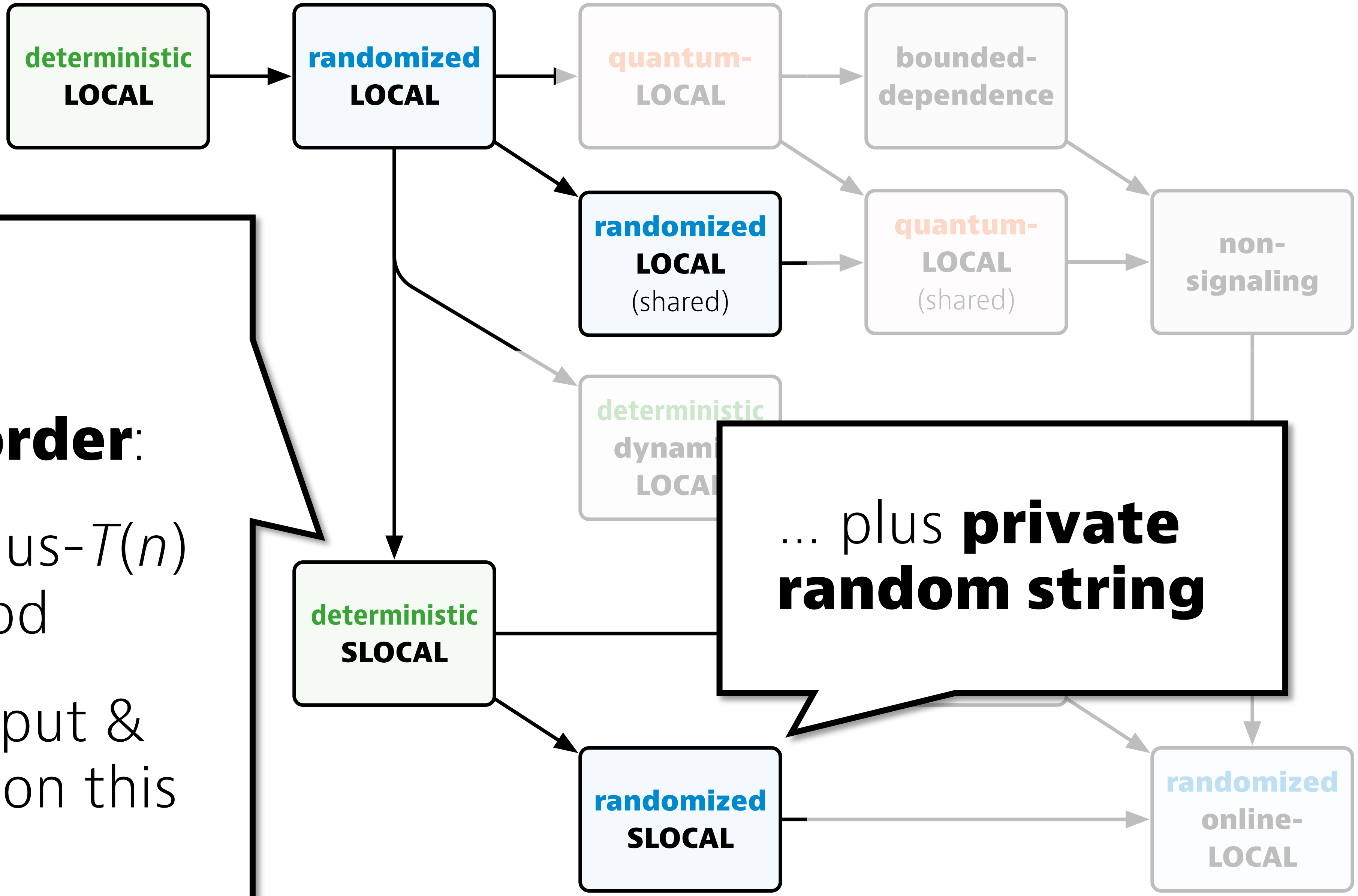
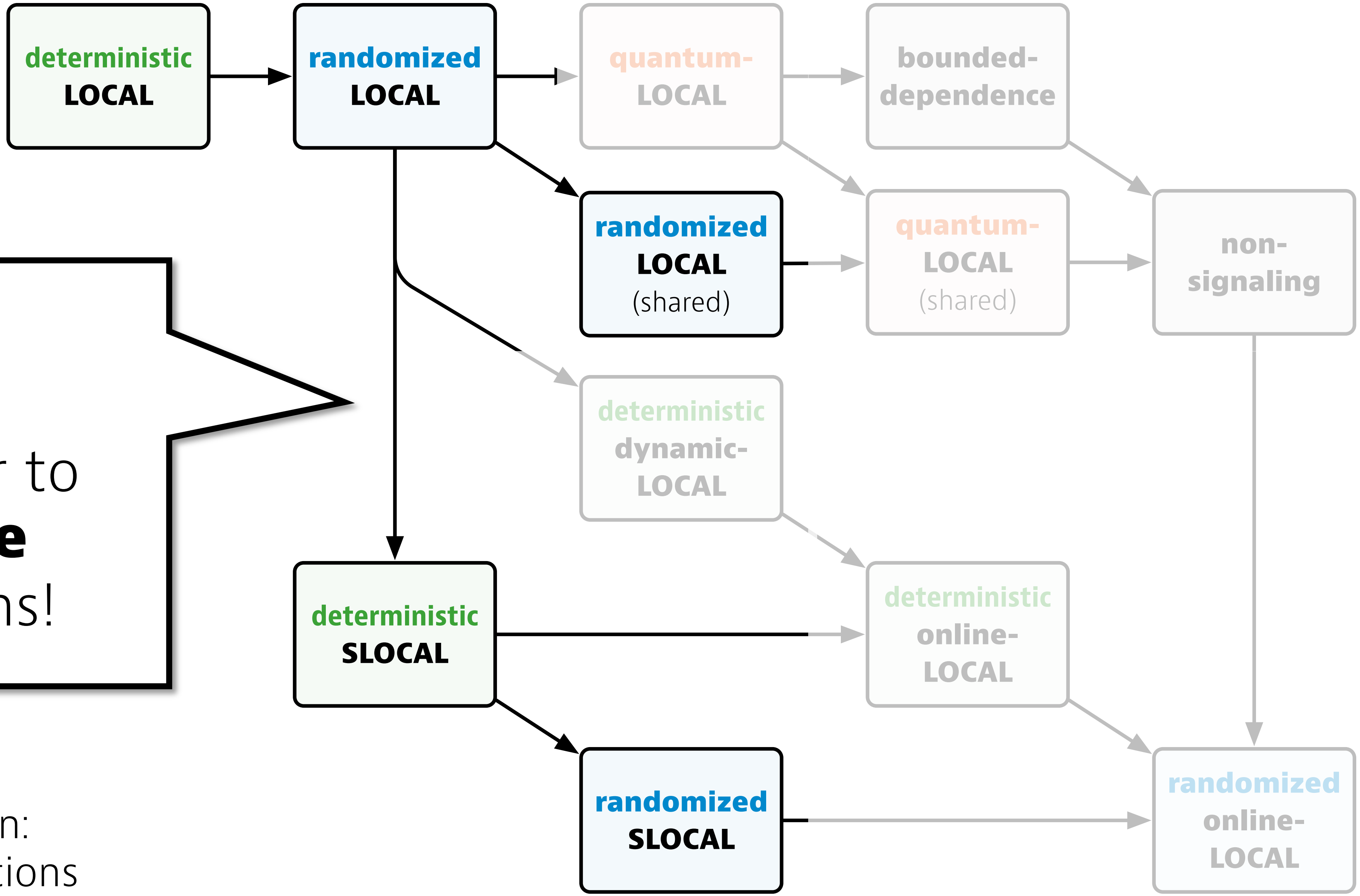
… plus all nodes also have access to the same **shared random string**

Each node in a **sequential, adversarial order**:

- looks at radius-$T(n)$ neighborhood
- picks its output & state based on this information





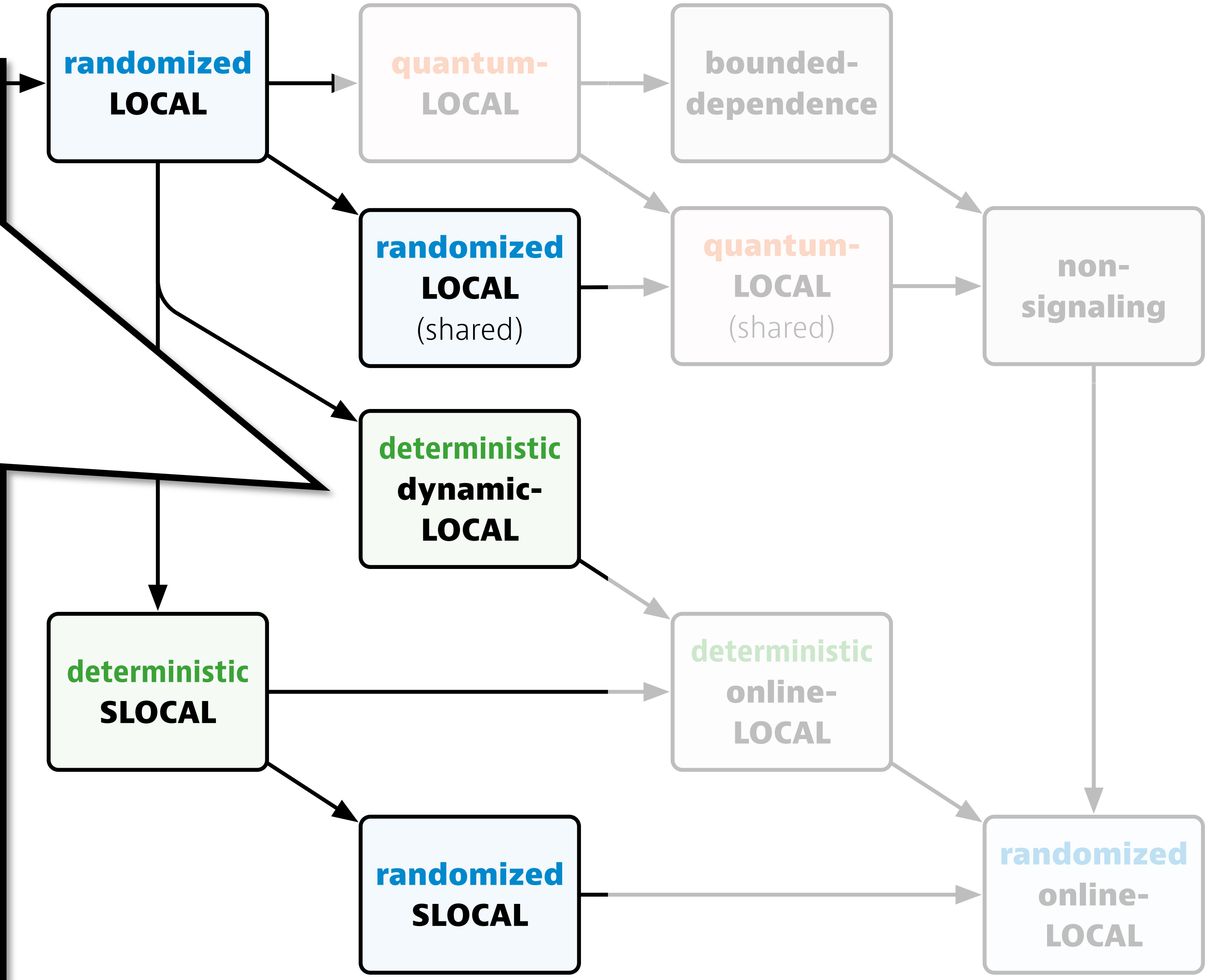
Sequential ordering gives enough power to **derandomize** local algorithms!

Ghaffari, Harris, Kuhn:
conditional expectations

Adversary adds nodes and edges one by one

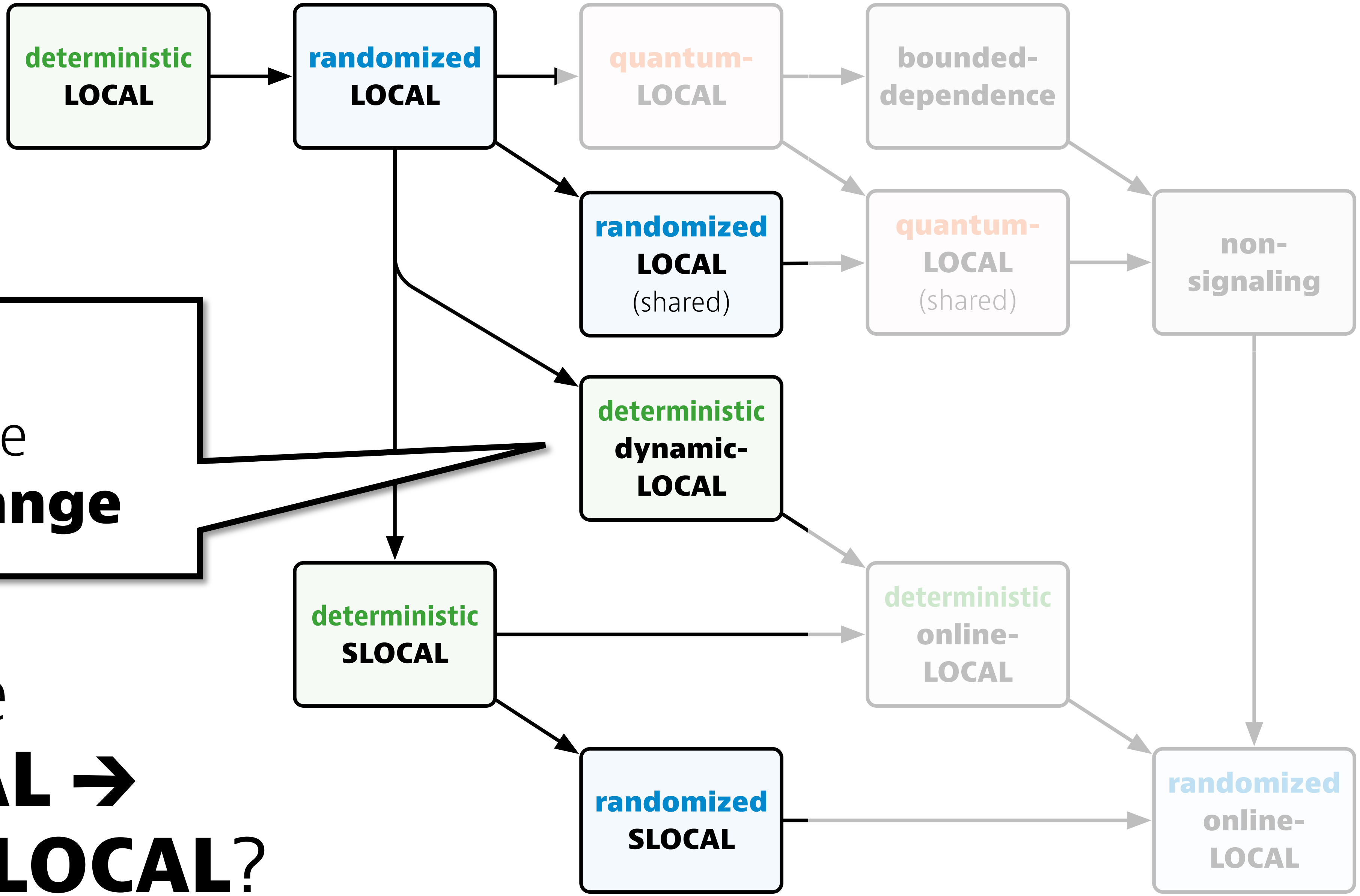
- We can **see everything**
- Have to maintain valid solution

