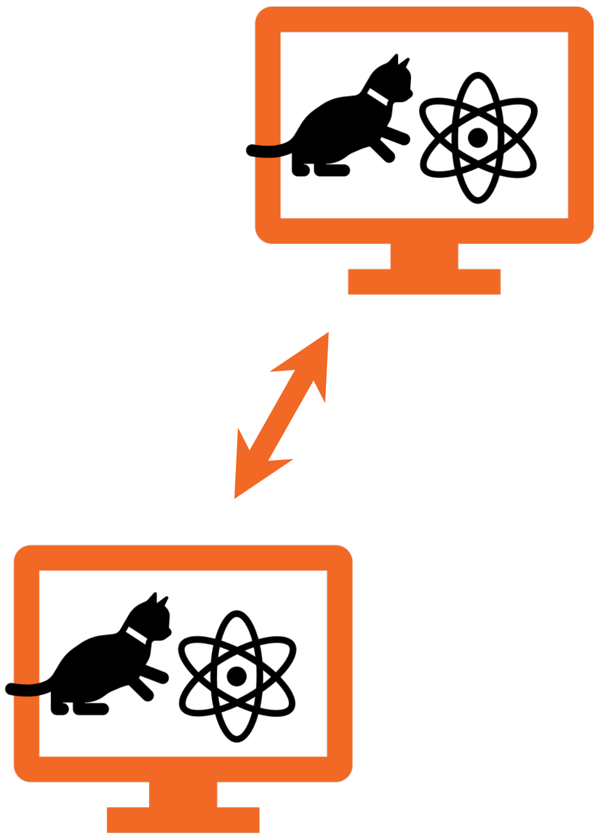
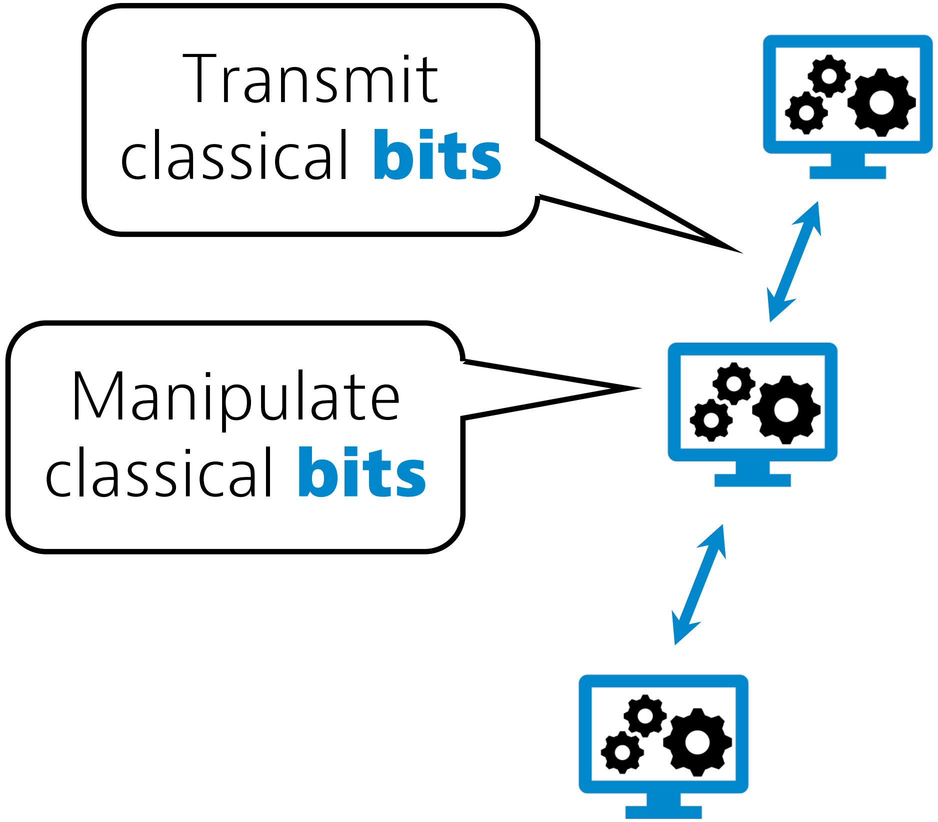
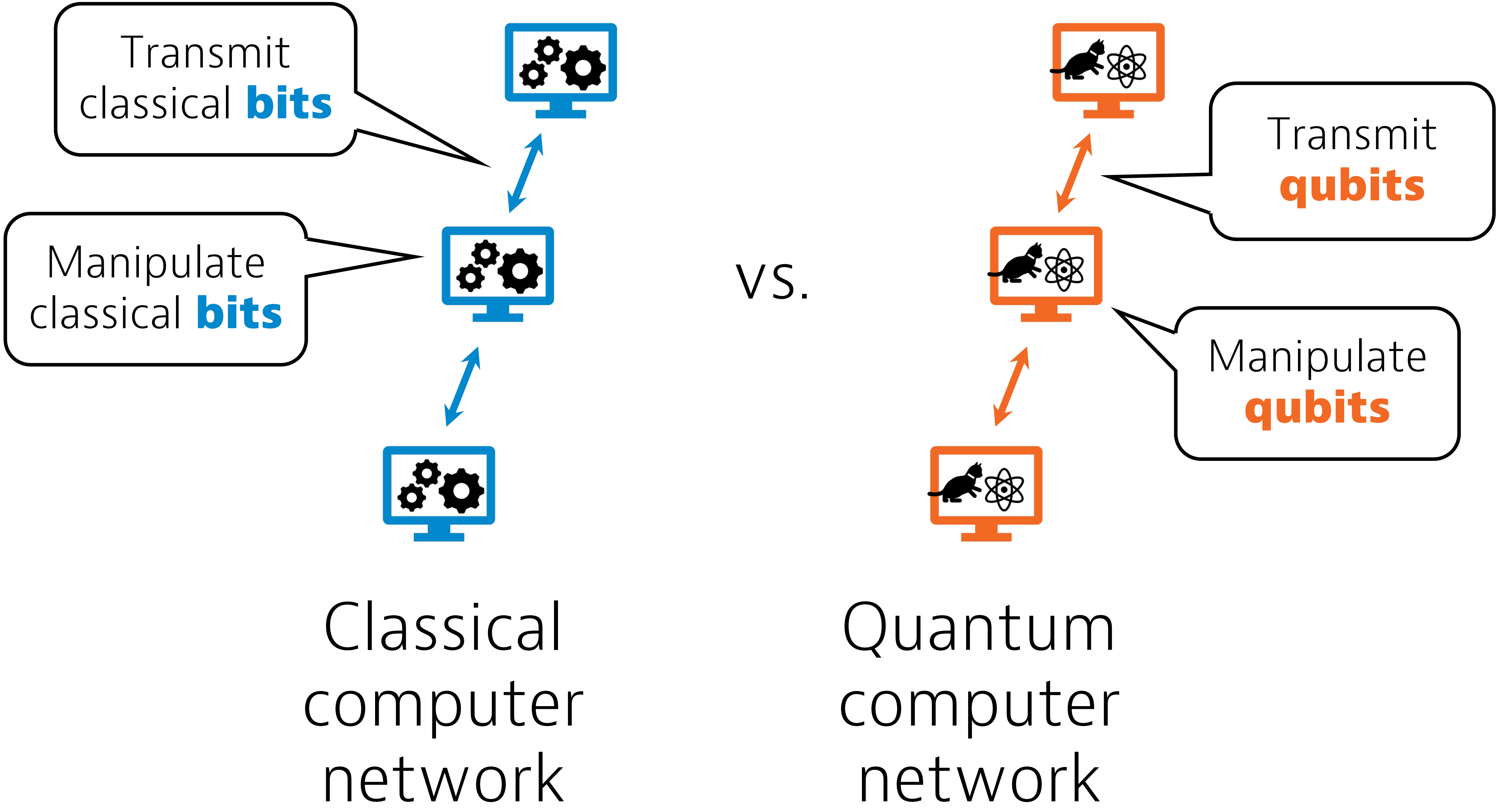
Distributed Quantum Advantage



Jukka Suomela
Aalto University



Classical computer network



Transmit classical bits
Manipulate classical bits
vs.
Transmit qubits
Manipulate qubits
Classical computer network
Quantum computer network

Do we gain anything?

Formally

- What task can be solved asymptotically faster:
  - in ***quantum-LOCAL*** vs. ***LOCAL*** ?
  - in ***quantum-CONGEST*** vs. ***CONGEST*** ?
- How do we reason about such questions?

Cornerstones

1. Quantum physics is **nonlocal**
2. But you **cannot violate causality**

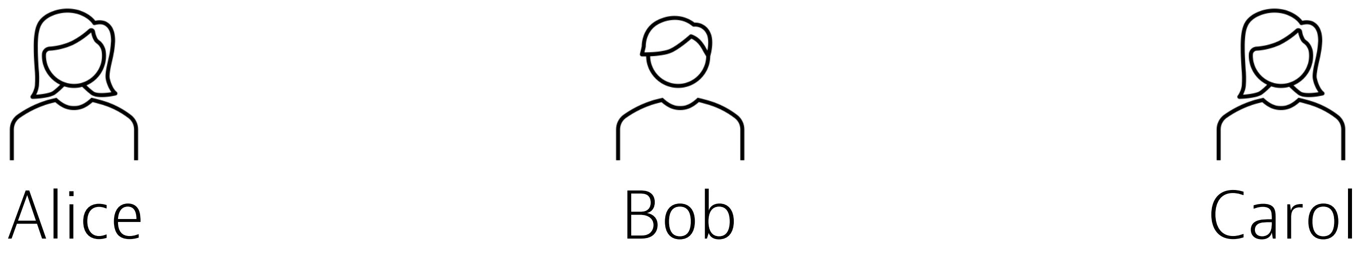
Quantum nonlocality

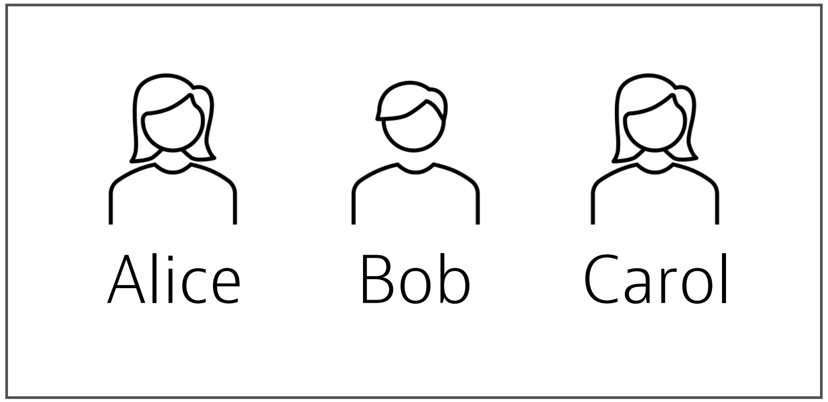
Quantum nonlocality

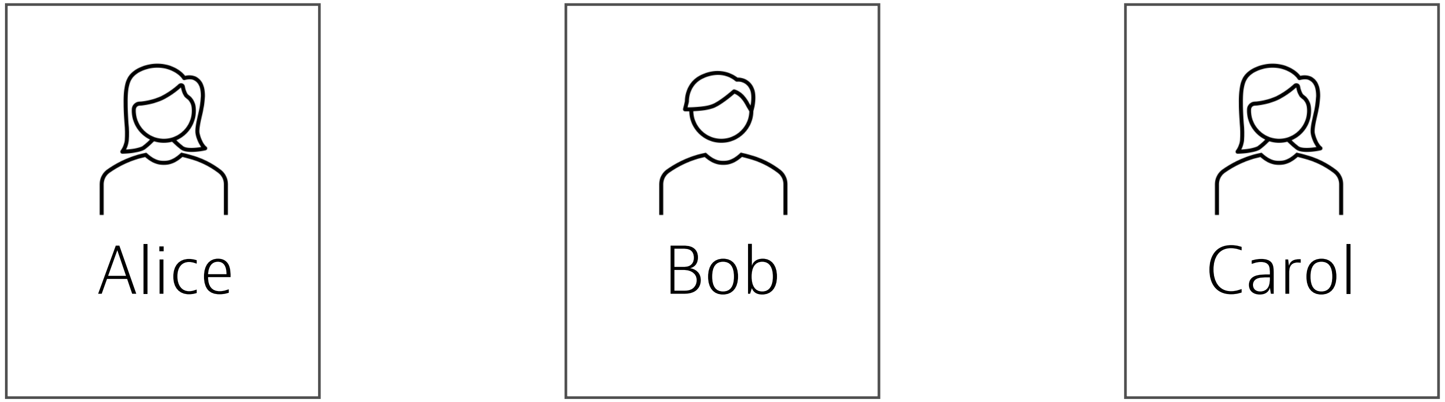
- I will **not** do a proper history overview
  - sorry, no *Einstein–Podolsky–Rosen paradox*
  - sorry, no *Bell inequalities*
- I will give a single concrete example that makes the point clear to a computer scientist: ***Greenberger–Horne–Zeilinger game***

