#### Jukka Suomela Aalto University Can we automate our own work - or show that it is hard?

# Computer science: *what can be automated?*

### 

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#### Today: can we automate our own work?

# Focus: theory of distributed computing

### Consider a typical theory paper in OPODIS, PODC, DISC...

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**1998 ACM Subject Classification** F.1.1 Models of and Classes

Keywords and phrases distributed computing, sp weak models, Thue–Morse sequence

Digital Object Identifier 10.4230/LIPIcs.OPODIS.2



In the classical centralised theory of computing, the helped us with understanding computability and c

- 1. Constant-space models (finite-state machines) patient.
- Space-limited complexity classes (e.g., PSPAC limited complexity classes (e.g., NP ⊆ PSPACE

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#### **Automatic Lower Bound**

#### **Automatic Upper Bound**

## Lost sanity?

### Toy example: Locally checkable problems in cycles



#### • Computer network: cycle of n computers

- globally consistent orientation
- each node has one "successor" and one "predecessor"



#### Setting

- Computer network: cycle of n computers
- Model of computing: LOCAL model
  - synchronous communication rounds
  - time = number of rounds until all nodes stop
  - unbounded message size
  - unlimited local computation
  - unique identifiers



#### Setting

- Computer network: cycle of n computers
- Model of computing: LOCAL model
- Problem: any discrete problem you can define with local constraints
  - finite number of output labels
  - relation that tells which label sequences are valid



• Computer network: cycle of n computers

0

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Example: maximal independent set



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#### Valid label sequences

- 2-coloring: 12, 21
- 3-coloring: 12, 21, 13, 31, 23, 32
- Independent set: 01, 10, 00
- *Maximal independent set:* **001, 010, 100, 101**
- Distance-2 coloring with 3 colors: 123, 132, 213, 231, 312, 321

#### Valid label sequences

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All possible output labelings in a window of size k

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- Distance-2 coloring with 3 colors: 123, 132, 213, 231, 312, 321

#### Fully automatic



• Write down the specification of any locally checkable problem X

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- Then you can *find efficiently* 
  - distributed round complexity of X
  - asymptotically optimal distributed algorithm for X



This algorithm solves X in time O(log\* n)

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Polynomial time (in the size of problem description)

0 1 0





Example: X = maximal independent set problem

0 1 0

0 1 0

:

1 0 1

0 0 1

0 1 0

0 0 1

0

0 1 0 1 0

Compatible neighborhoods for adjacent nodes

0 1 0

0 0 1

0

1



Compatible neighborhoods for adjacent nodes



0 1 0

1 0





0 1 0

0 0 1

0

1

1










































distance-2 coloring



















## self-loop J solvable in O(1) rounds

## **Algorithm:** Constant output (e.g. here all-0)





**Proof:** No self-loop  $\rightarrow$  any solution breaks symmetry everywhere  $\rightarrow$  can be used to find 3-coloring  $\rightarrow$  not possible in  $o(\log^* n)$  rounds

























distance-2 coloring































































































## $k = 5, 6, 7, 8, \dots$

"Flexible": for all k ≥ k<sub>0</sub> there is a selfreturning walk of length k



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Decidable in polynomial time
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solvable in
O(log\* n) rounds

#### **Algorithm:**

- split in blocks of length  $\geq k_0$
- use the flexible configuration at each block boundary
- fill in between boundaries by following a self-returning walk

"Flexible": for all  $k \ge k_0$ there is a selfreturning walk of length k solvable in



solvable in O(log\* n) rounds **Proof:** Not flexible  $\rightarrow$  must use the same non-flexible configuration at least twice far from each other; not compatible for all distances  $\rightarrow$  global coordination needed  $\rightarrow$  not possible in o(n) rounds

























independent set





*O*(log\* *n*)







independent set





*O*(log\* *n*)





O(n)

# Fully automatic

- Write down the specification of any locally checkable problem X
- Then you can *find efficiently* 
  - distributed round complexity of X
  - asymptotically optimal distributed algorithm for X



This algorithm solves X in time O(log\* n)

# "Oh but doing it for **this** case is of course trivial..."

# But what are other cases in which algorithm design & lower-bound proofs can be automated?



Grids





### solution ≈ execution history of a **finite automaton**







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 $\left( \right)$ 

()

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Many questions undecidable

# Undecidable # hopeless

Any algorithm **A** that solves a locally checkable problem X fast can be written as  $\mathbf{A} = \mathbf{B} \circ \mathbf{C}_{\mathbf{k}}$ 

- $C_k$  = distance-k coloring
- **B** = finite function that maps colored neighborhoods to local outputs

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**Proof idea:** Coloring  $\approx$  locally unique identifiers. If *A* fails with such fake identifiers, it also fails in some small graph with some real identifiers.

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# For each *k* = 1, 2, 3, ...:

- check all possible candidate functions **B**
- if any of them is good  $\rightarrow$  fast algorithm found!

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#### **Undecidability:** *don't know when to stop if fast algorithms don't exist*

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C<sub>k</sub> = distance-k
 B = finite funct
 Computational complexity:
 neighborhoods
 typically doubly-exponential in k

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- We can make it more feasible in practice:
  - more "compact" normal forms, e.g. distance-k coloring  $\rightarrow$  ruling set
  - represent "candidate B is good for this value of k" as a Boolean formula and use modern SAT solvers to find such a B

- Example: *4-coloring in grids*
- Computers were much faster than human beings in figuring out that this is solvable in  $O(\log^* n)$  rounds

[Brandt et al., PODC 2017]



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### Grids

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Many questions undecidable (but there is hope!)

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# **Grids + beyond**

solution ≈ execution history of a **Turing machine** 

Bad news apply to any graph family that contains large grids

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### **Grids + beyond**

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What is here between paths and grids?

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# Trees Bounded treewidth High girth

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### **Grids + beyond**

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lots of open questions, no known obstacles! Trees Bounded treewidth High girth

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# Big picture: **towards meta-computational research questions**

# Meta questions

- **Traditional questions:** what is the best distributed algorithm for solving problem X ?
- Meta-computational questions: can we design an (efficient) *meta-algorithm* that finds the best distributed algorithm for *any problem X* in some problem family *F* ?
• Classification: "Any problem in this family belongs to one of these classes"

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- locally checkable problems in cycles have complexity O(1) or  $\Theta(\log^* n)$  or  $\Theta(n)$
- locally checkable problems in general graphs belong to one of four broad classes



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- Detect patterns, generalize

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- Detect p How? Are there general techniques we can apply without much thinking?

### General techniques

Automatic Lower Bound
Automatic Upper Bound

Example: *round elimination* technique
<u>github.com/olidennis/round-eliminator</u>
applicable to any locally checkable problem

[Brandt, PODC 2019] [Olivetti, PODC 2020]

### General techniques

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applicable to any locally checkable problem

 Does not always work — but when it works, you get algorithms and/or lower bound proofs for free!

[Brandt, PODC 2019] [Olivetti, PODC 2020]

#### **Success stories**

- Lower bound for maximal matching and maximal independent set
- Six people and one computer program
  - enabled rapid hypothesis testing and exploration of possible proof strategies

#### [Brandt et al., FOCS 2019]

# Conclusions

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#### Opportunities for human-computer collaboration!

• theory researchers who can write programs are going to have a competitive edge!