We can **change** our output only within distance $T(n)$ from a point of change

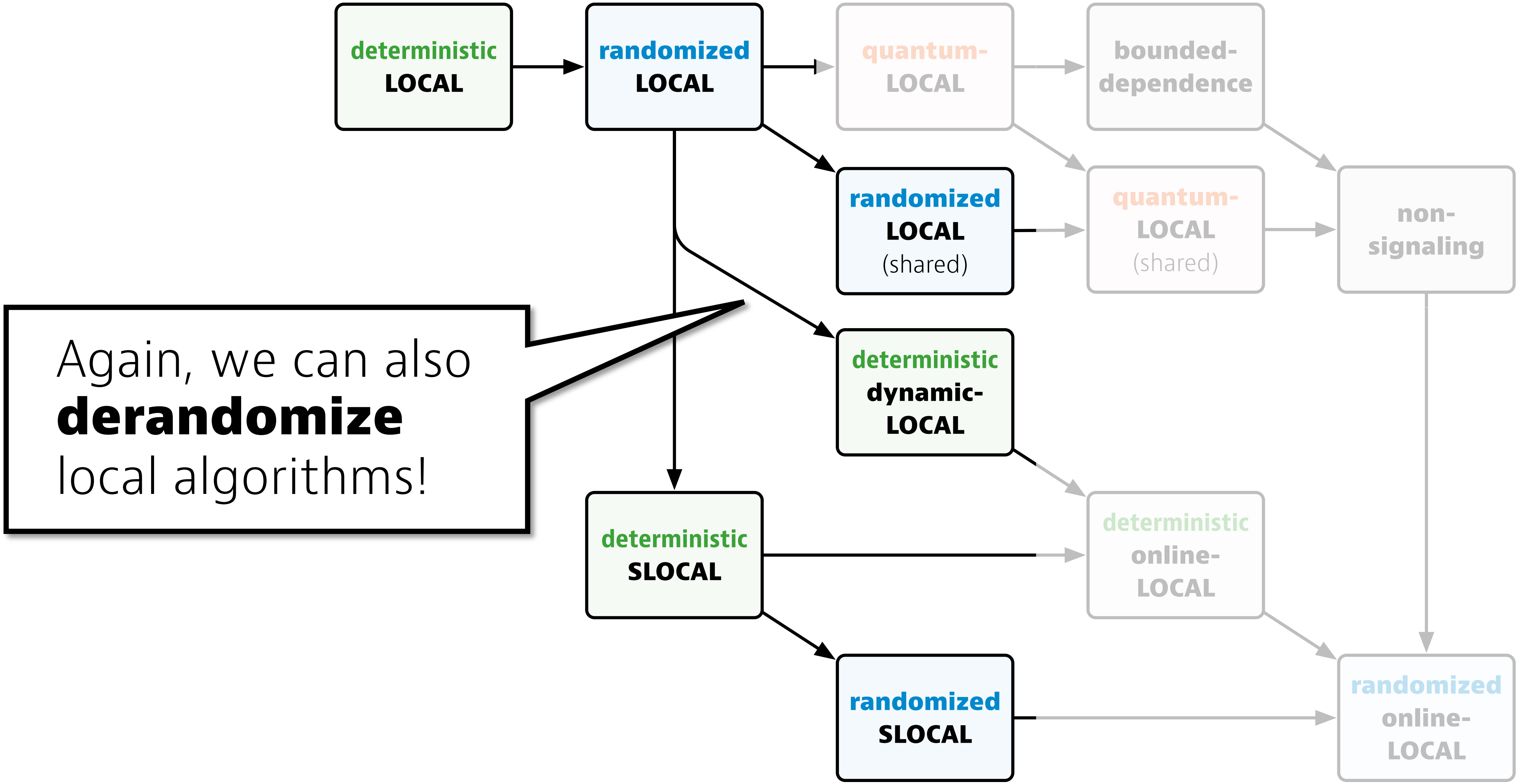


Limited what we can **see**

Limited what we can **change**



Why do we have **LOCAL → dynamic-LOCAL**?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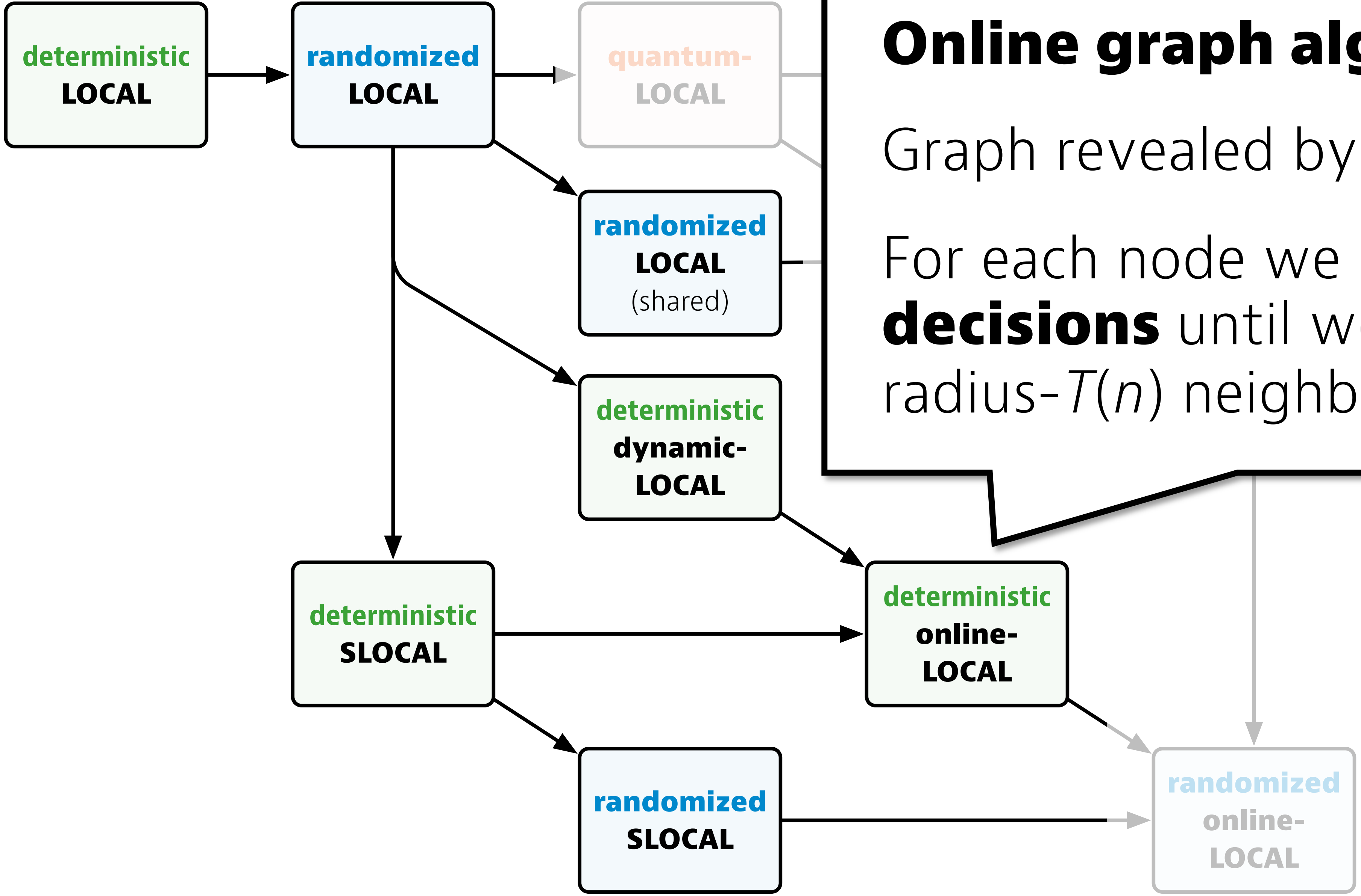
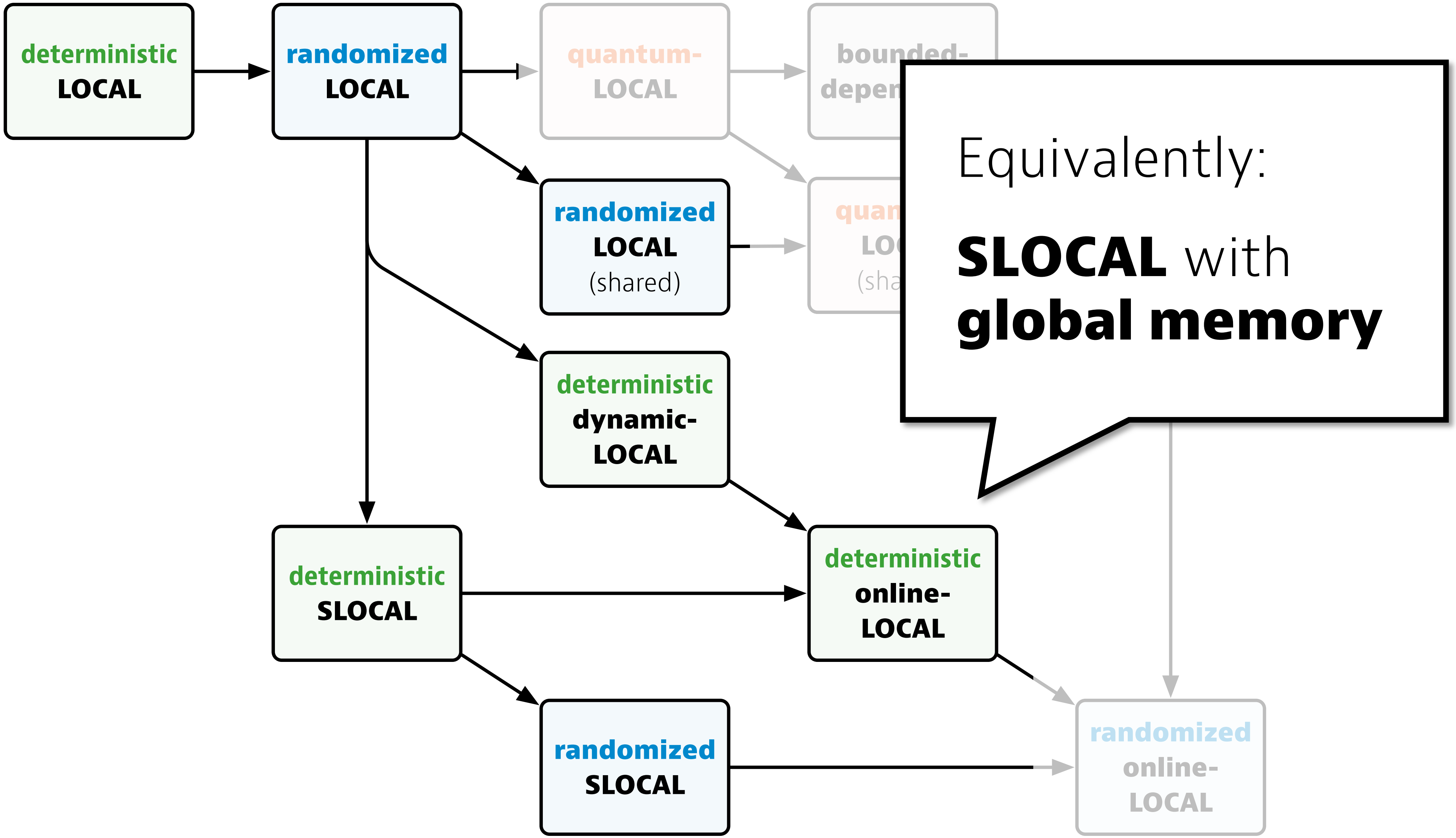
Again, we can also **derandomize** local algorithms!