GHZ game

Greenberger,
Horne, Zeilinger




Alice
Bob
Carol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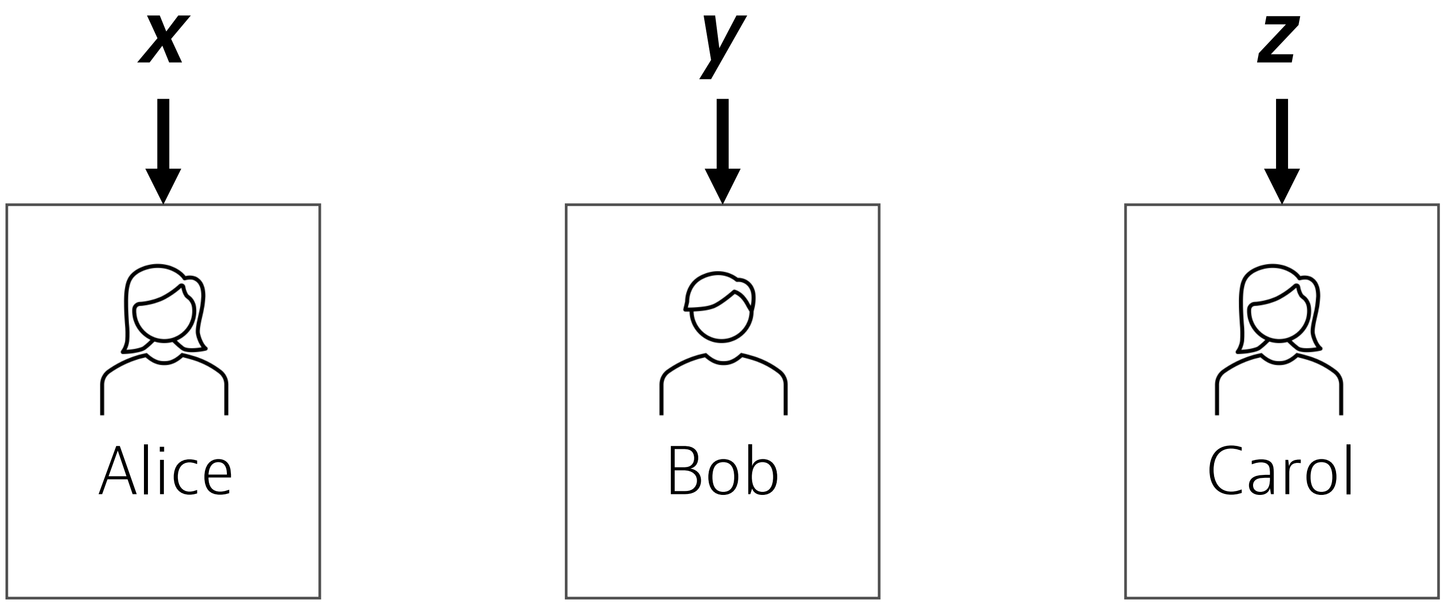
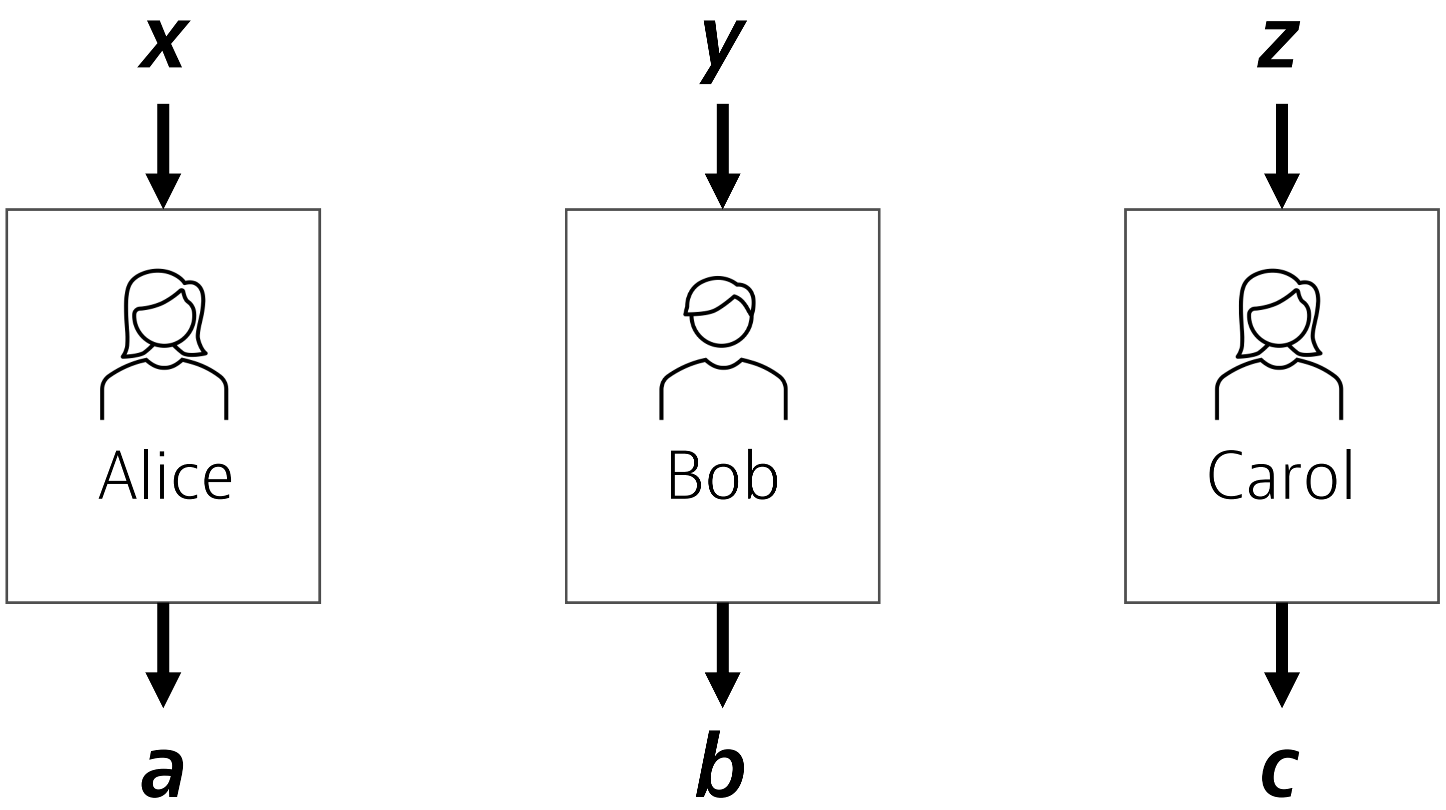
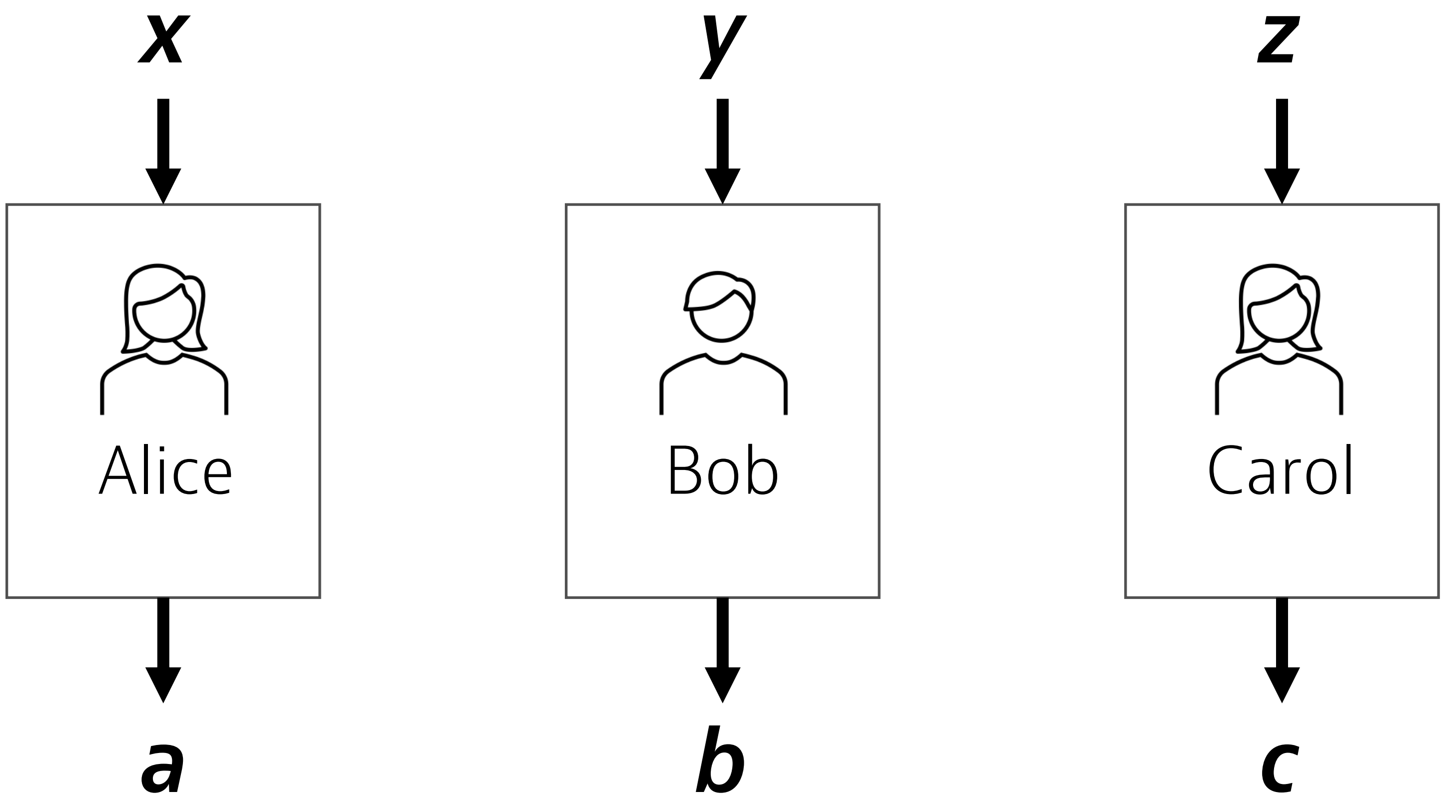
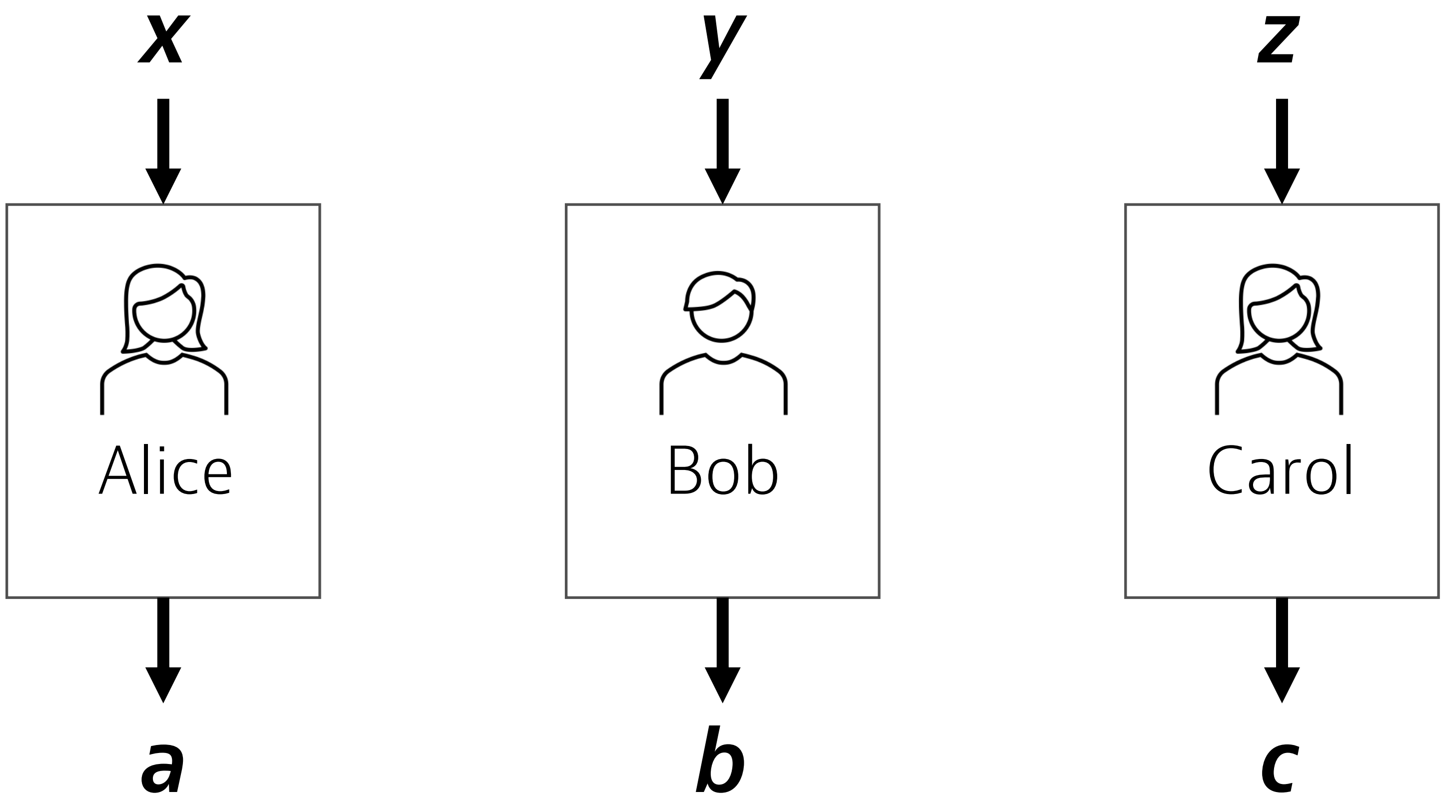

