Approximating relay placement in sensor networks

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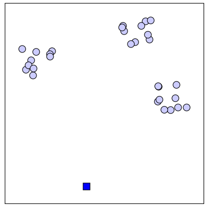
Objective

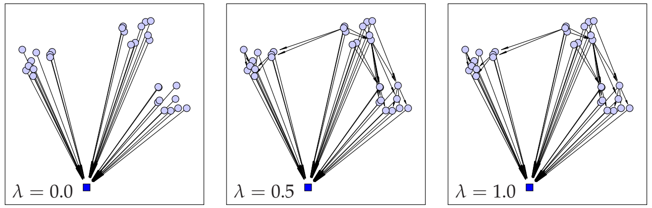
- Maximise balanced data gathering $\lambda \min_i q_i + (1-\lambda) \operatorname{avg}_i q_i$
  - q_i: amount of data gathered from sensor node i
  - $\lambda \in [0, 1]$: balance parameter
- Energy-constrained nodes
- Model of wireless communication:
  - Reception cost $= \rho$
  - Transmission cost $\propto d^\alpha$ (d distance, α constant)



Input (circles are sensors, the box is the sink)

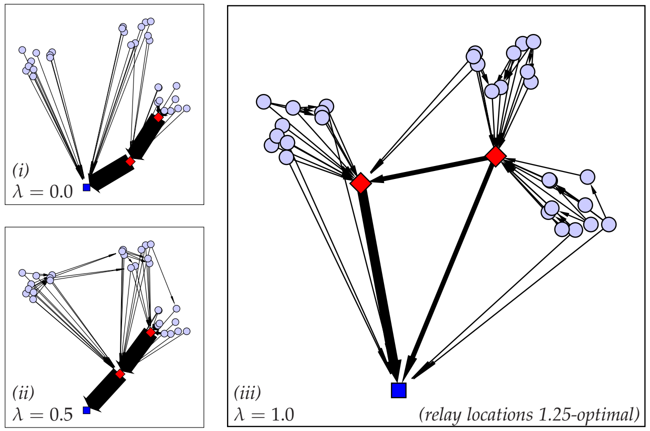
Balanced data gathering problem

Find optimal data flows [1, 3]



Relay placement problem

Find optimal relay locations and data flows [5]



Hard to approximate

Definitions

- *Euclidean problem*: relays placed anywhere in the plane
- *Sensor-upgrade problem*: larger batteries installed in some sensor nodes

Previous work

- The Euclidean problem, the sensor-upgrade problem, and many others are NP-hard [5]
- More complicated cases are NP-hard to approximate within any constant factor [5]

New results

- The Euclidean problem and the sensor-upgrade problem are NP-hard to approximate within small constant factors
- Even if the parameters of the communication cost model are fixed to physically realistic values, inapproximability within factors as high as 10 can be obtained:

			$\alpha = 2.0$	$\alpha = 3.0$	$\alpha = 4.0$
Euclidean	$\lambda = 1.0$	$\rho > 0$	3.24	5.85	10.56
		$\rho = 0$	2.99	5.19	8.99
	$\lambda = 0.5$	$\rho > 0$	1.52	1.70	1.82
		$\rho = 0$	1.49	1.67	1.79
Sensor-upgrade	$\lambda = 1.0$	$\rho > 0$	3.99	7.99	15.99
		$\rho = 0$	1.14	1.14	1.14
	$\lambda = 0.5$	$\rho > 0$	1.59	1.77	1.88
		$\rho = 0$	1.06	1.06	1.06

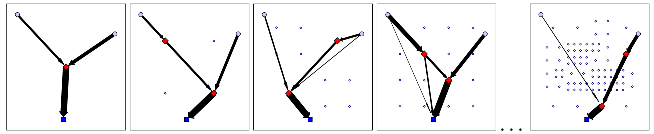
Some examples of the inapproximability ratios

Proof outline

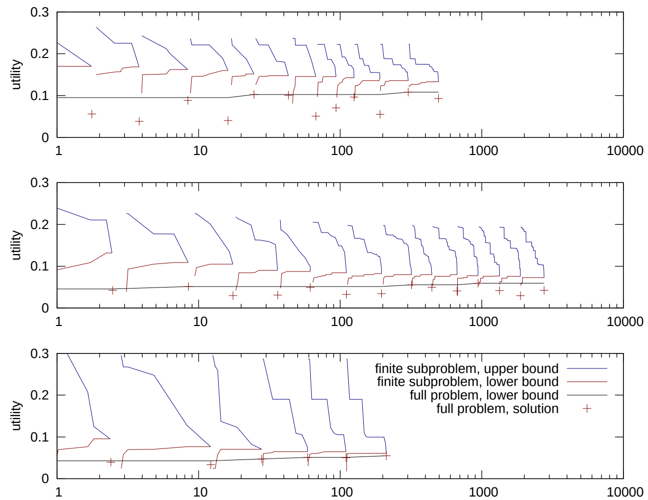
- Same basic idea as in a proof of the inapproximability of *k-centre clustering* [2]
- Consider *3-planar vertex covering* (vertex covering in planar graphs with maximum degree 3). This problem is NP-complete [4]
- Transform an instance of 3-planar vertex covering to an equivalent *embedded* instance of 3-planar vertex covering
- Reduce the decision problem to the problem of *approximating* relay placement

But we can certainly try!

- Heuristic algorithm for finding *both lower and upper bounds* for the Euclidean problem
- Tighten the upper bound by partitioning the plane into cells
- Make division denser in the areas where relays are placed:



Placing 2 relays (small diamonds are the centre points of the cells)



Timings for the examples in Figures (i), (ii), (iii); in seconds

References

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