Online graph algorithms

Graph revealed by adversary

For each node we can **delay decisions** until we see its radius-$T(n)$ neighborhood

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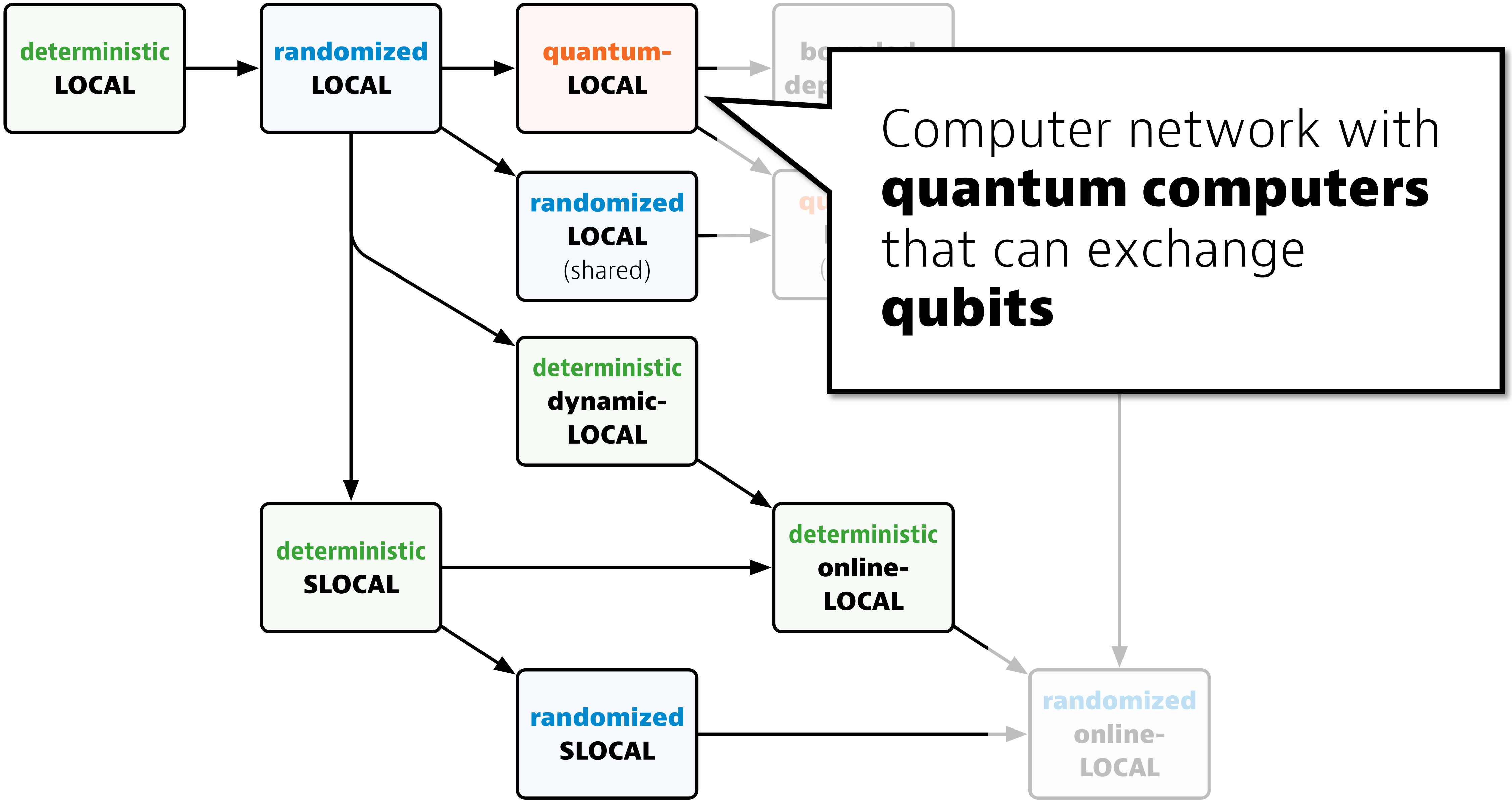
Equivalently:
SLOCAL with global memory



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randomized LOCAL (shared)
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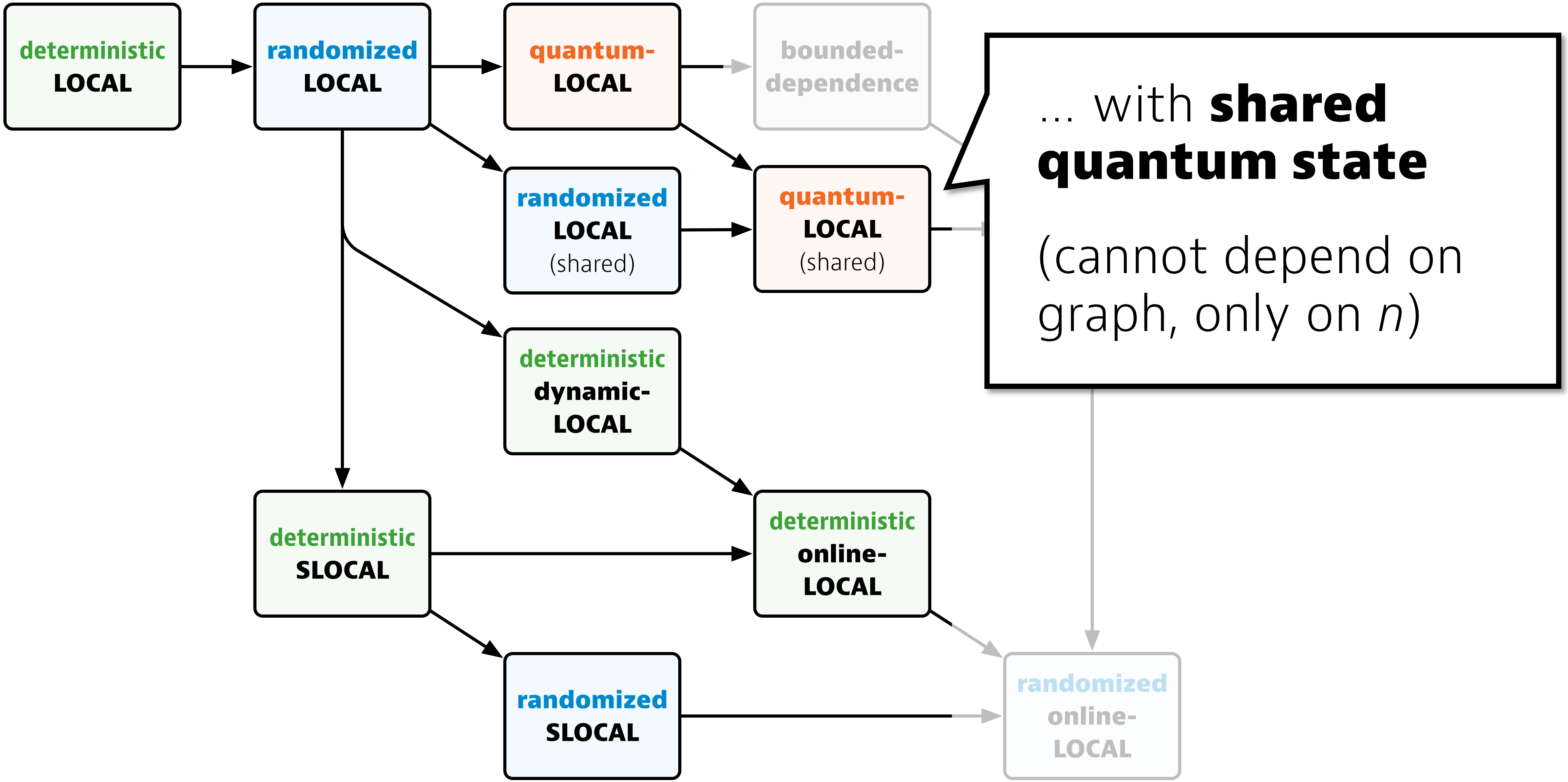
randomized SLOCAL
randomized online-LOCAL
Computer network with **quantum computers** that can exchange **qubits**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quantum-LOCAL (shared)
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deterministic online-LOCAL
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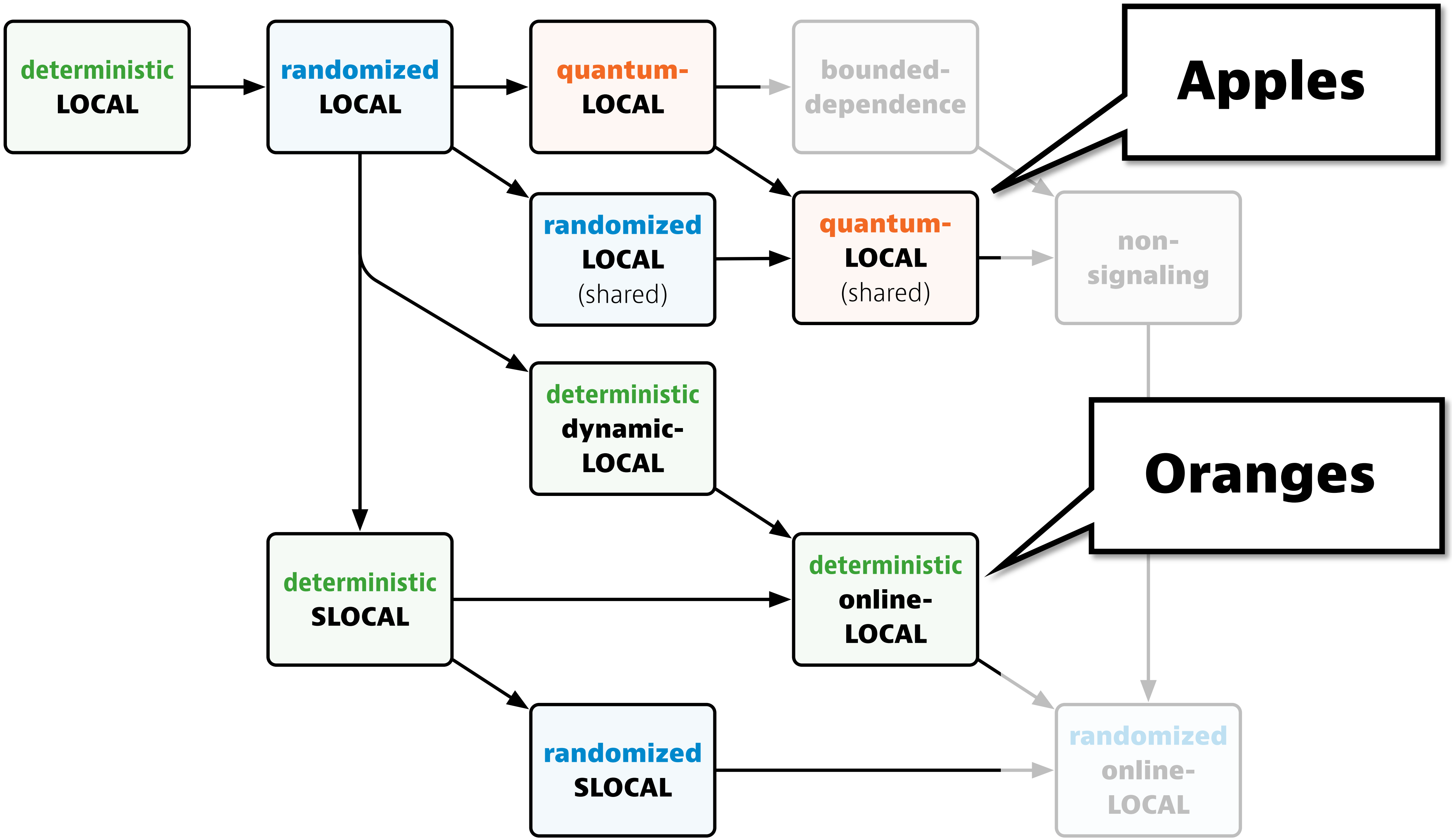
randomized online-LOCAL
… with **shared quantum state**
(cannot depend on graph, only on *n*)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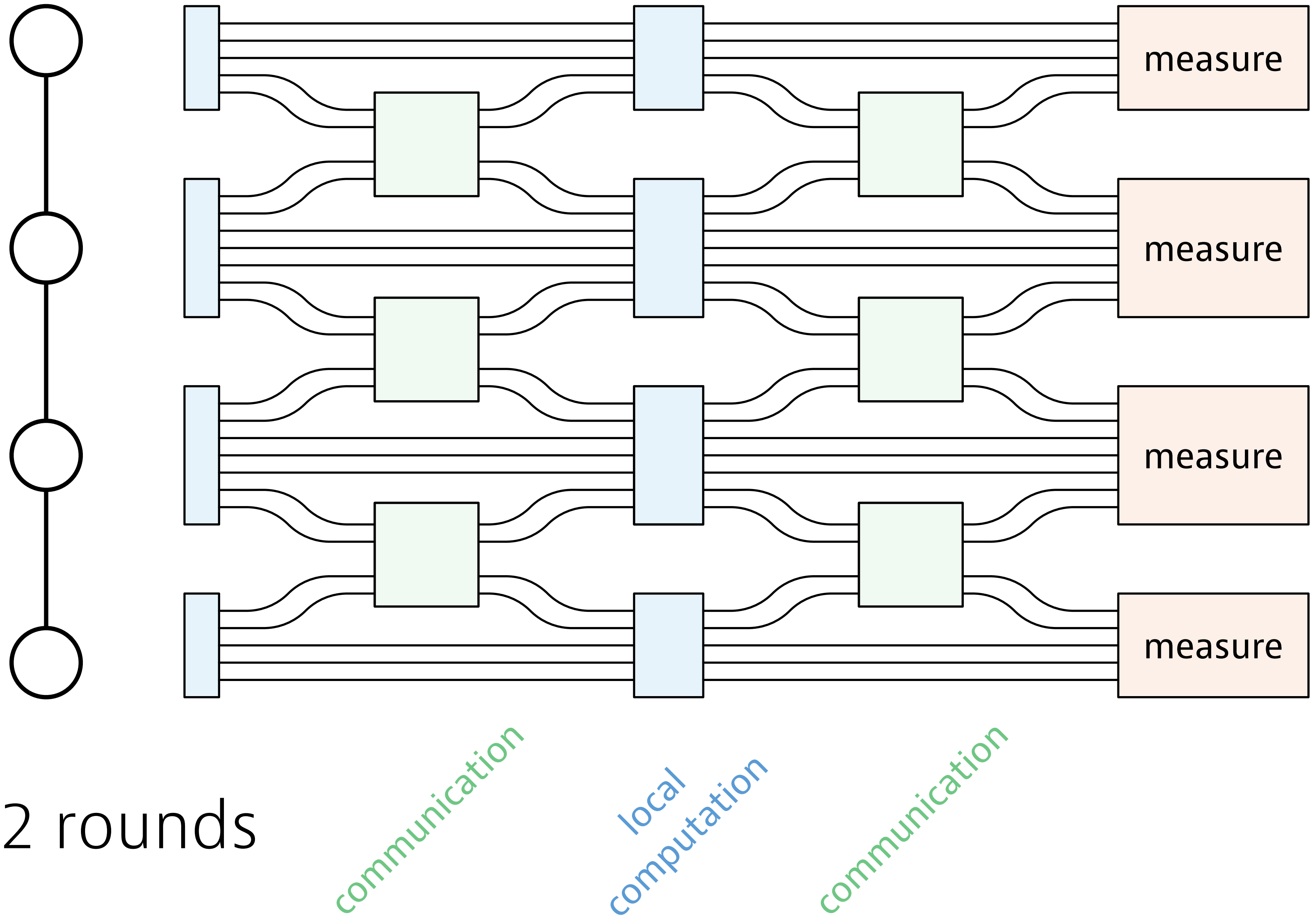
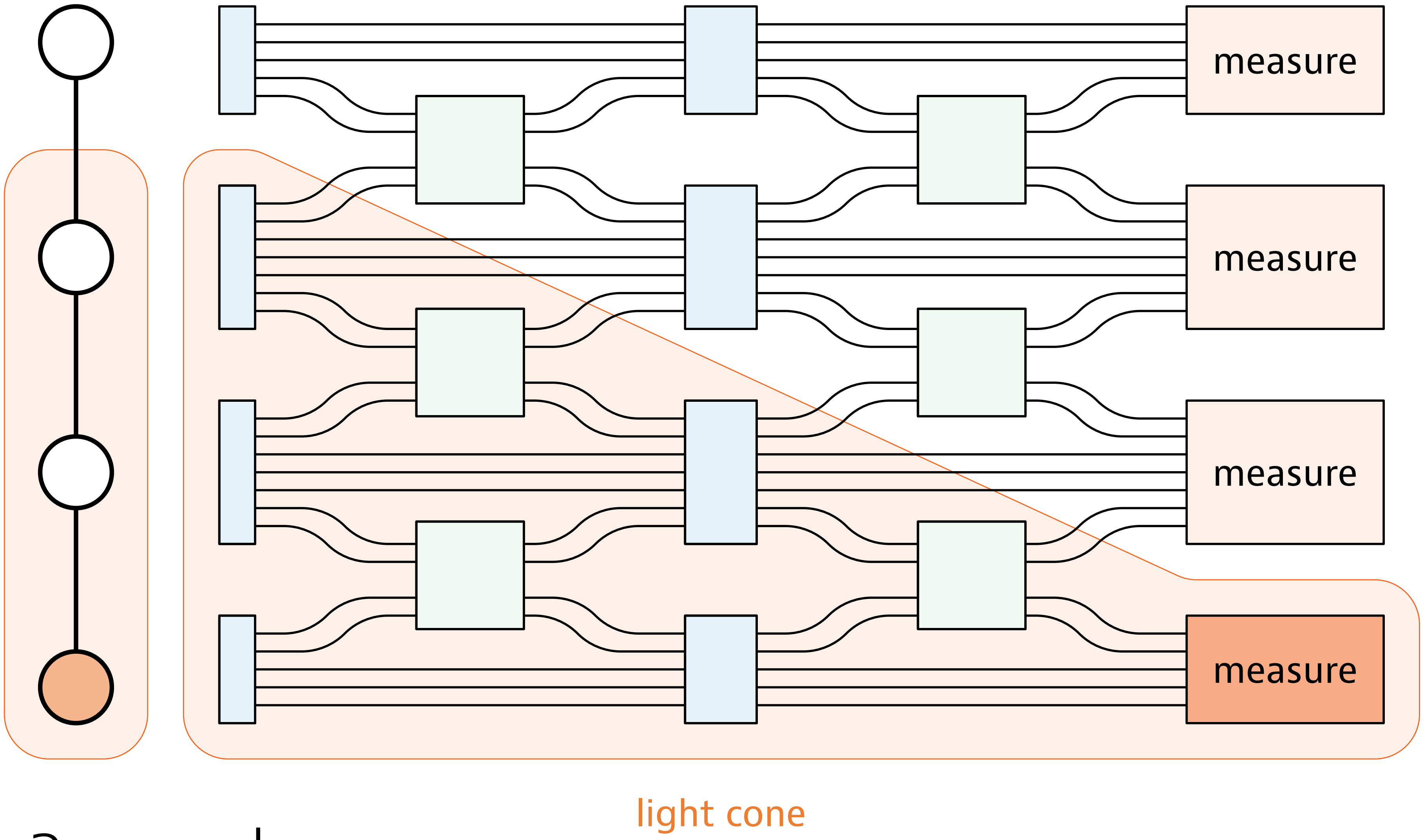
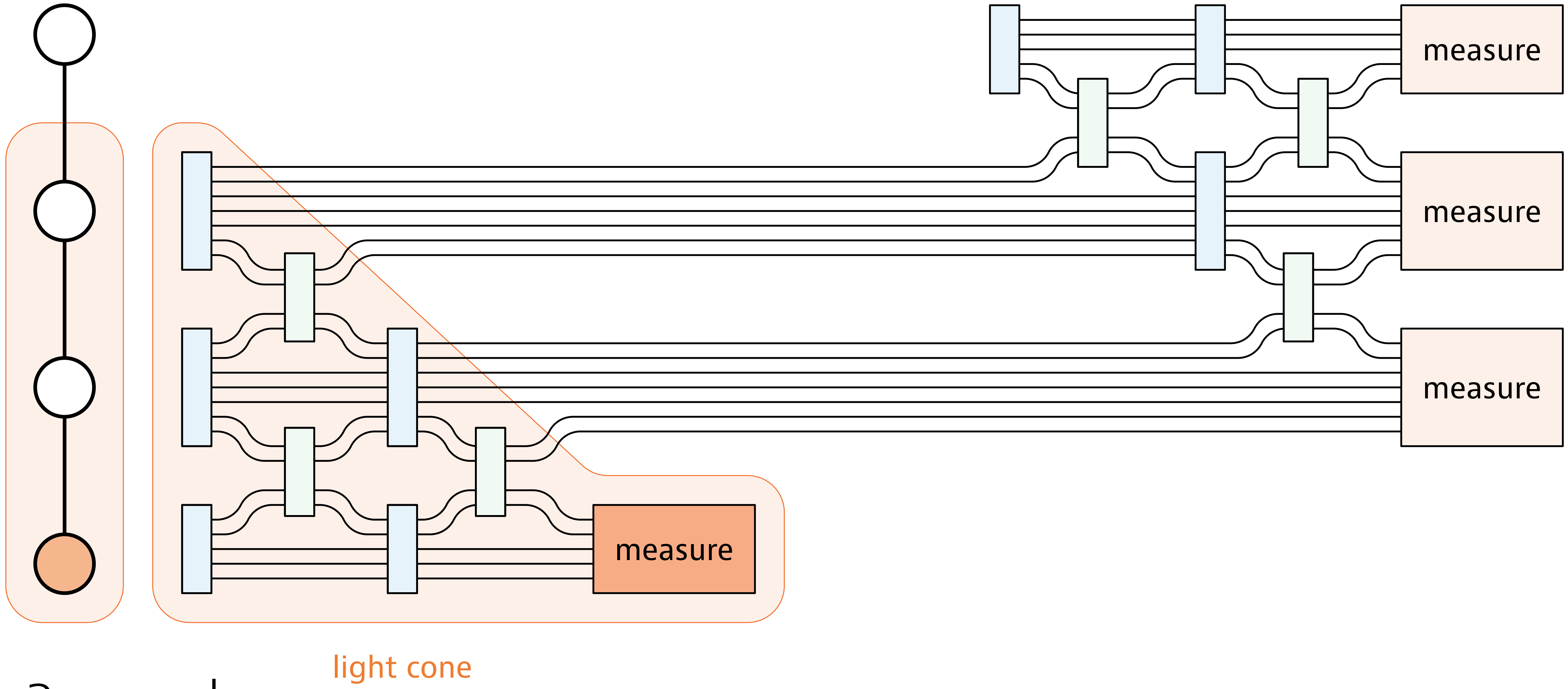