x
y
z
Alice
Bob
Carol


x
y
z
Alice
Bob
Carol
a
b
c

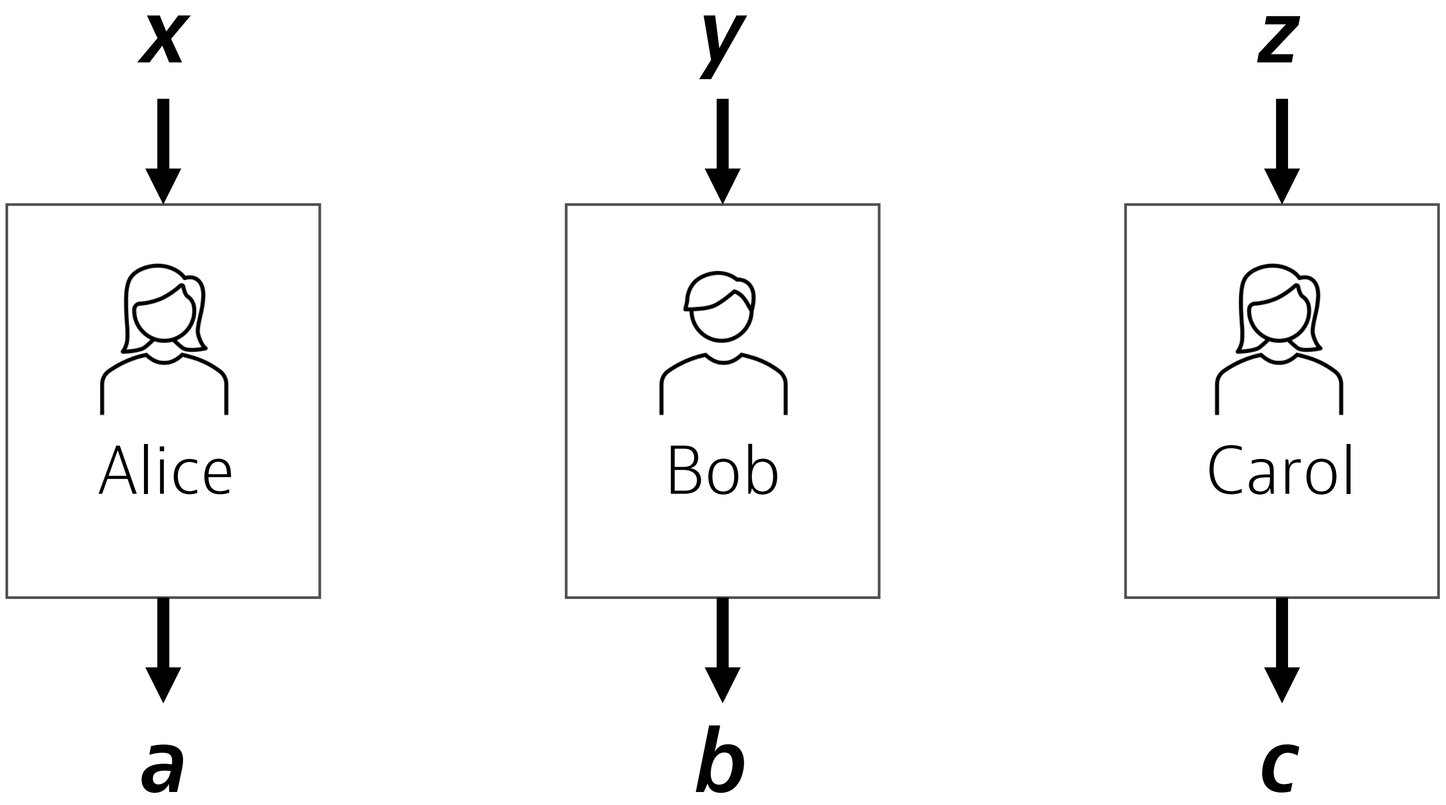
$x + y + z$	$a + b + c$
0	
1	
2	
3	



$x + y + z$	$a + b + c$
0	even
1	
2	odd
3	

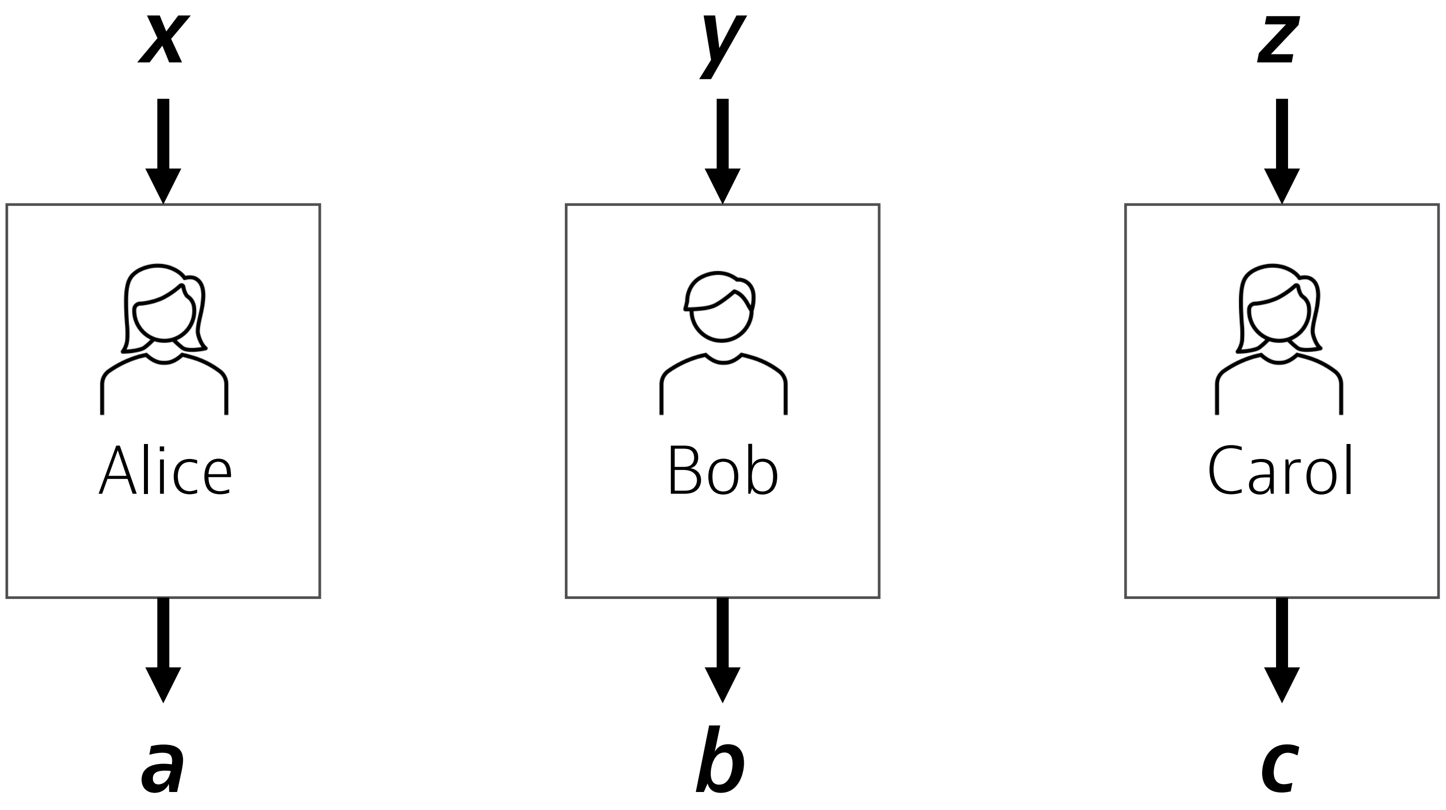


$x + y + z$	$a + b + c$
0	even
1	any
2	odd
3	any

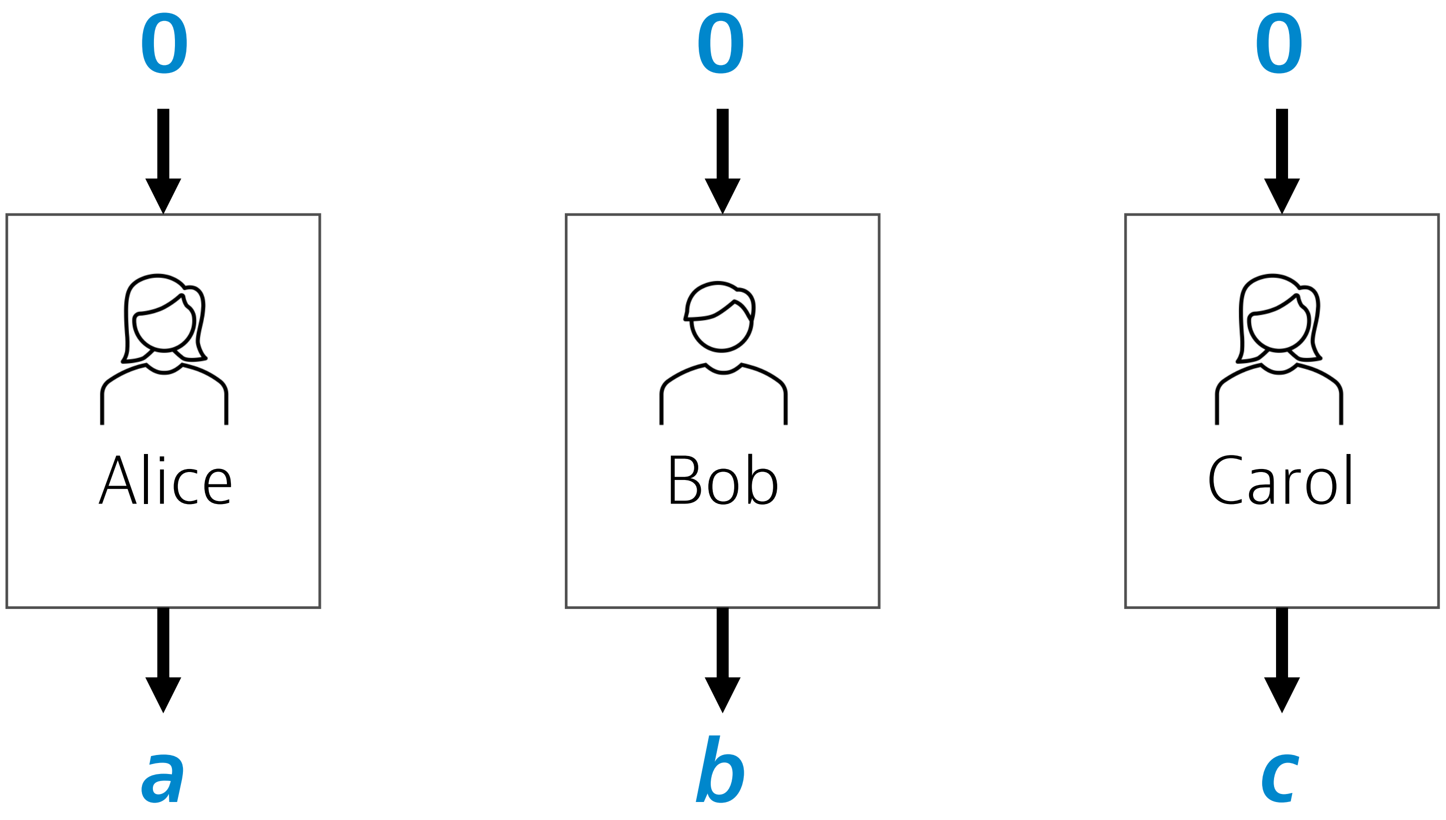


$x + y + z$	$a + b + c$
0	even
1	any
2	odd
3	any

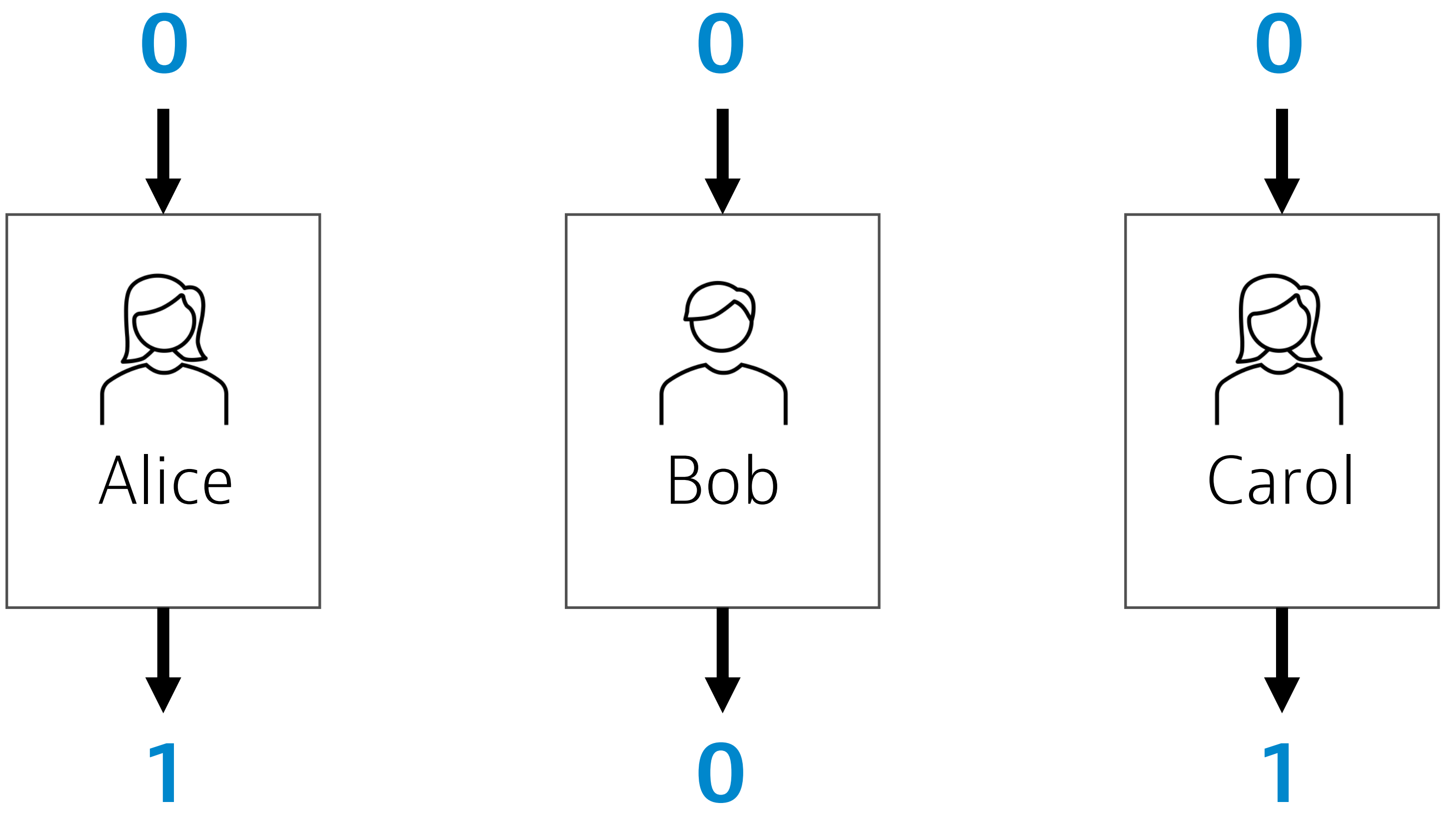
No classical solution!