measure
measure
measure
measure
communication
local
computation
communication

2 rounds



2 rounds



2 rounds

Non-signaling model

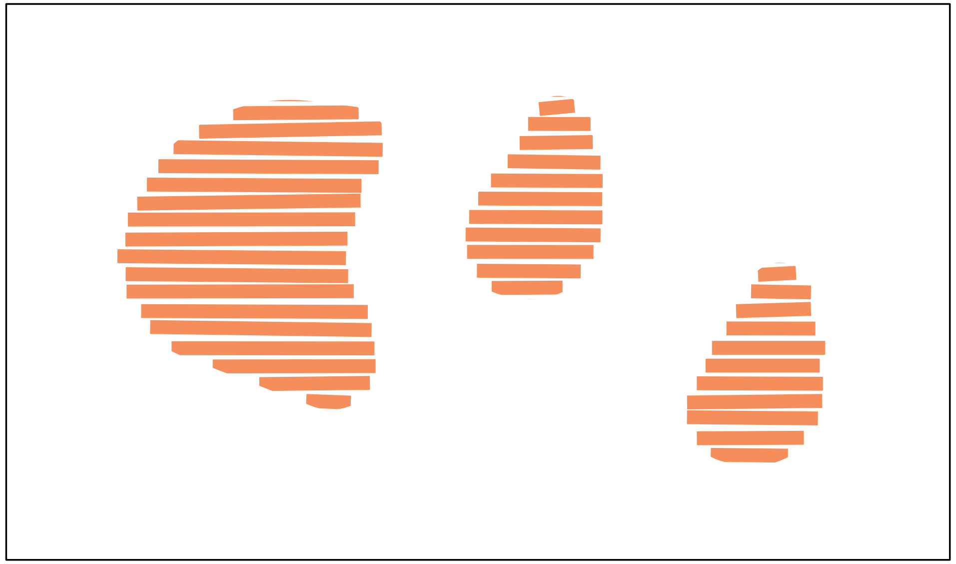
- Quantum LOCAL can't violate causality
- Key idea: **define** a model so that it can do **anything** except violating causality

Non-signaling model

Definition (*non-signaling distribution*):

- fix any **set of nodes *X*** …

Gavoille, Kosowski, Markiewicz 2009
Arfaoui, Fraigniaud 2014



Non-signaling model

Definition (*non-signaling distribution*):

- fix any **set of nodes *X***
- changes in the input **more than *T* hops away** from ***X*** do not influence the output distribution of ***X***