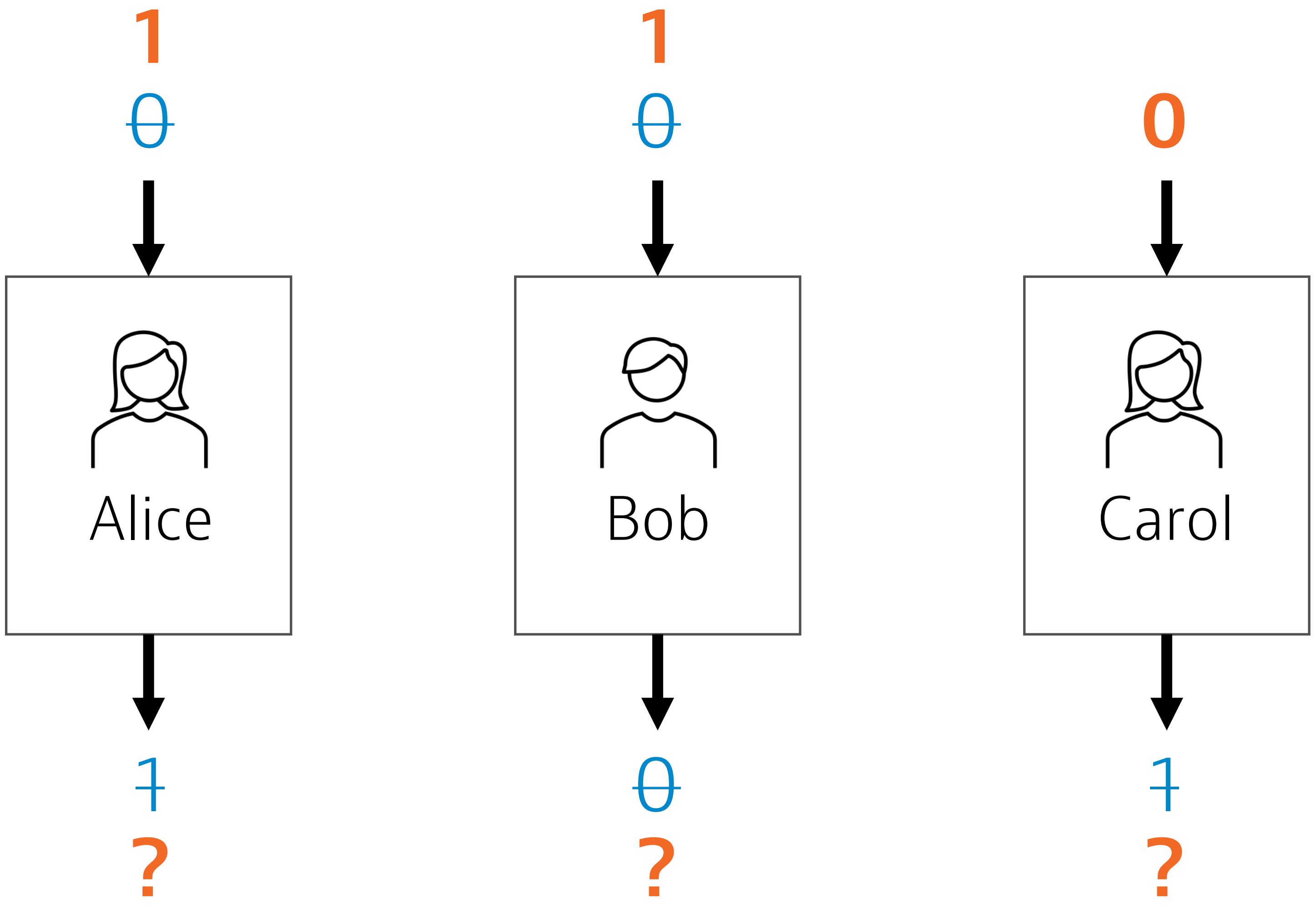
$x + y + z$	$a + b + c$
0	**even**
1	any
2	odd
3	any



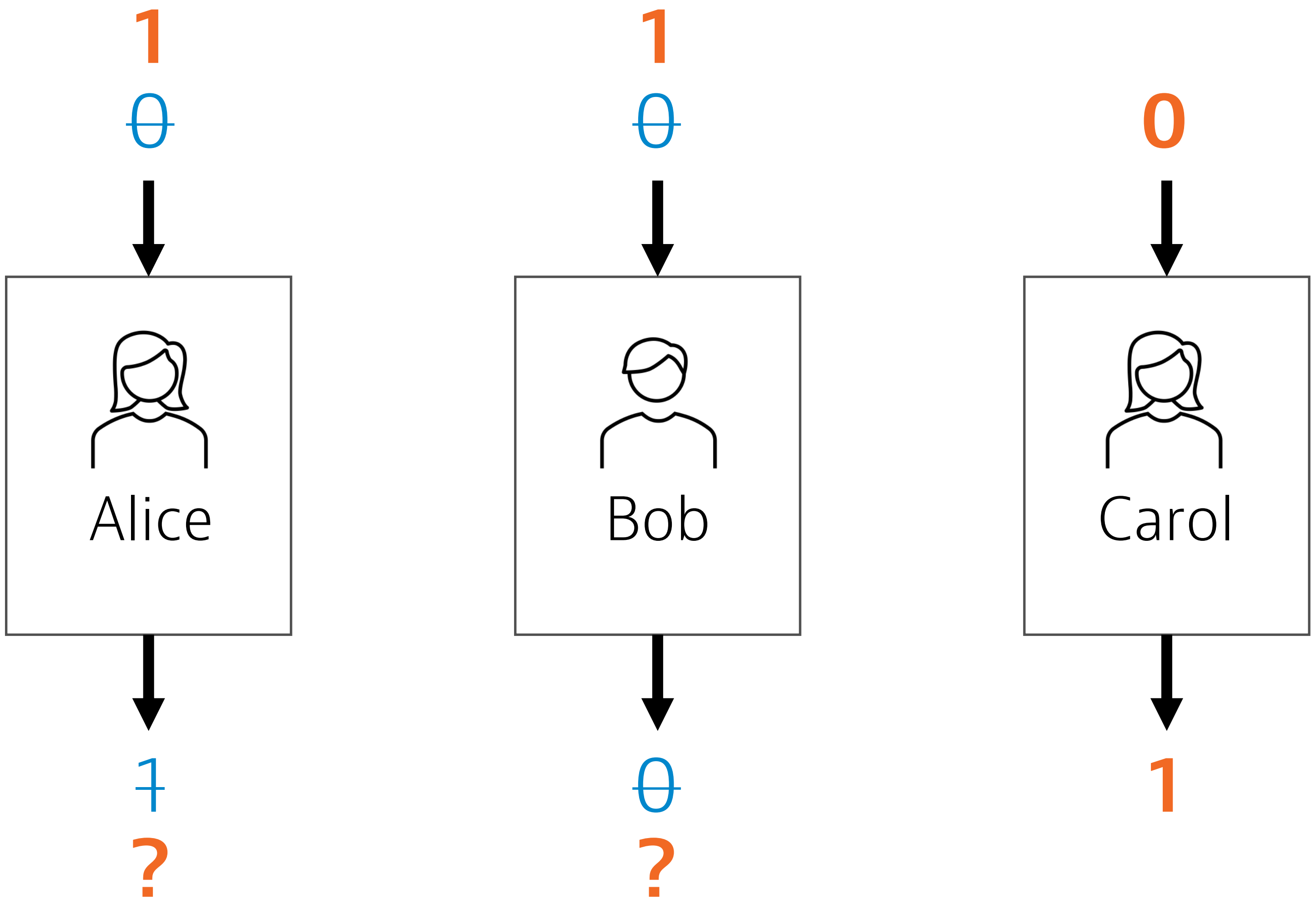
$x + y + z$	$a + b + c$
0	**even**
1	any
2	odd
3	any



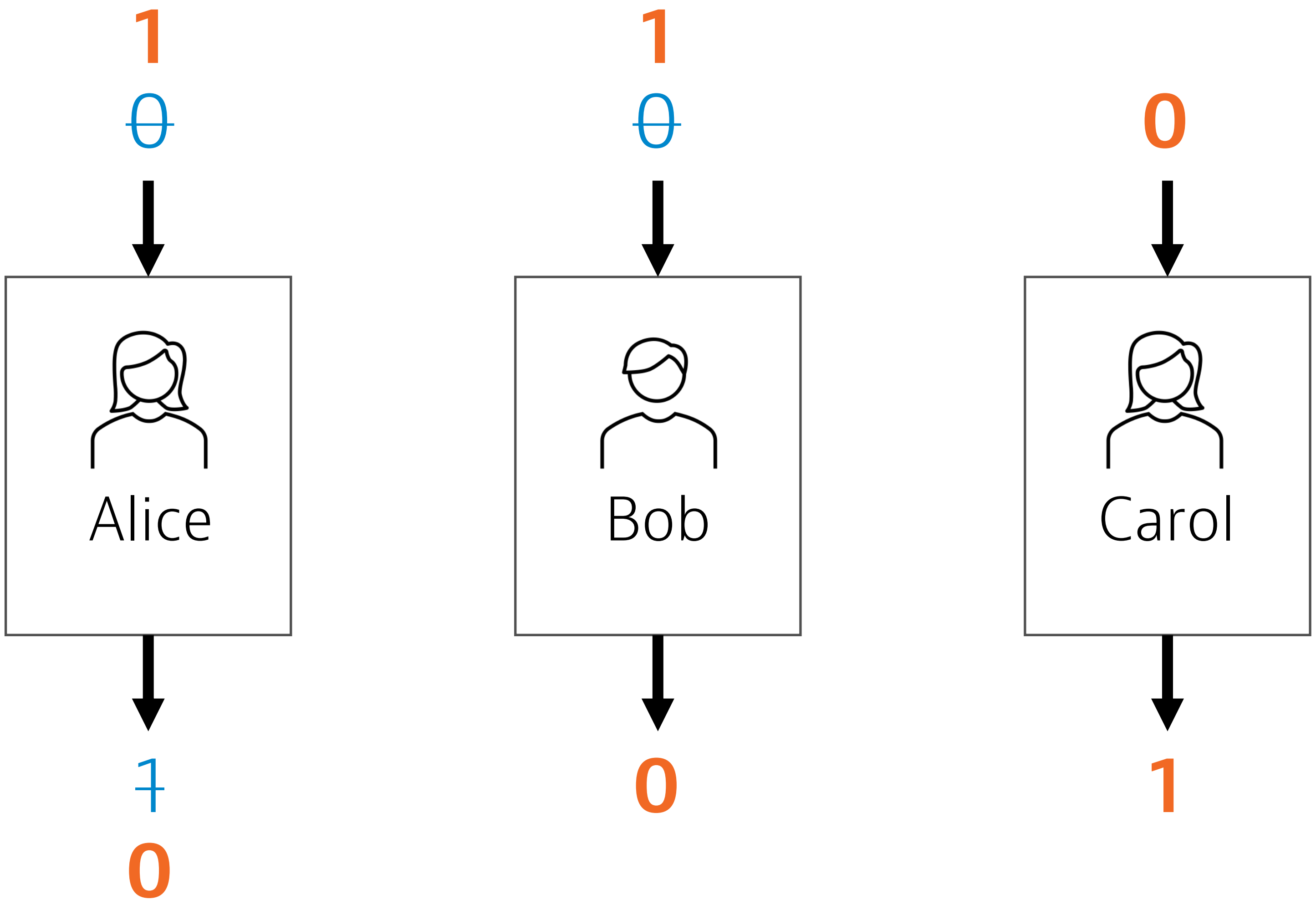
$x + y + z$	$a + b + c$
0	**even**
1	any
2	**odd**
3	any



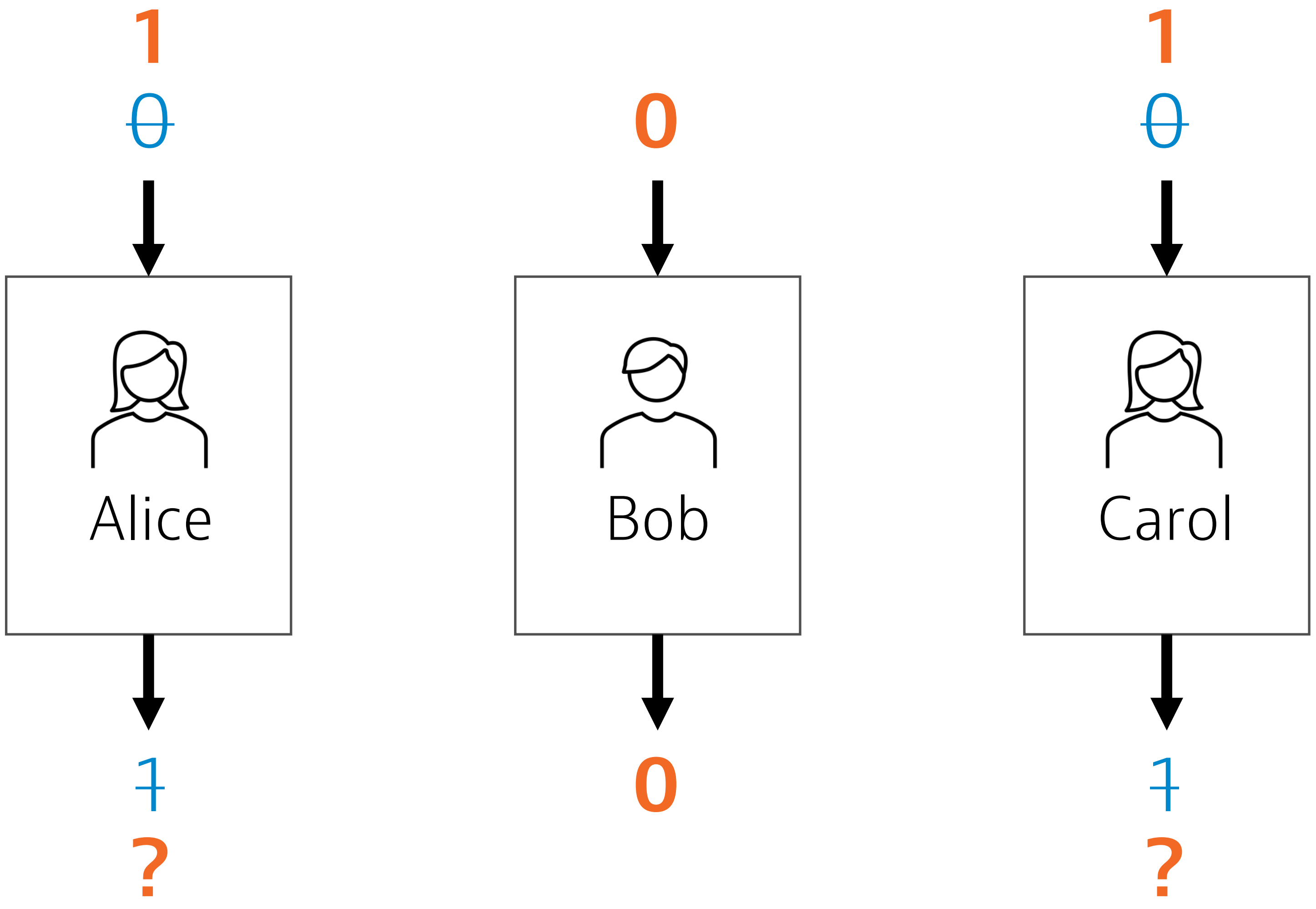
$x + y + z$	$a + b + c$
0	**even**
1	any
2	**odd**
3	any