Gavoille, Kosowski, Markiewicz 2009
Arfaoui, Fraigniaud 2014

Classical probability theory

Classical (randomized) distributed algorithms

↓

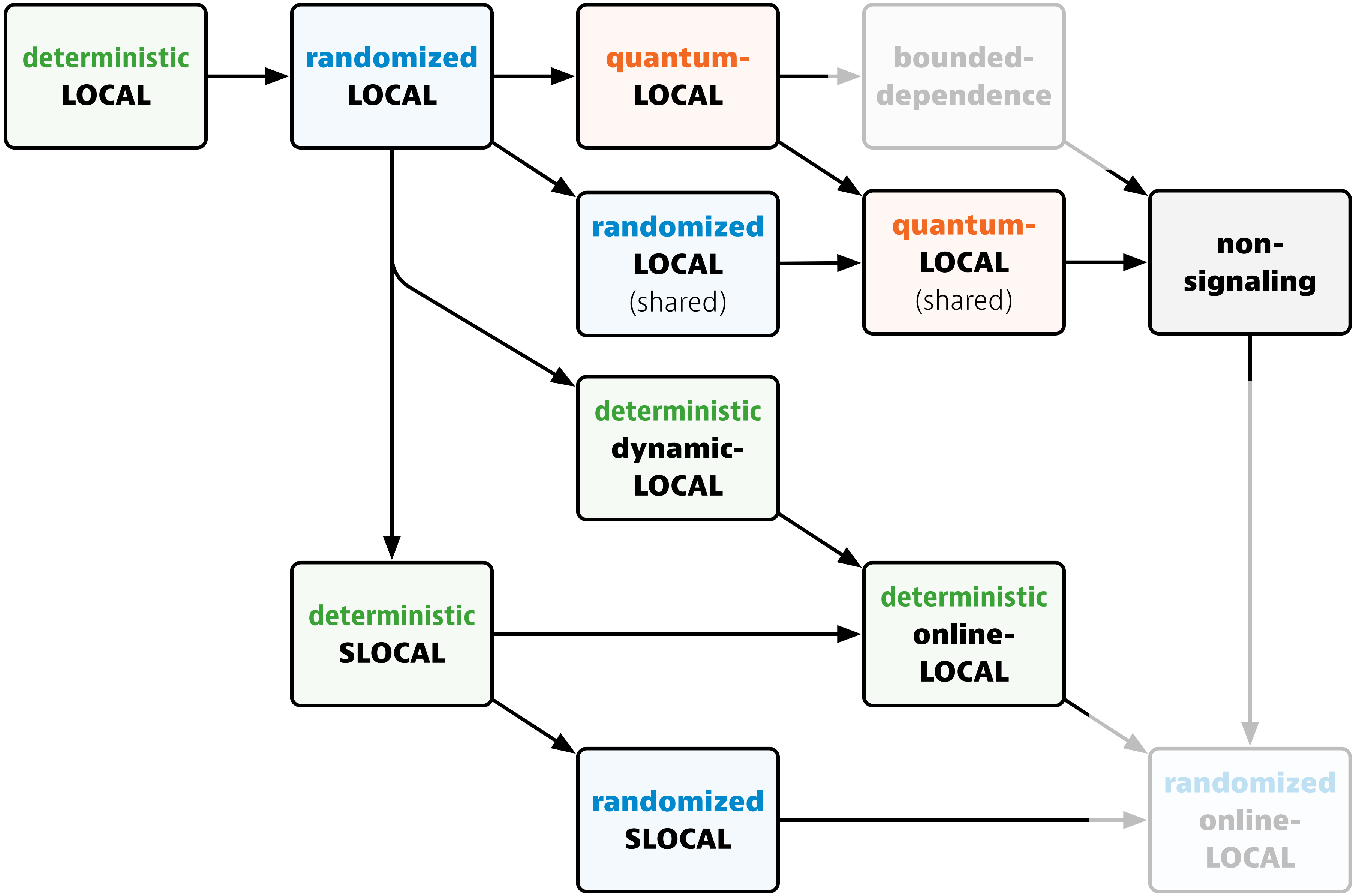
Weird quantum things

Quantum distributed algorithms

↓

Classical probability theory

Non-signaling “algorithms”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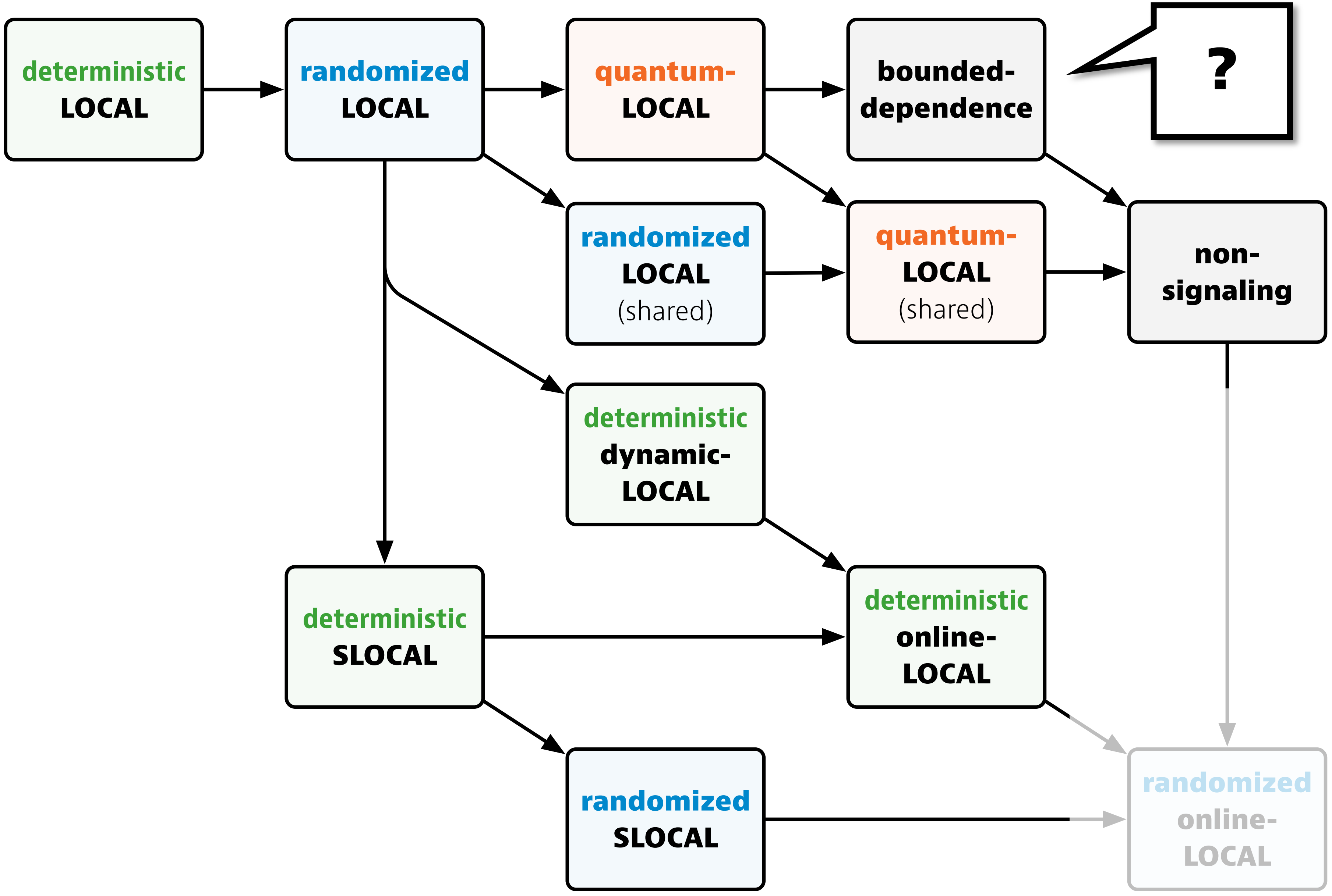
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non-signaling
deterministic dynamic-LOCAL
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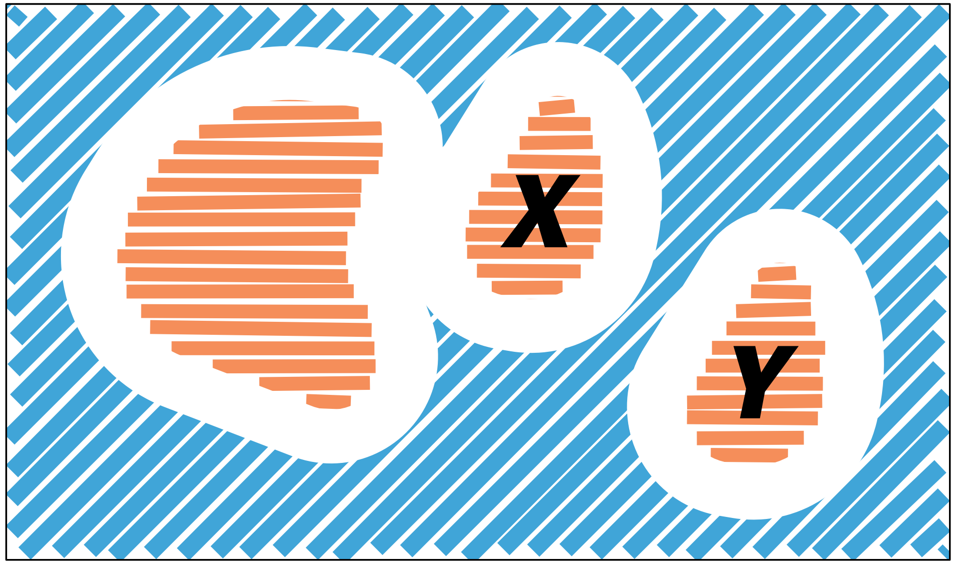
deterministic online-LOCAL
randomized SLOCAL
randomized online-LOCAL

Bounded dependence

- **Finitely dependent distributions**
  - X and Y far from each others → independent
  - usually "far" = some constant
- For clarity, we call it here **bounded dependence model** when "far" = $T(n)$



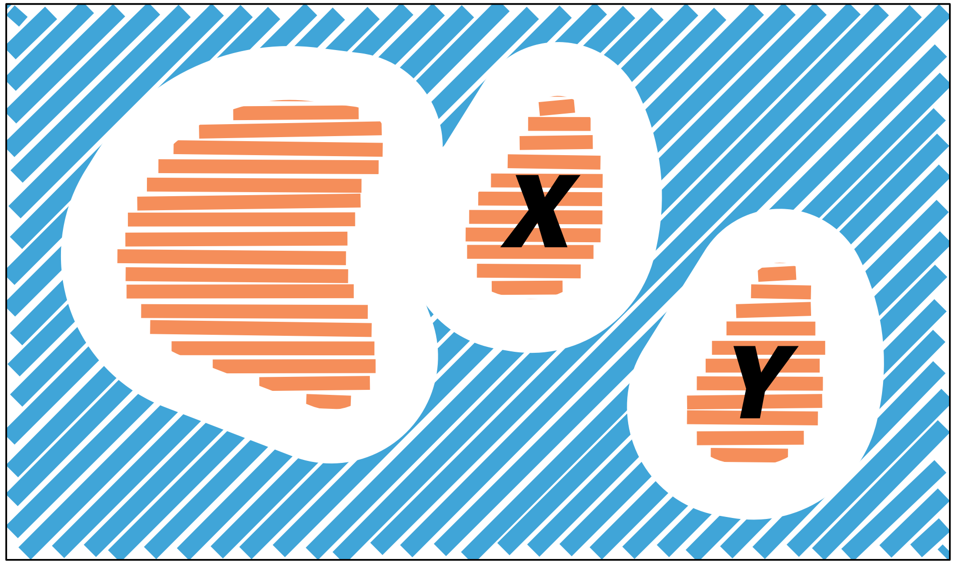
Bounded dependence

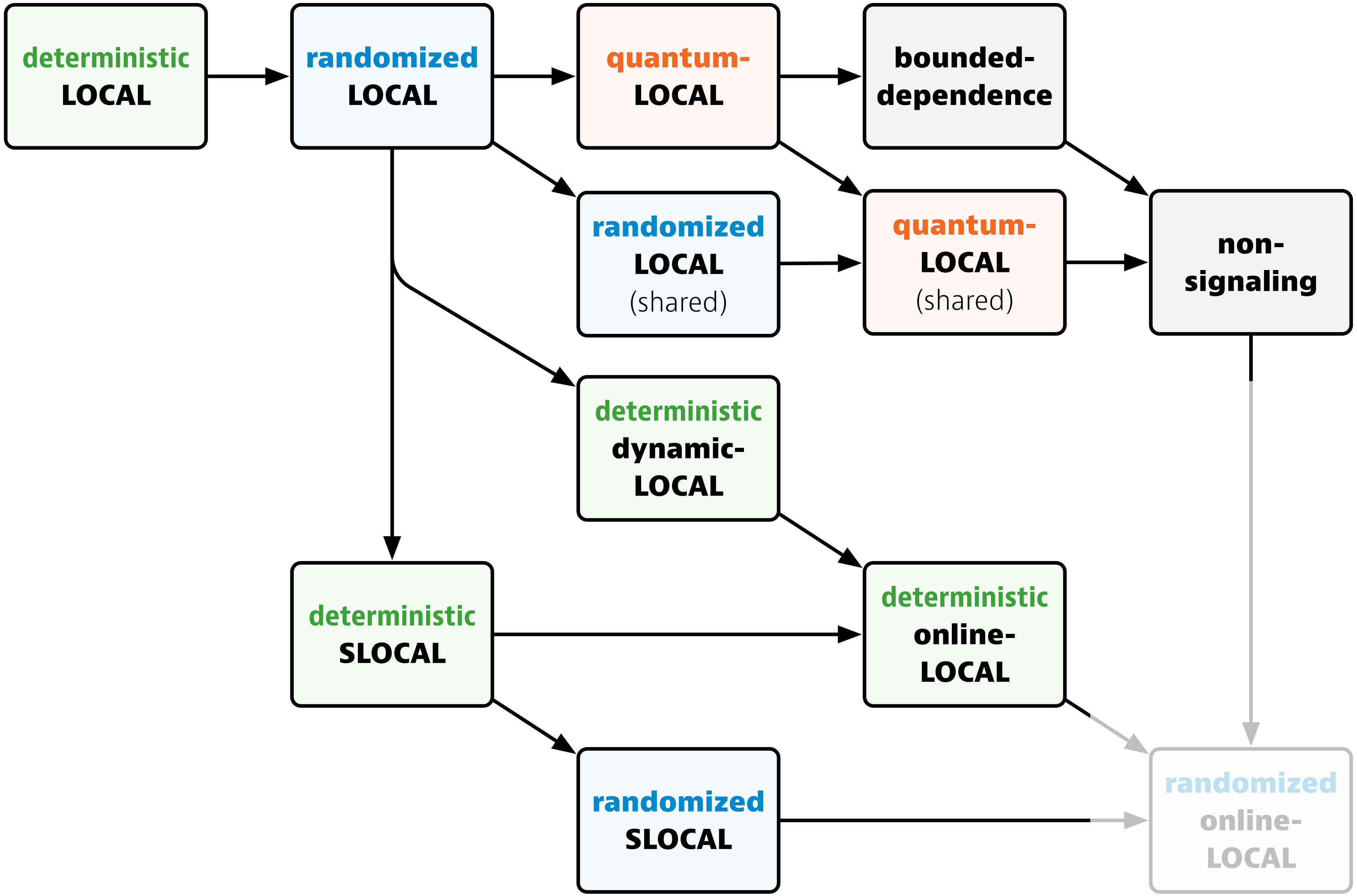
Quantum-LOCAL without shared quantum state:

Output of $T(n)$-round algorithm

↓

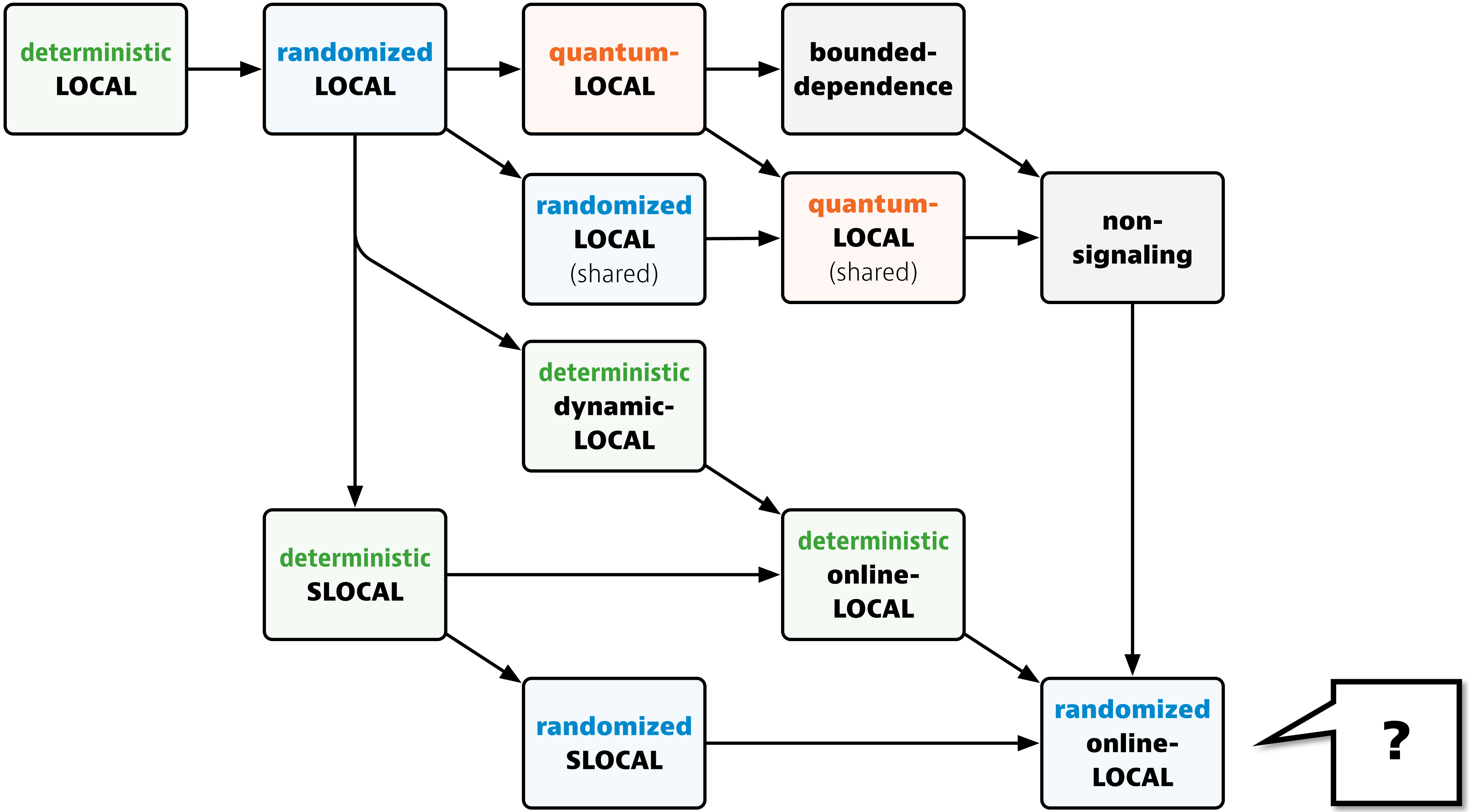
Not just non-signaling but also bounded dependence for distance ≈ $T(n)$





Constant-round quantum-LOCAL algorithms output finitely dependent distributions!