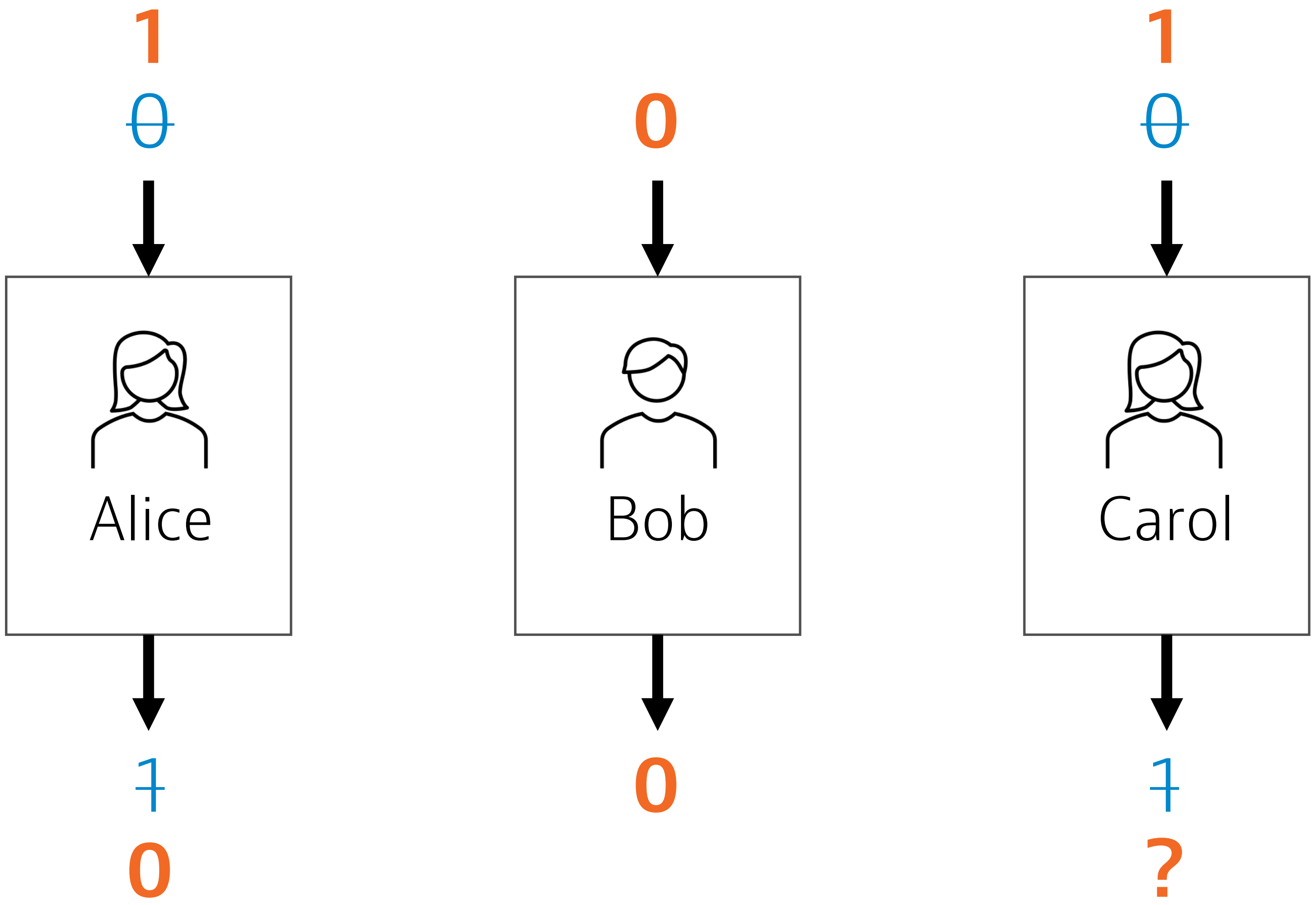
$x + y + z$	$a + b + c$
0	**even**
1	any
2	**odd**
3	any



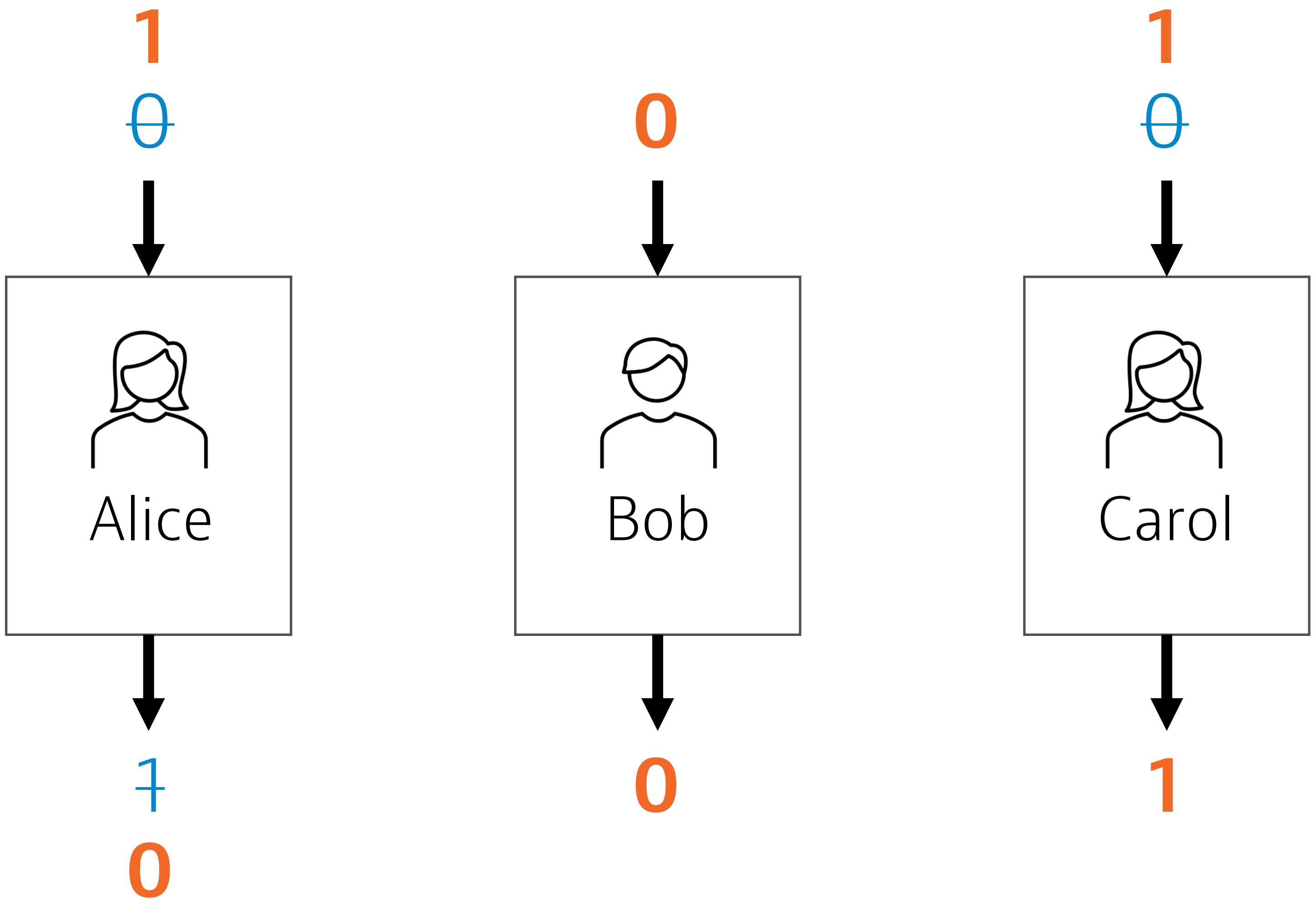
$x + y + z$	$a + b + c$
0	even
1	any
2	odd
3	any



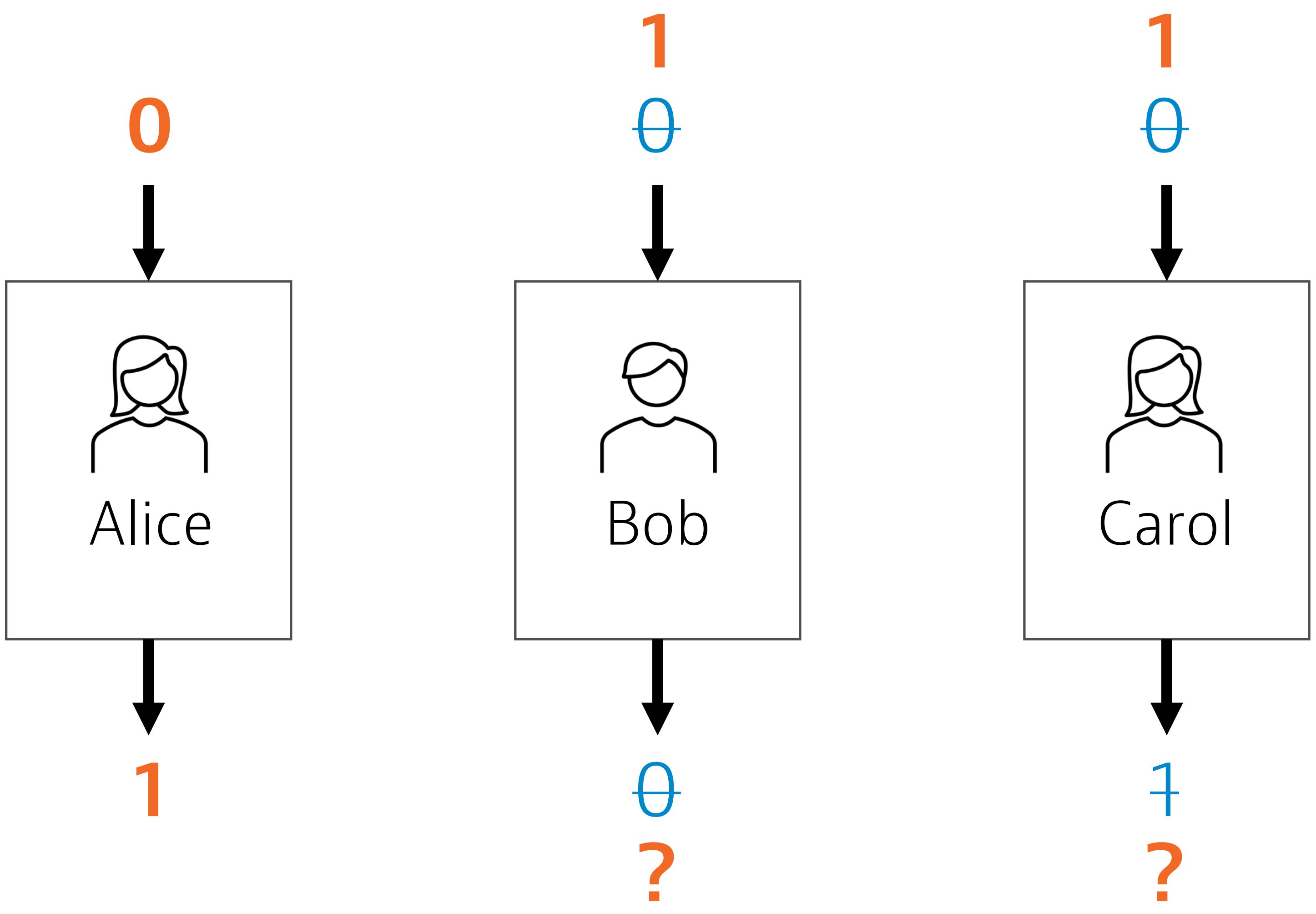
$x + y + z$	$a + b + c$
0	**even**
1	any
2	**odd**
3	any



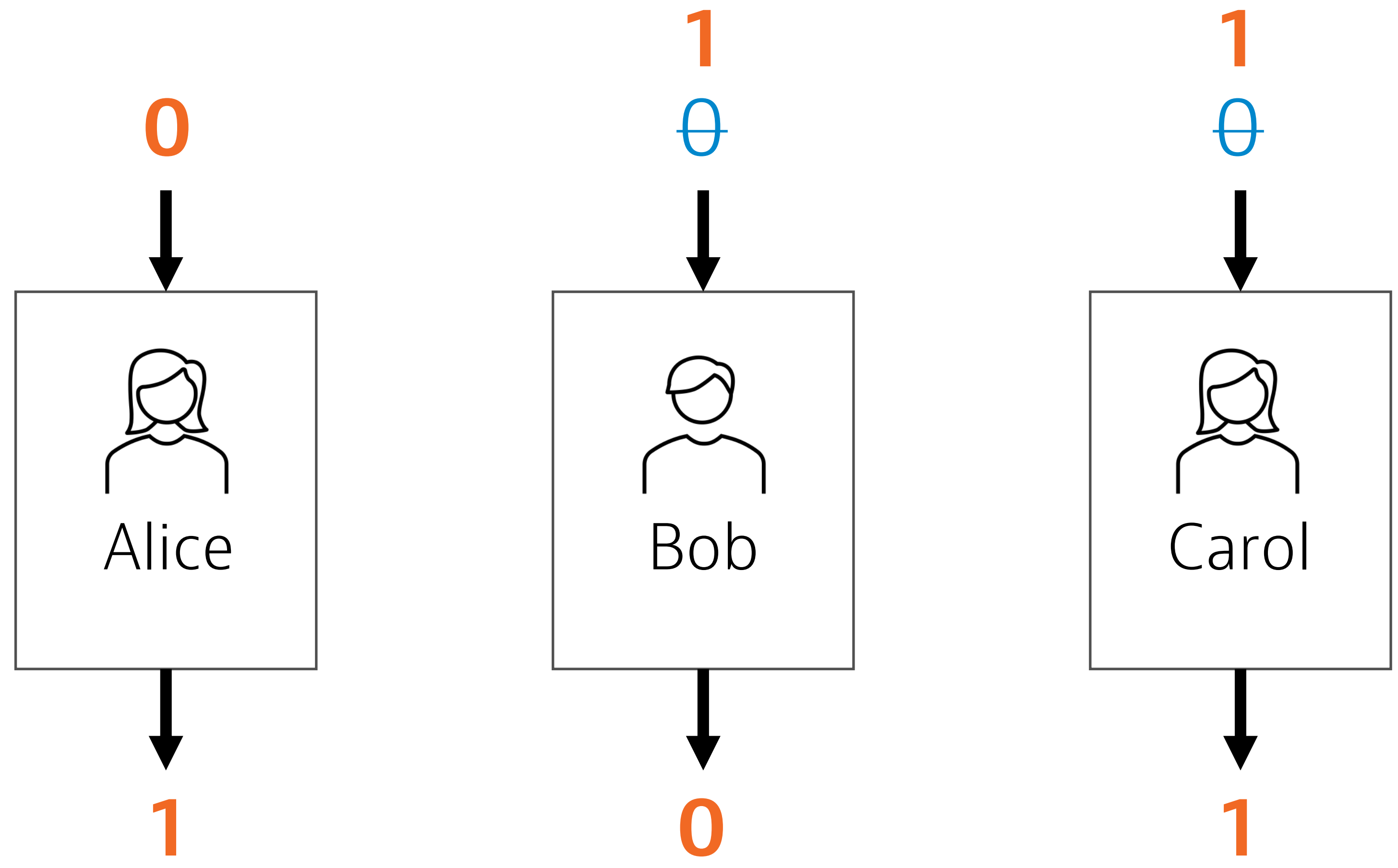
$x + y + z$	$a + b + c$
0	**even**
1	any
2	**odd**
3	any