Useful tool for creating interesting finitely dependent distributions?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deterministic SLOCAL
deterministic online-LOCAL
randomized SLOCAL
randomized online-LOCAL
?

Rand. online-LOCAL

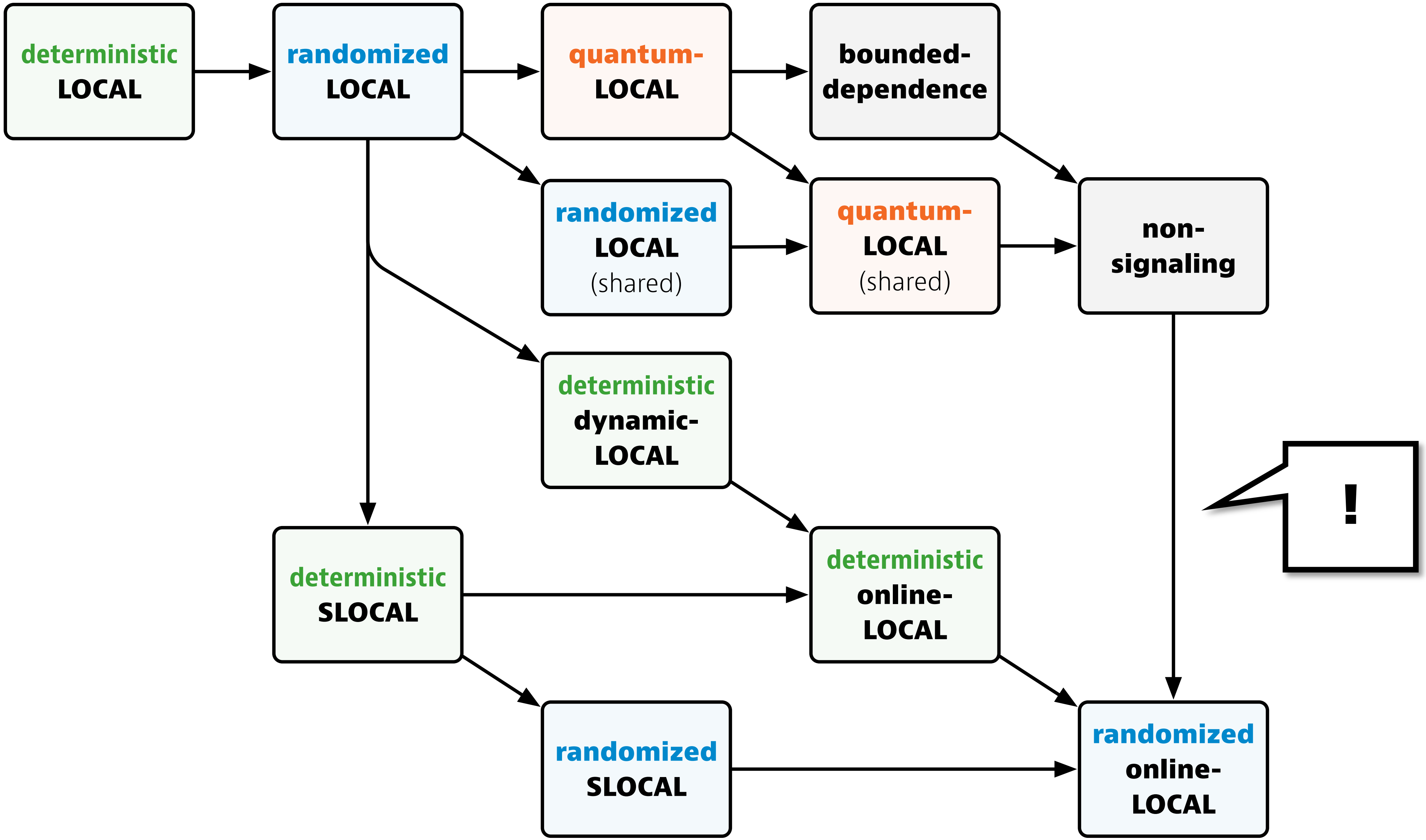
- Adversary fixes a **graph** + **order** in which nodes are revealed

Oblivious adversary

- For each node *v*
  - algorithm sees radius-*T* neighborhood of *v*
  - algorithm must choose the label of *v*
- Algorithm can **remember** everything, algorithm can use **randomness**

Rand. online-LOCAL

- **Trivial:**
  - randomize online-LOCAL can simulate deterministic online-LOCAL
- **Suprise:**
  - randomized online-LOCAL ***can simulate any non-signaling distribution*** (with the same asymptotic locality)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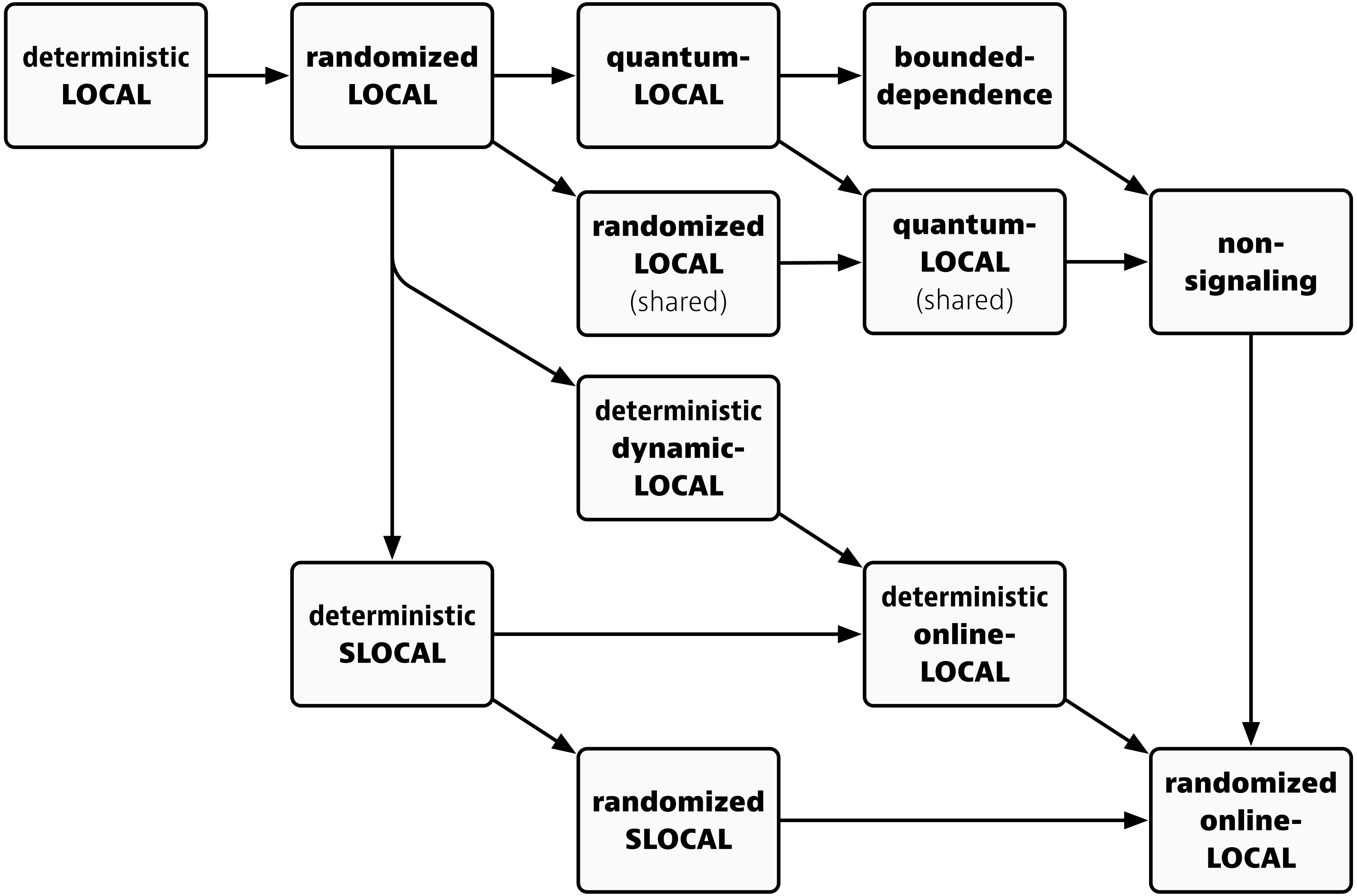
randomized SLOCAL
randomized online-LOCAL
!

Some results...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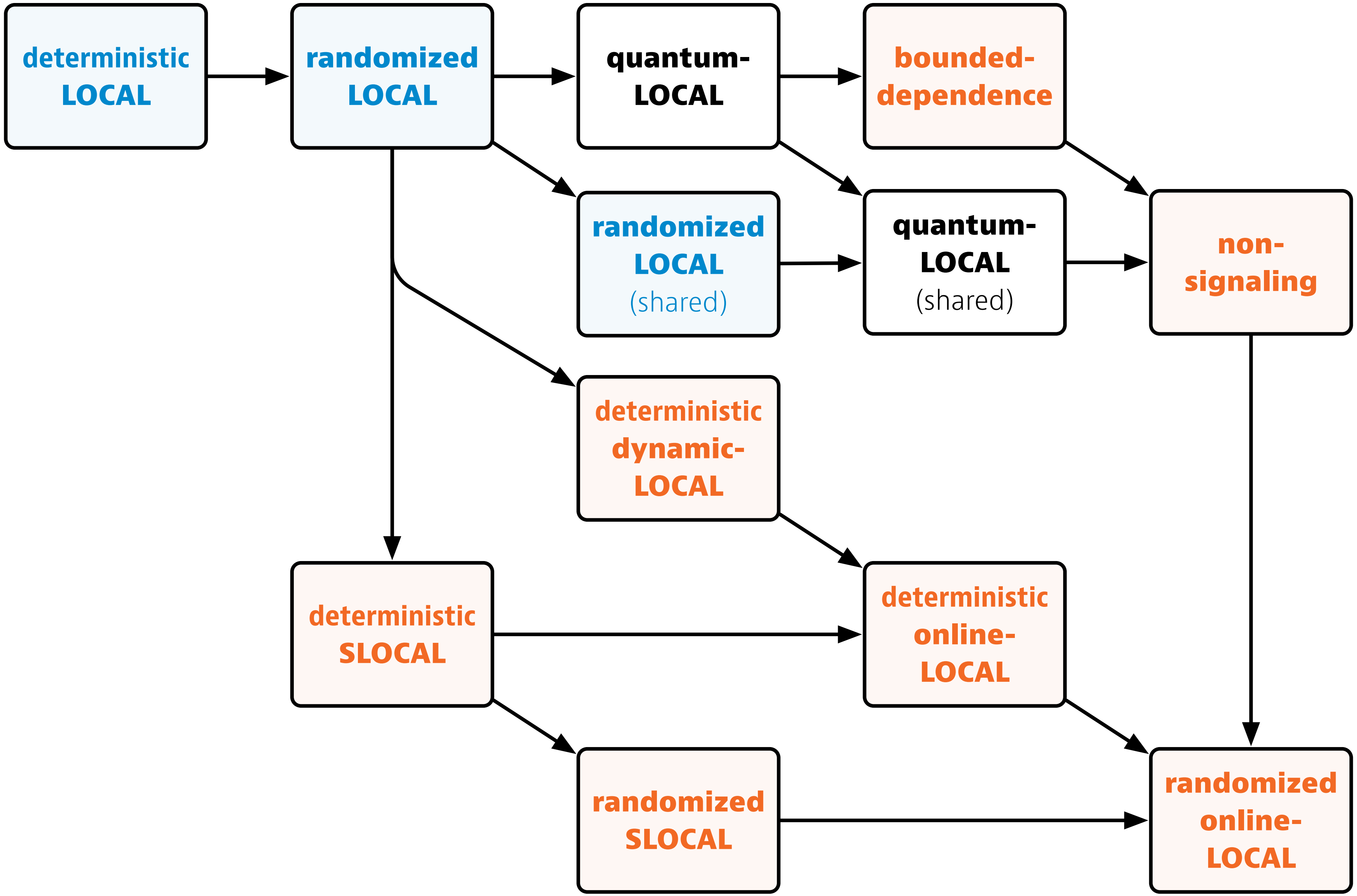
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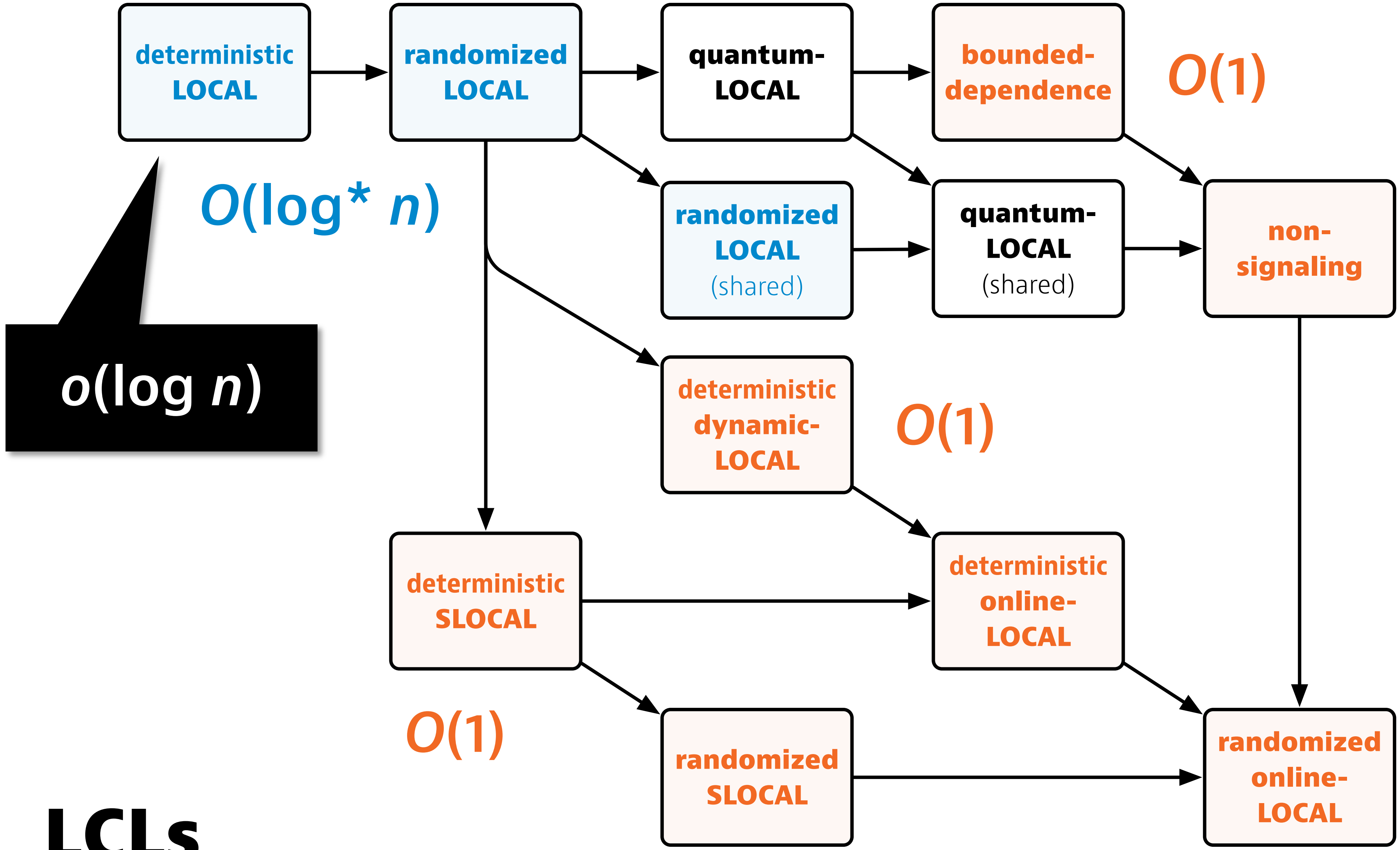
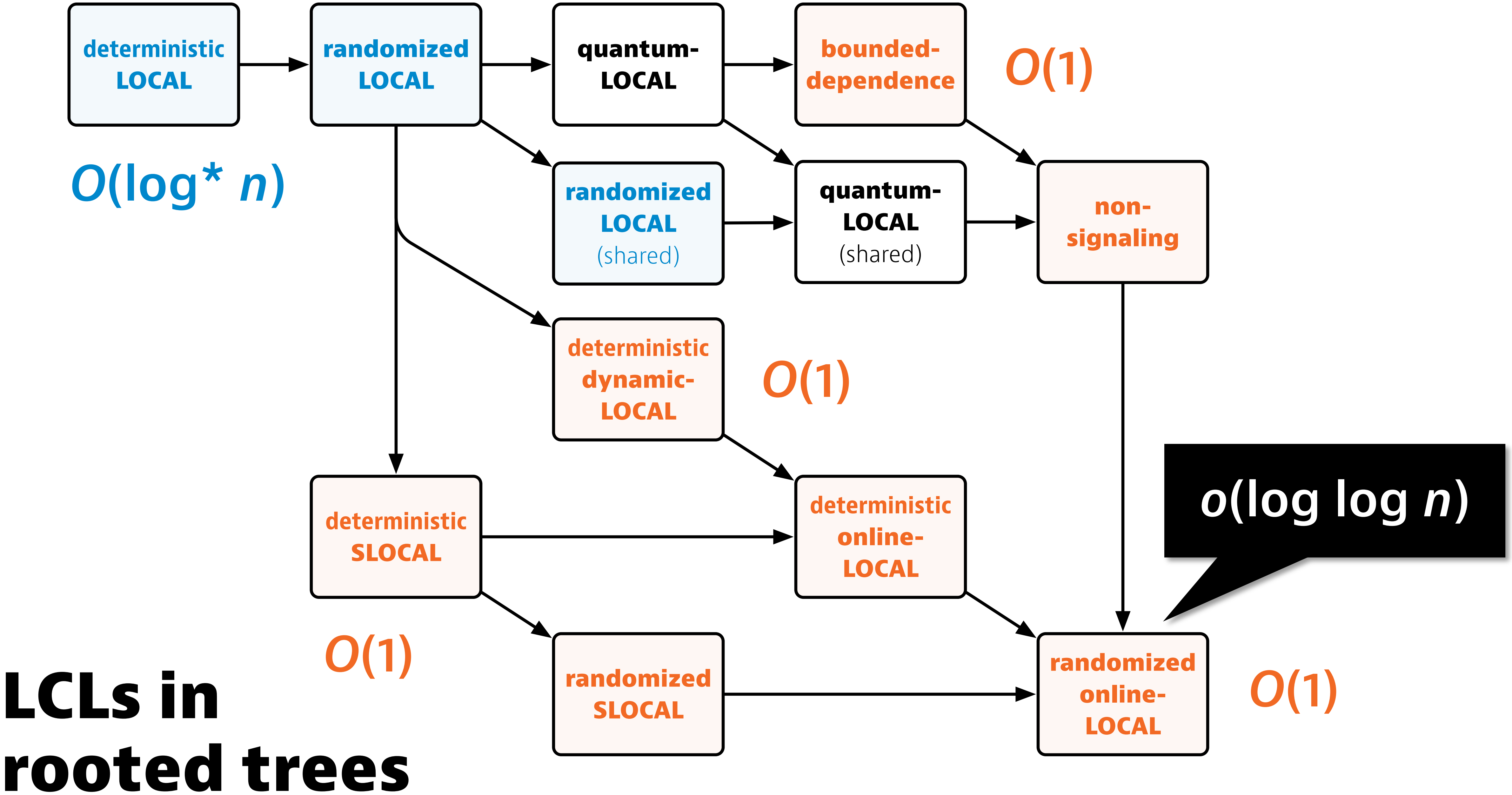
LCLs



Finitely
dependent
coloring
in paths
→ rooted trees
→ graphs
→ power graphs
→ fake IDs
→ solve any LCL