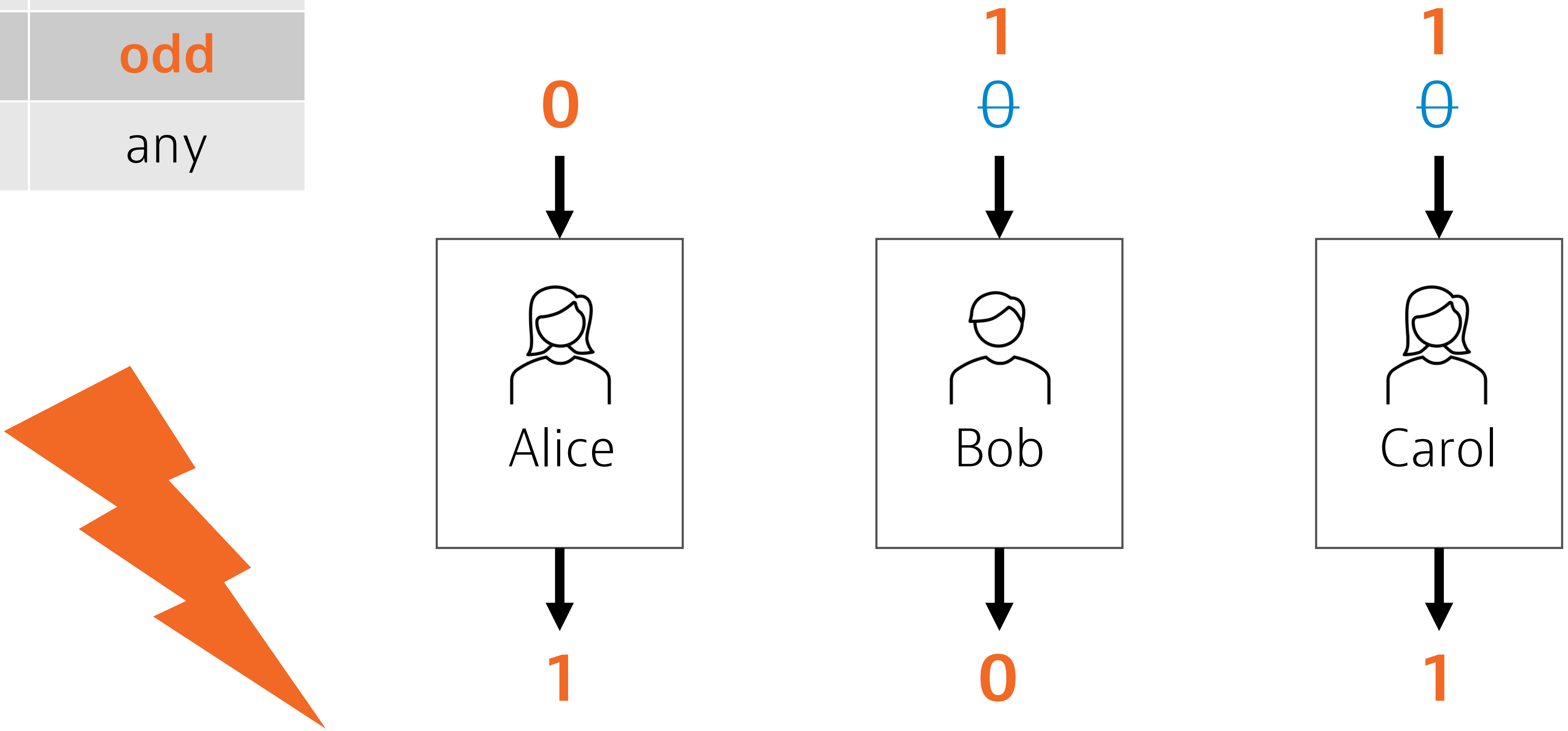
$x+y+z$	$a+b+c$
0	**even**
1	any
2	**odd**
3	any



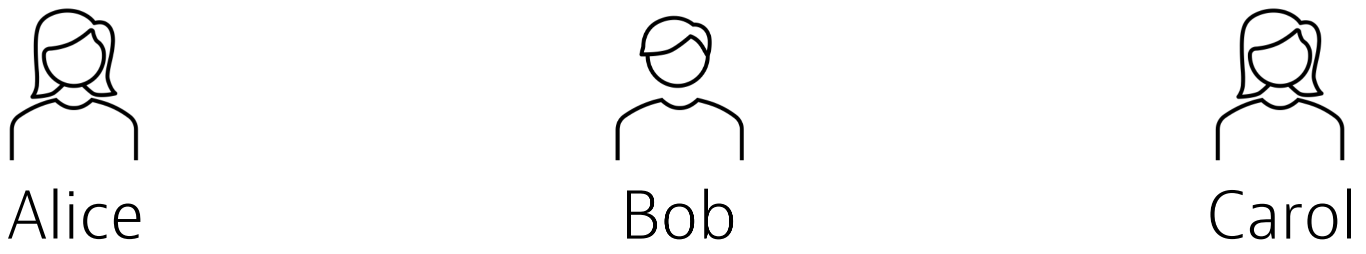
$x + y + z$	$a + b + c$
0	**even**
1	any
2	**odd**
3	any

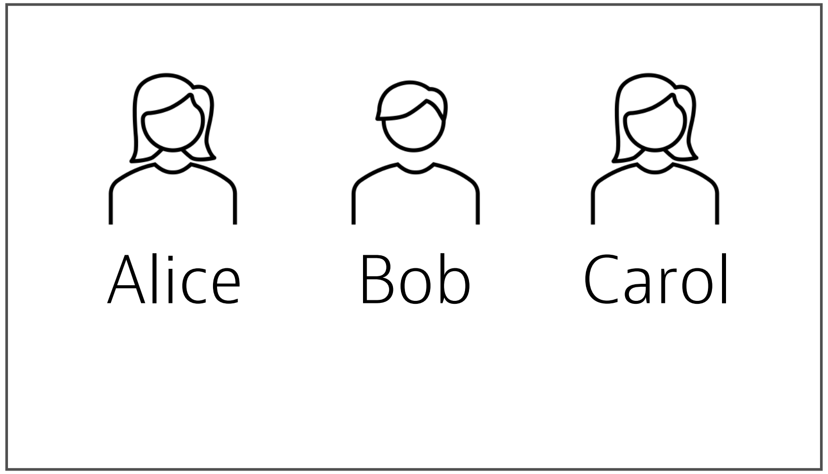


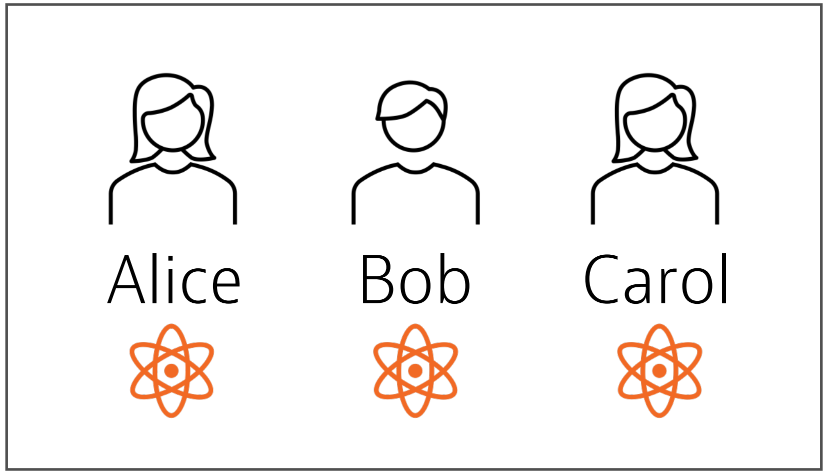
$x + y + z$	$a + b + c$
0	**even**
1	any
2	**odd**
3	any

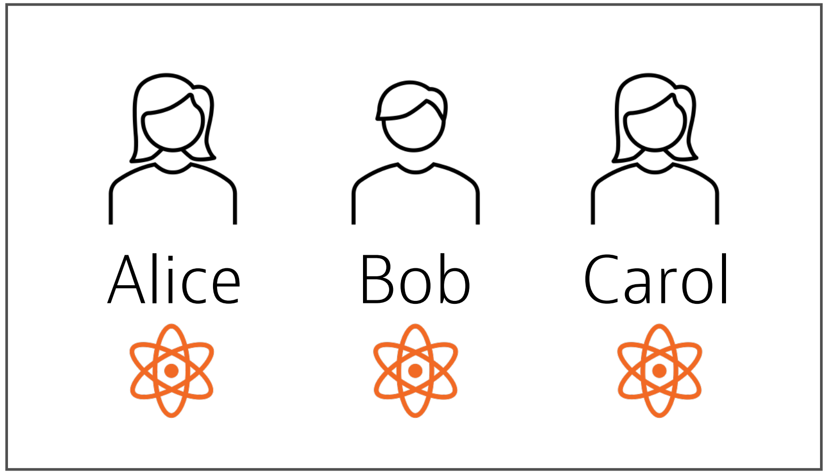


Quantum strategy

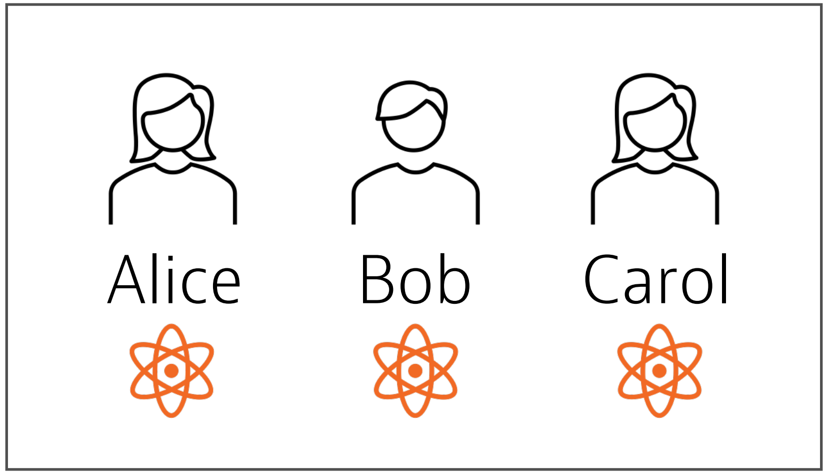


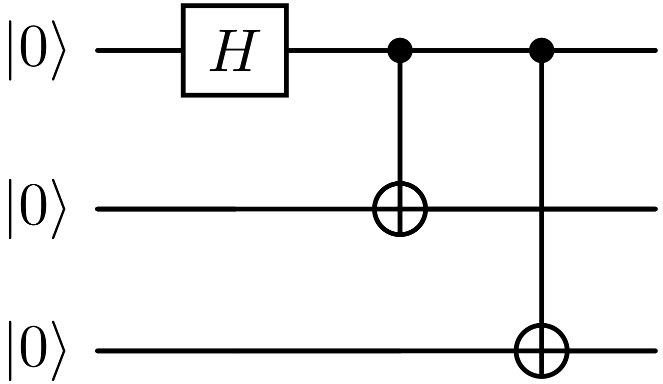

Alice
Bob
Carol


Alice
Bob
Carol

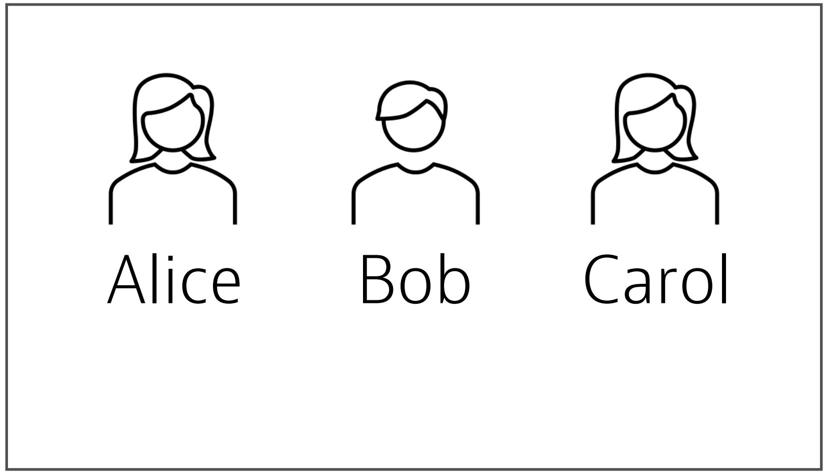


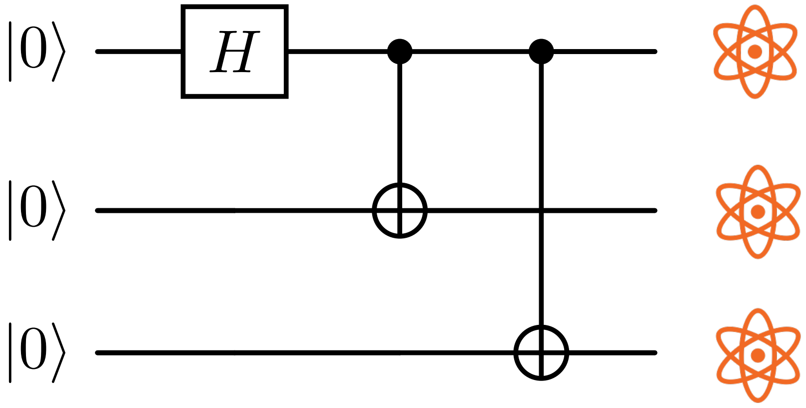
$$|\mathrm{GHZ}\rangle = \frac{|000\rangle + |111\rangle}{\sqrt{2}}$$



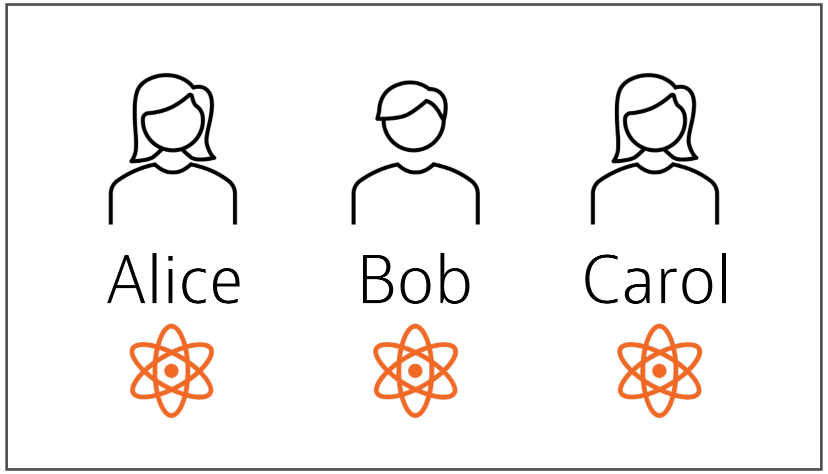


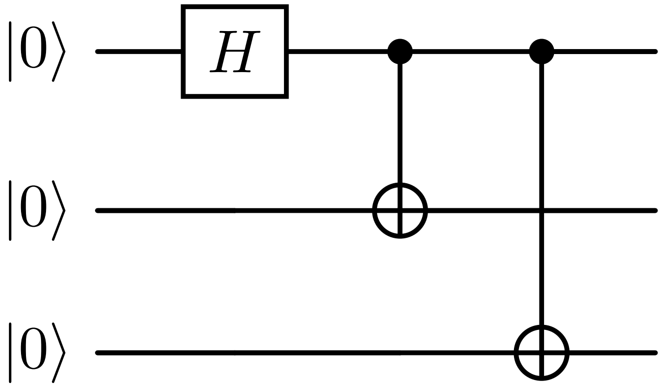
$$|\mathrm{GHZ}\rangle = \frac{|000\rangle + |111\rangle}{\sqrt{2}}$$



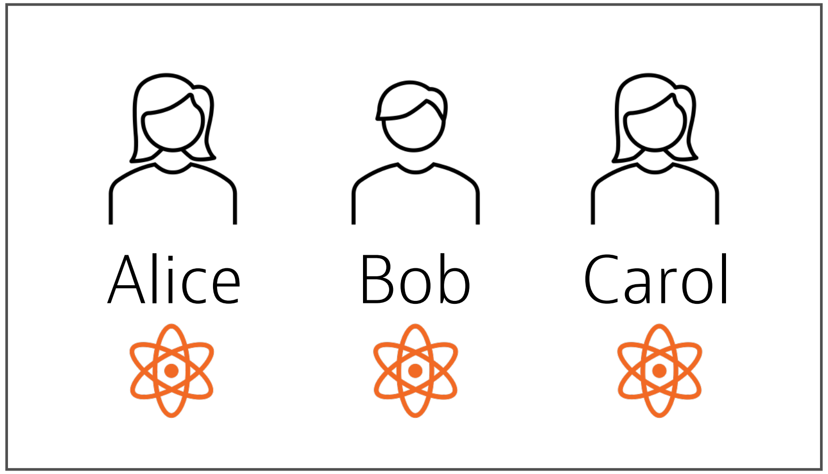


$$|\mathrm{GHZ}\rangle = \frac{|000\rangle + |111\rangle}{\sqrt{2}}$$

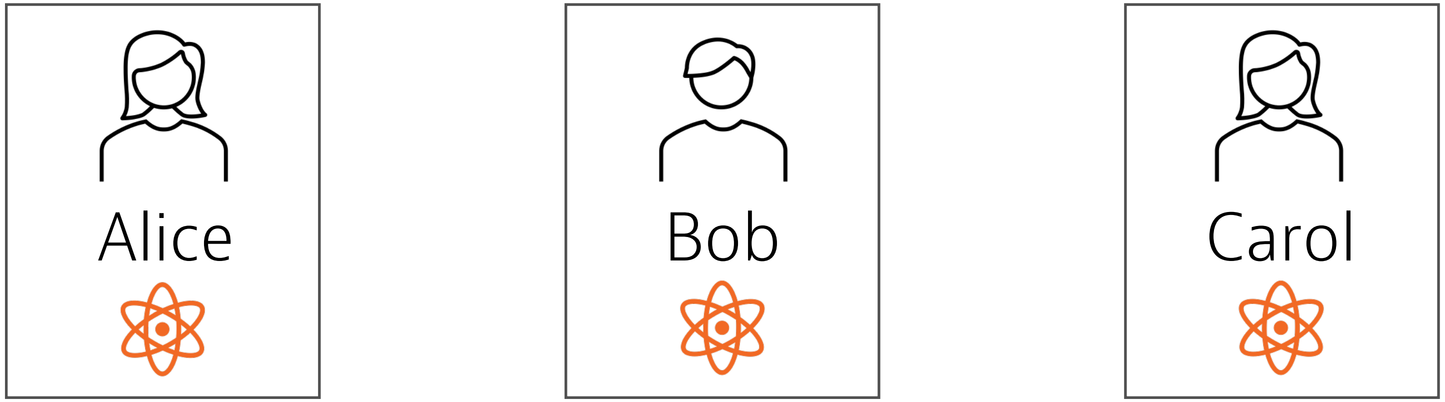


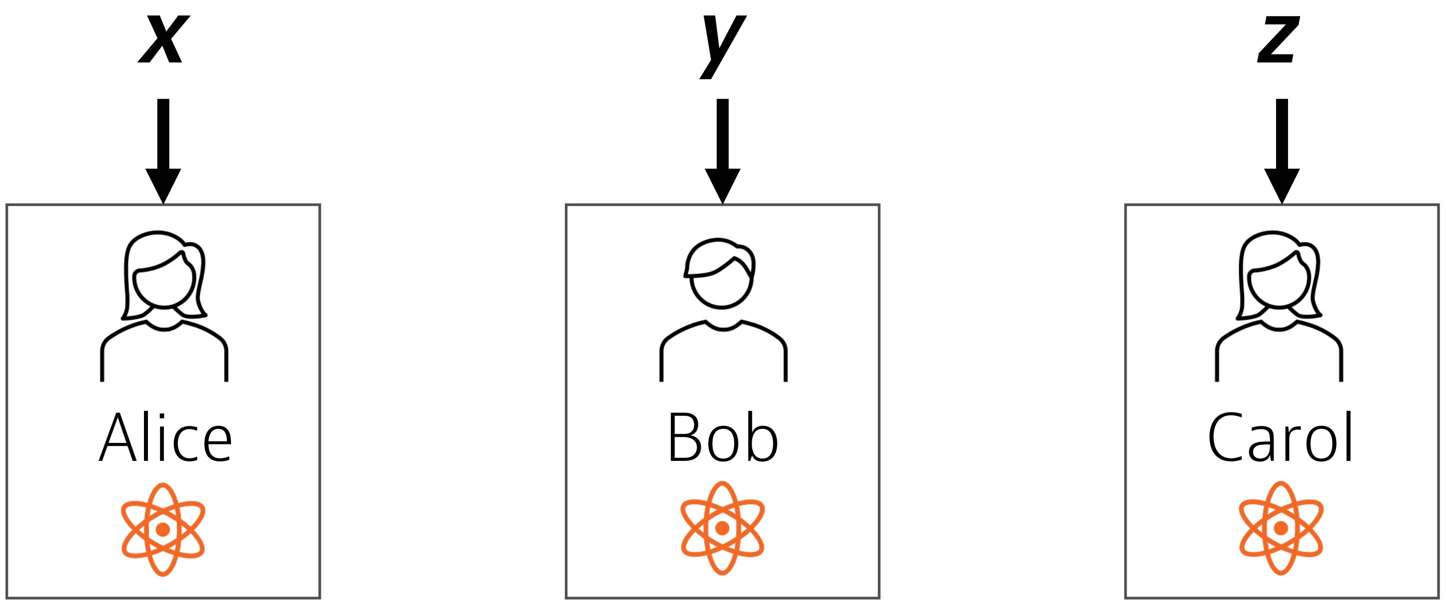


$$|\mathrm{GHZ}\rangle = \frac{|000\rangle + |111\rangle}{\sqrt{2}}$$

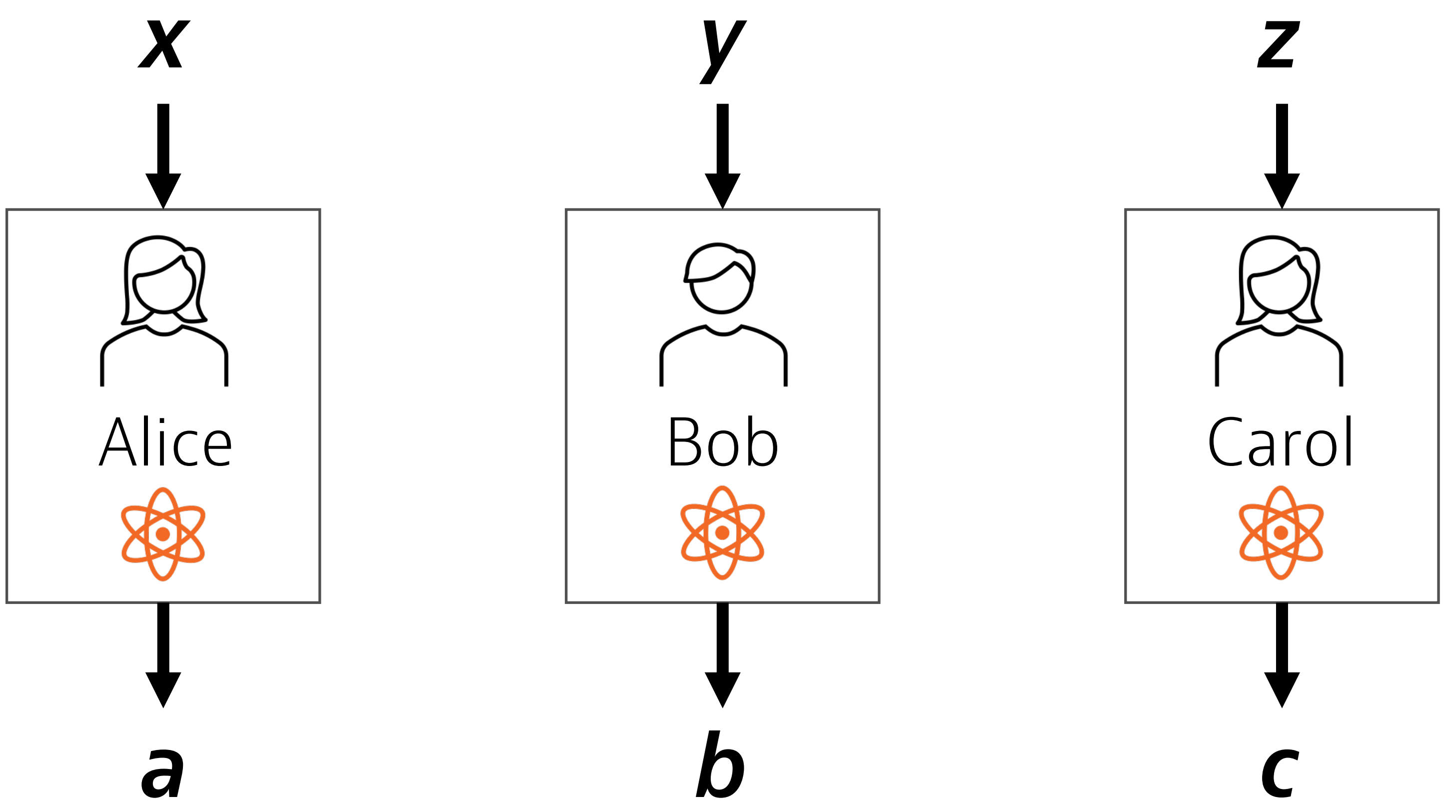


*Example: three **photons** with entangled polarization — all horizontal or all vertical*