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randomized LOCAL (shared)
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non-signaling
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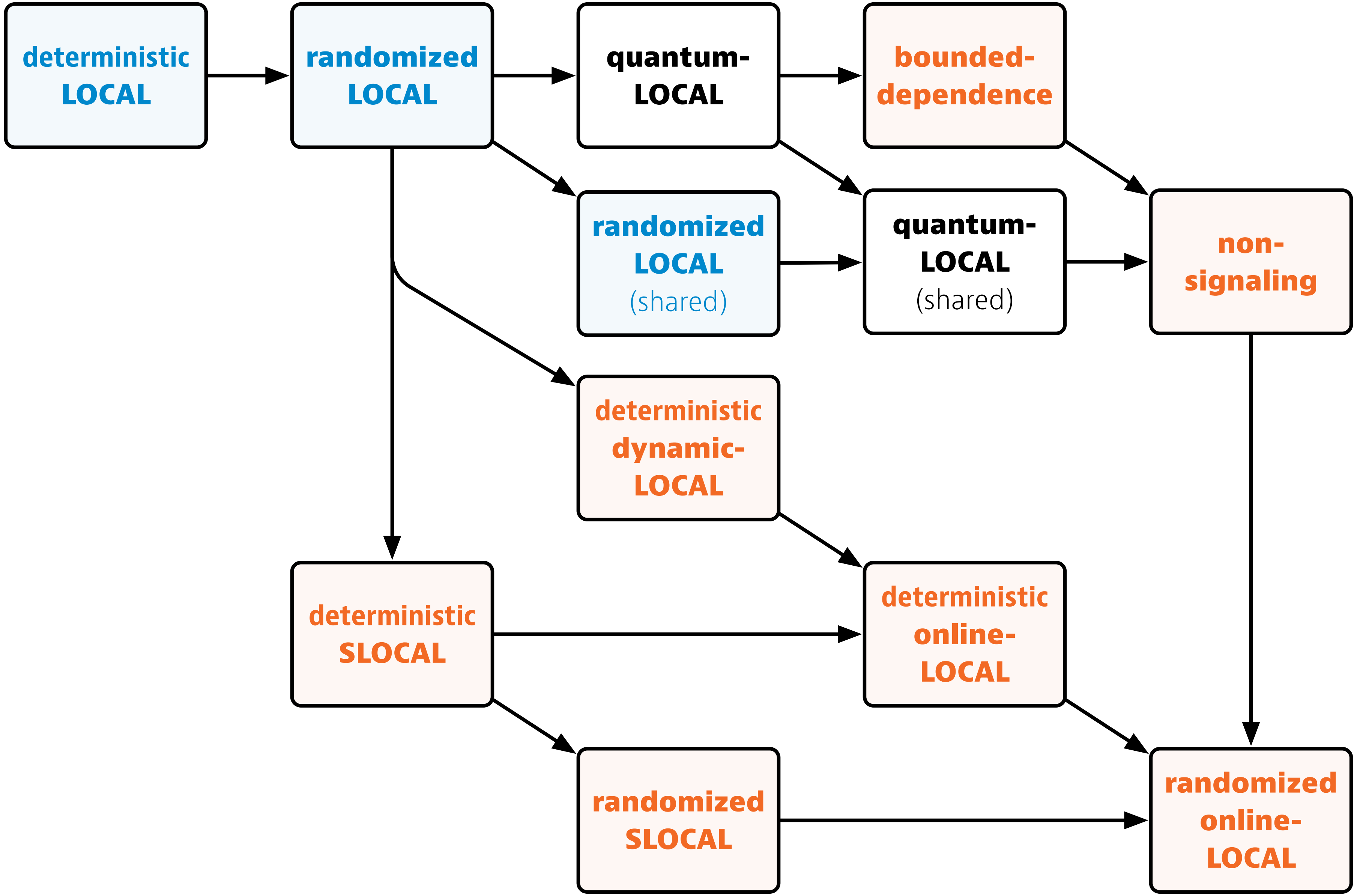
O(1)
o(log log n)
deterministic SLOCAL
deterministic online-LOCAL
O(1)
randomized SLOCAL
randomized online-LOCAL
O(1)
LCLs in rooted trees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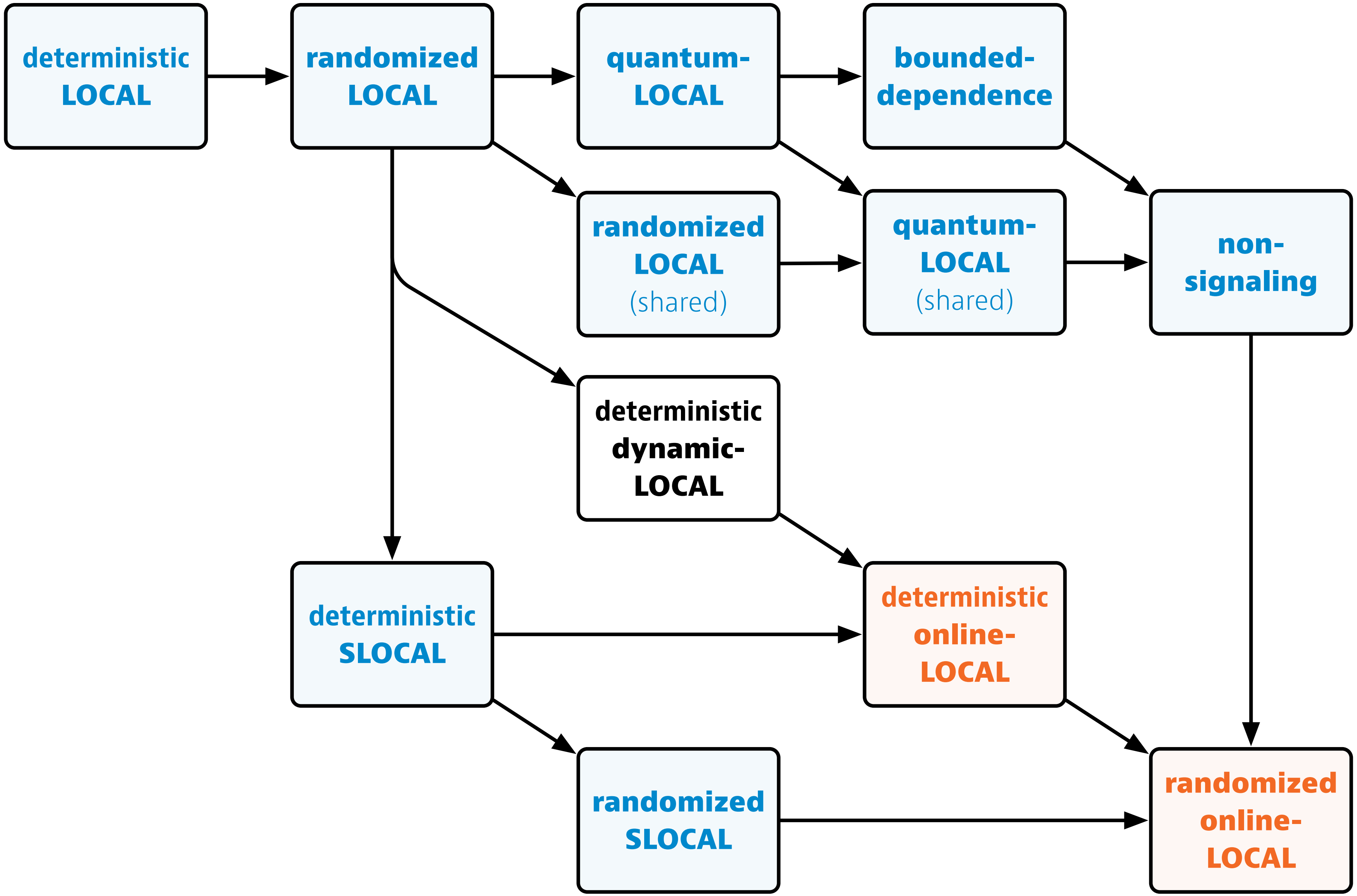
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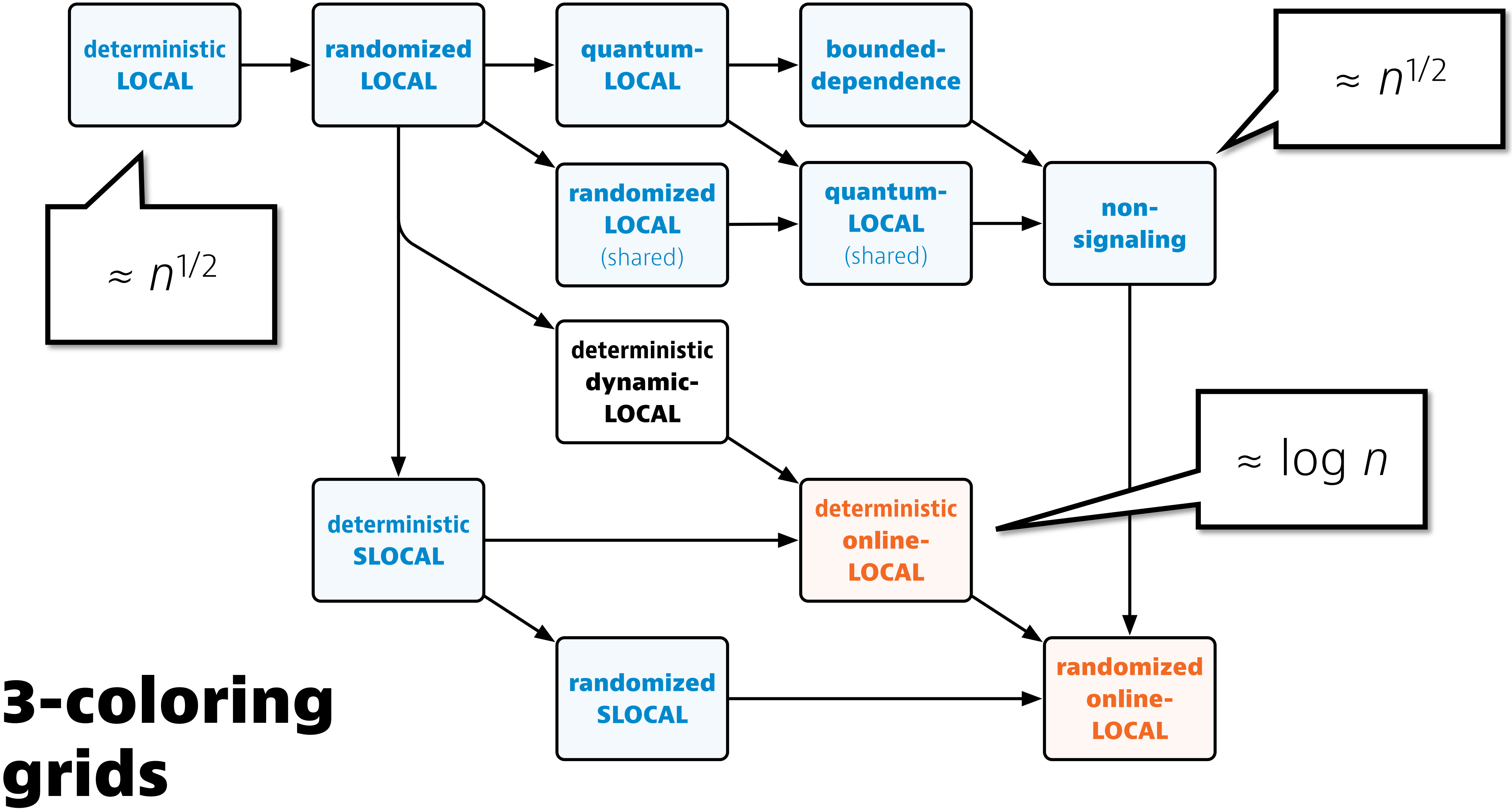
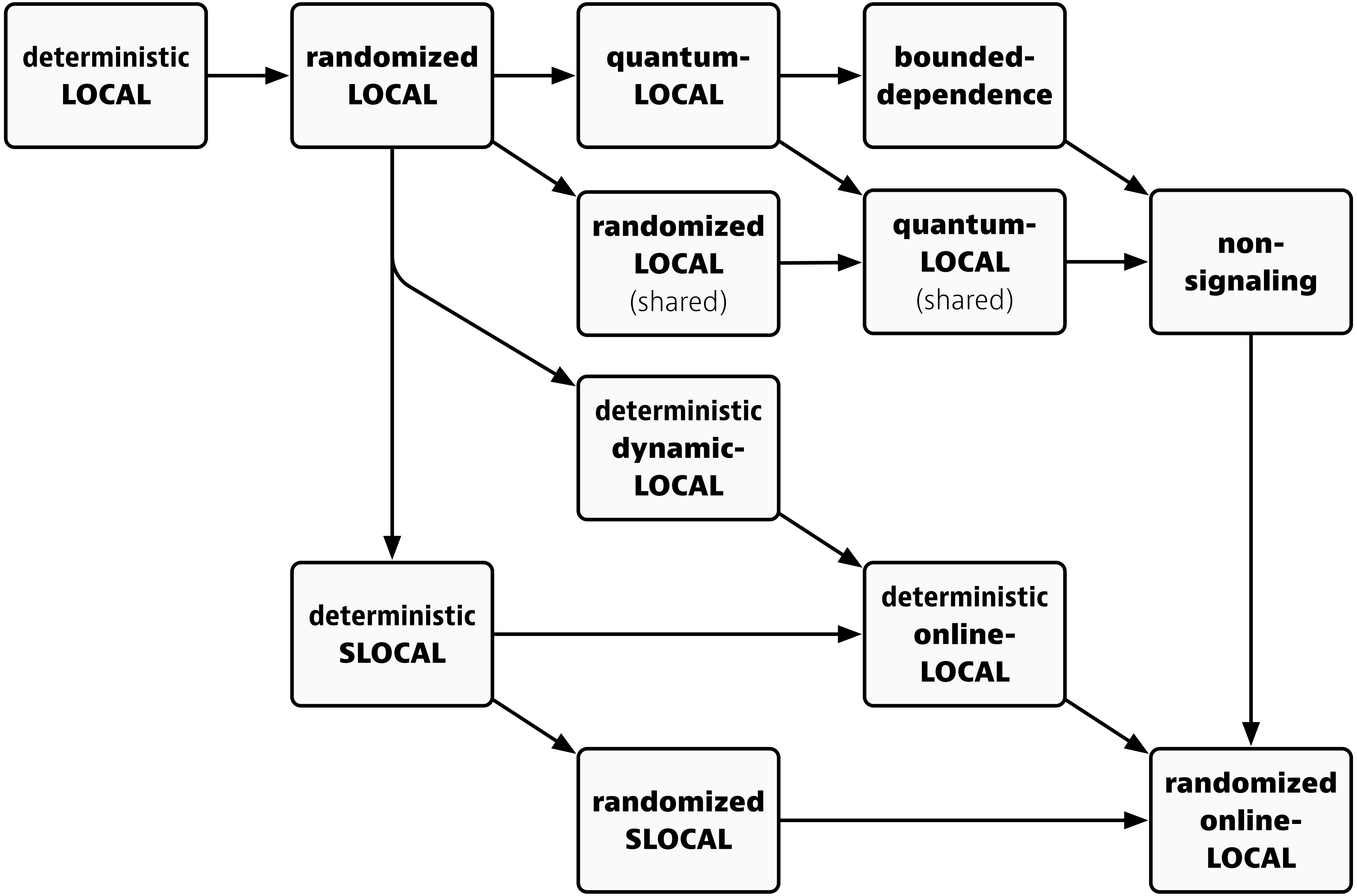
randomized SLOCAL
randomized online-LOCAL



deterministic LOCAL
randomized LOCAL
quantum-LOCAL
bounded-dependence
≈ $n^{1/2}$
≈ $n^{1/2}$
randomized LOCAL (shared)
quantum-LOCAL (shared)
non-signaling
deterministic dynamic-LOCAL
≈ log n
deterministic SLOCAL
deterministic online-LOCAL
randomized SLOCAL
randomized online-LOCAL
3-coloring grids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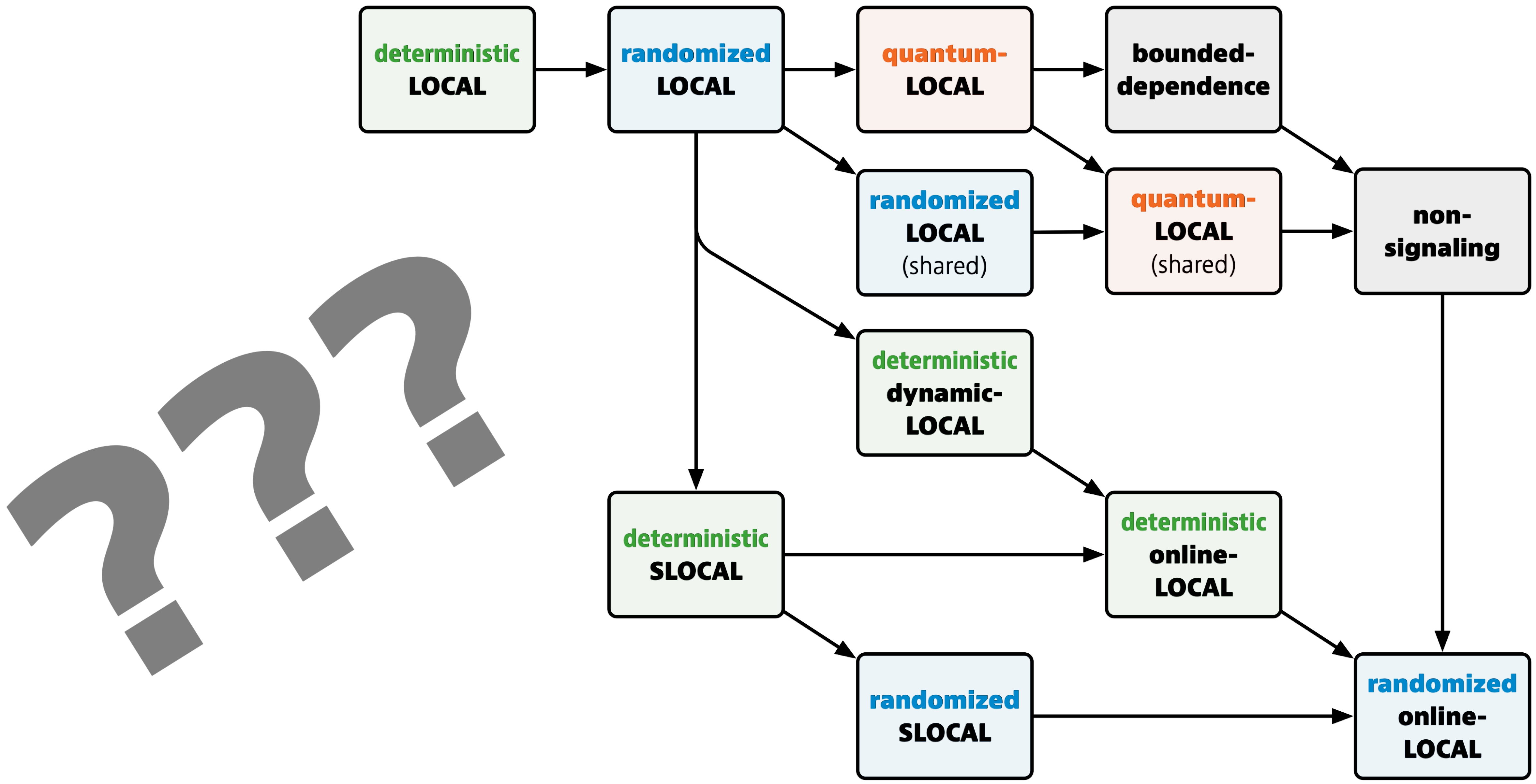

deterministic LOCAL
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Open questions

- **Is there any distributed quantum advantage for LCLs?**
  - example: can you ***color cycles*** in $O(1)$ rounds in ***quantum-LOCAL***?
- **Does global memory ever help in trees?**
  - could we ***simulate online-LOCAL*** in LOCAL model in unrooted trees?
  - or can we separate these for some LCL in trees?

Open questions

- **Lower bounds for sinkless orientation**
  - deterministic LOCAL: ≈ log *n*
  - randomized LOCAL: ≈ log log *n*
  - deterministic SLOCAL: ≈ log log *n*
  - randomized SLOCAL: ≈ log log log *n*
  - quantum-LOCAL: no lower bounds!
  - ***we cannot exclude finitely dependent distributions for sinkless orientation!***

Open questions

- **New lower-bound proof techniques?**
  - *existential graph-theoretic arguments* and propagation arguments seem too weak to tackle e.g. sinkless orientation
  - *round elimination* does not work in quantum and beyond
  - how far can we stretch Marks' technique?
  - current best hope: new *simulation results*?

References & pointers

arXiv:2109.06593 Locality in online, dynamic, sequential, and distributed graph algorithms *(ICALP 2023)*

arXiv:2307.09444 No distributed quantum advantage for approximate graph coloring *(STOC 2024)*

arXiv:2403.01903 Online locality meets distributed quantum computing

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