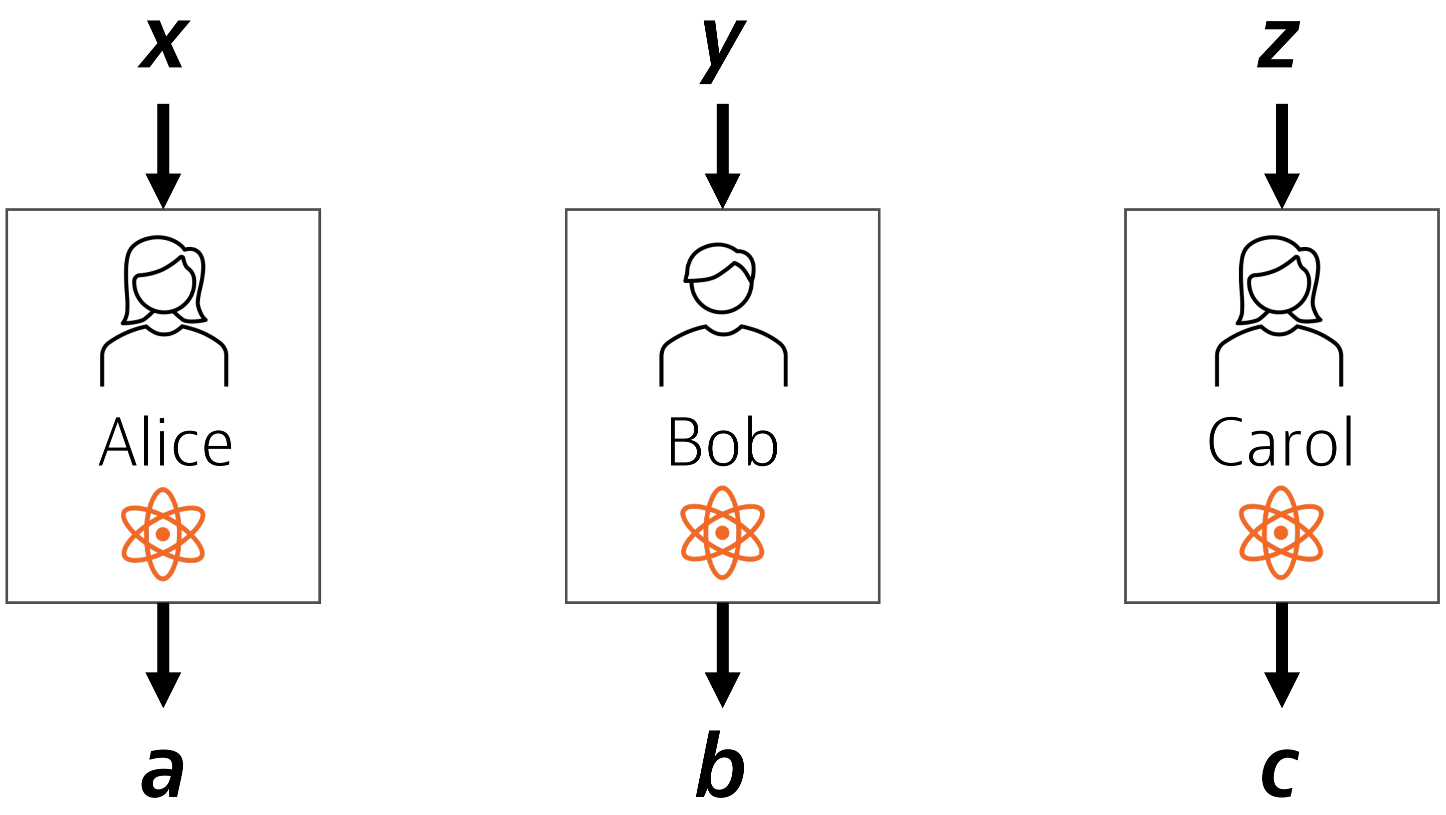

Alice
Bob
Carol



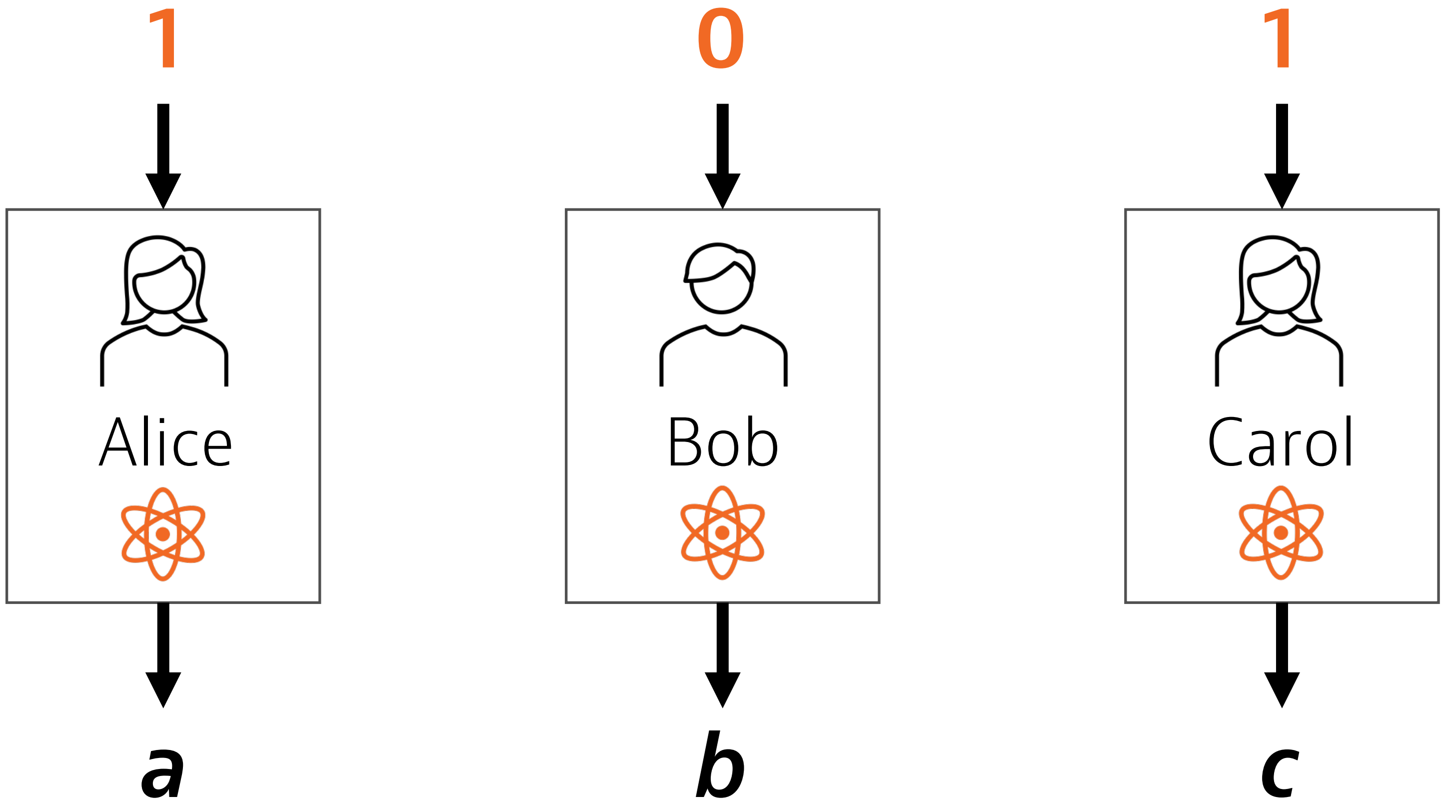
x
y
z
Alice
Bob
Carol


x
y
z
Alice
Bob
Carol
a
b
c

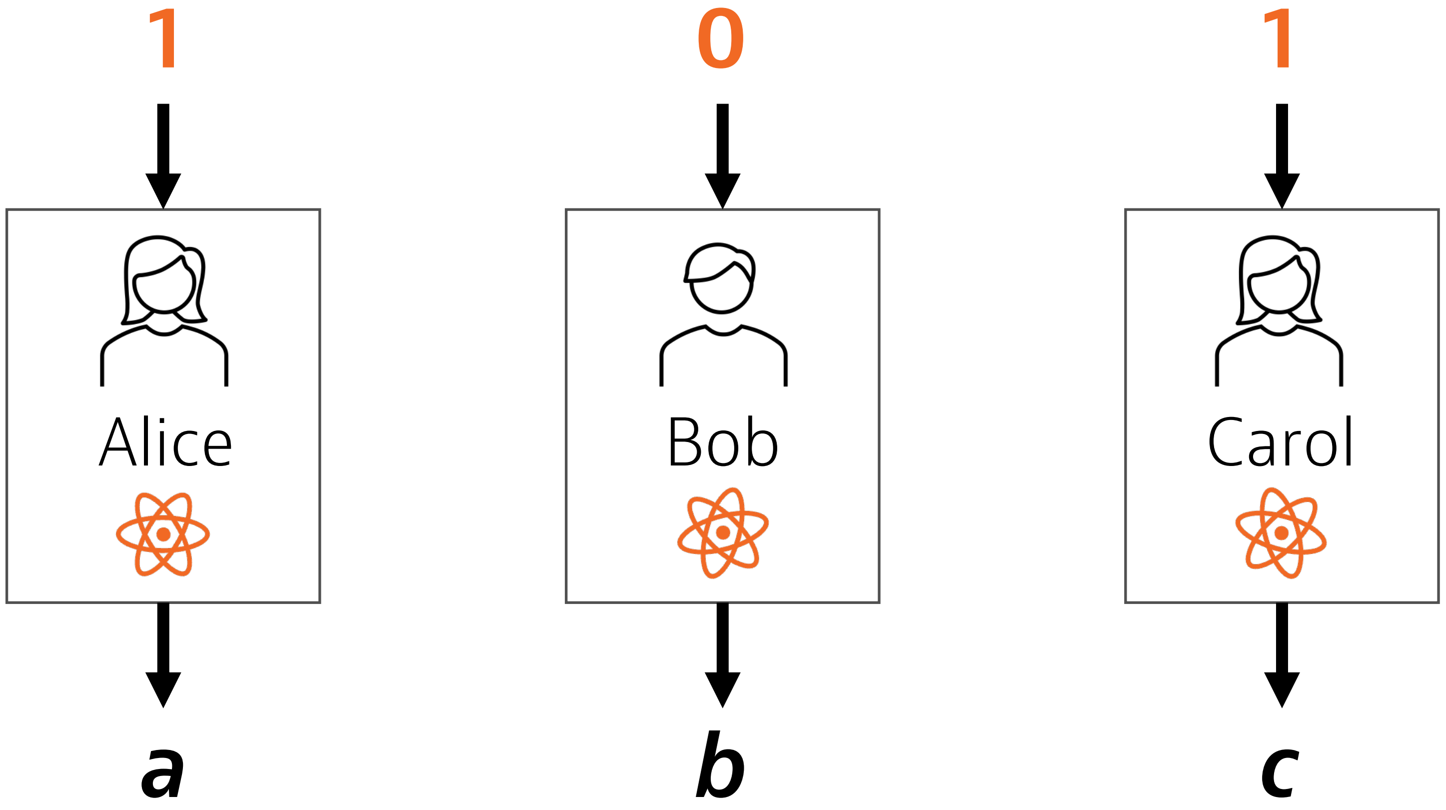
$x + y + z$	$a + b + c$
0	even
1	any
2	odd
3	any



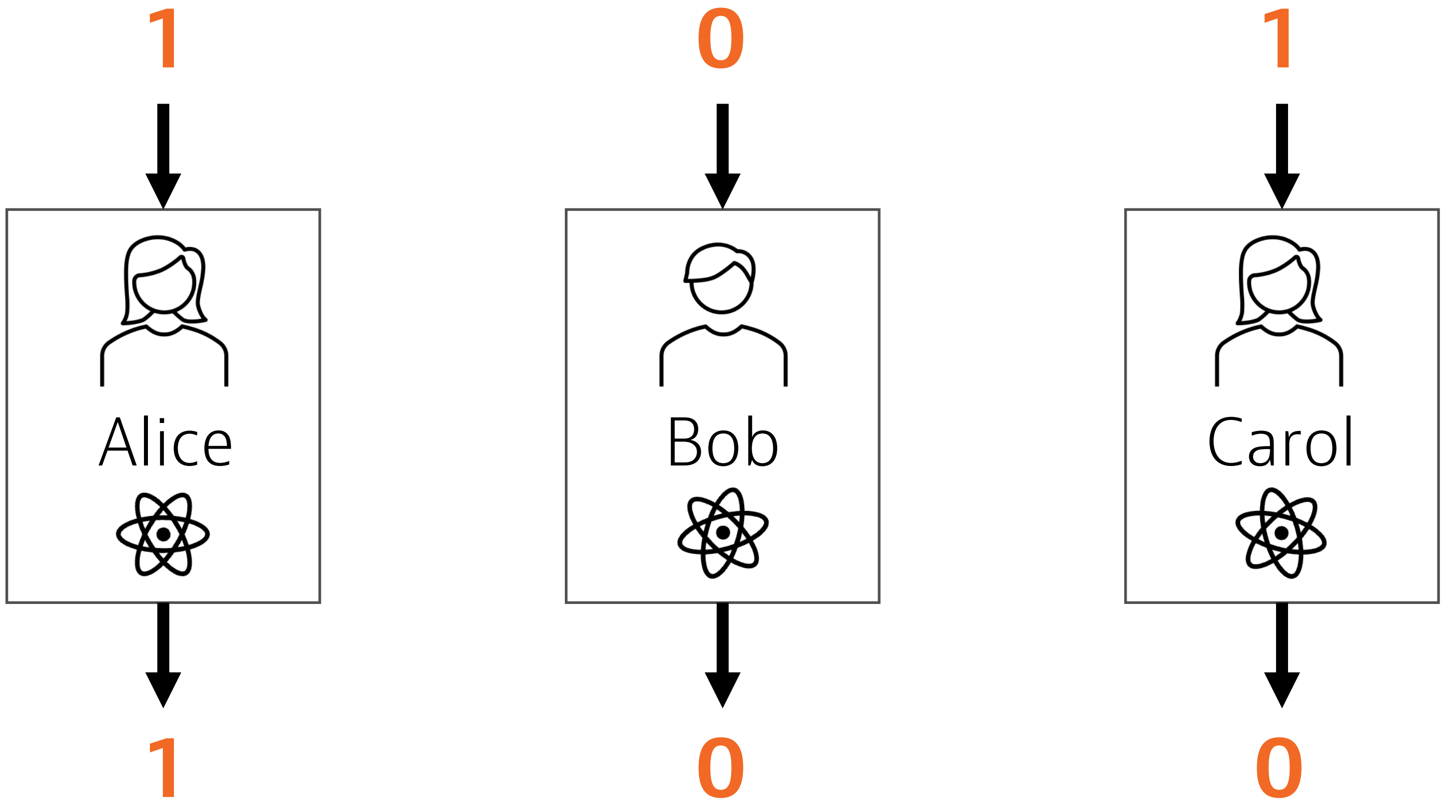
$x + y + z$	$a + b + c$
0	even
1	any
2	odd
3	any



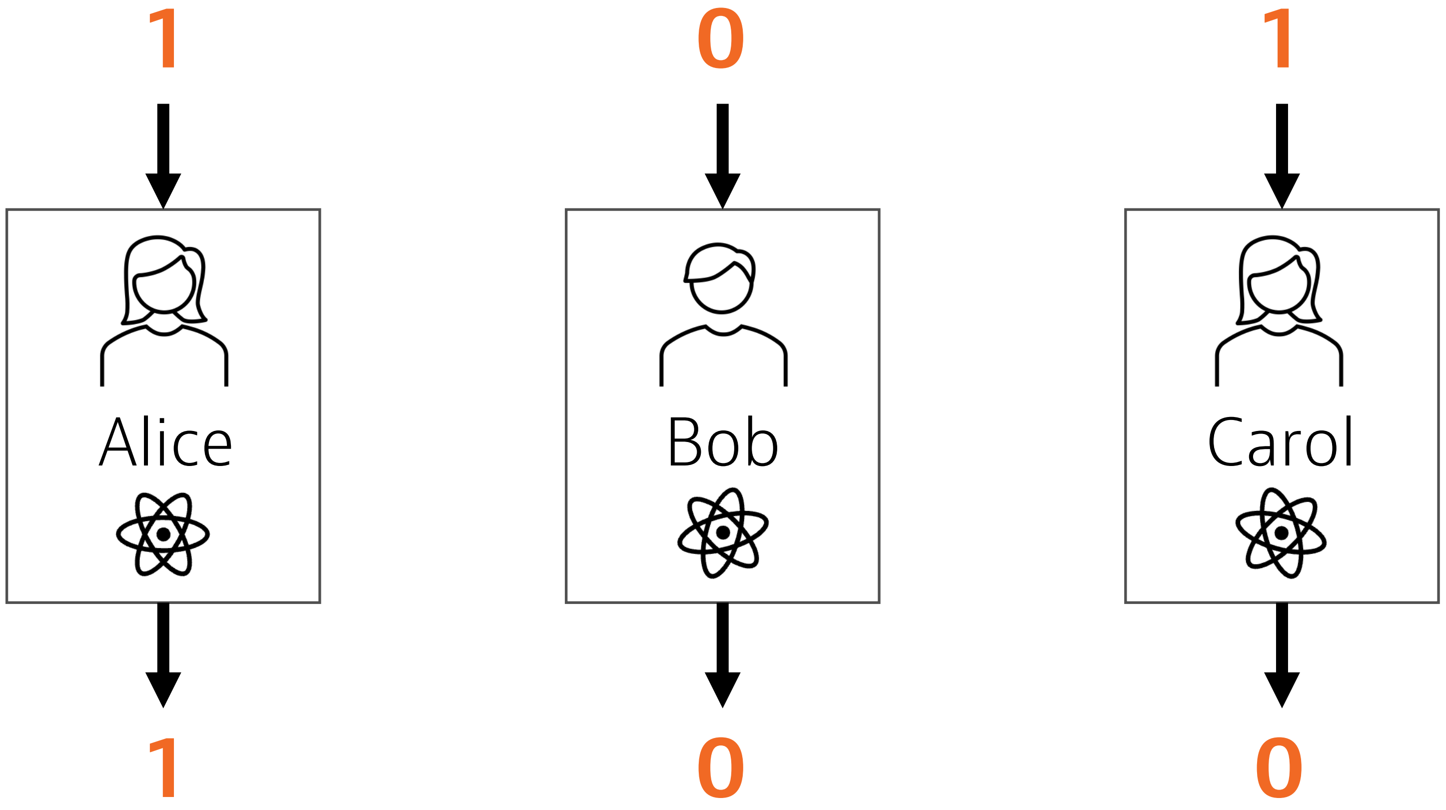
$x + y + z$	$a + b + c$
0	even
1	any
2	odd
3	any



$x + y + z$	$a + b + c$
0	even
1	any
2	odd
3	any

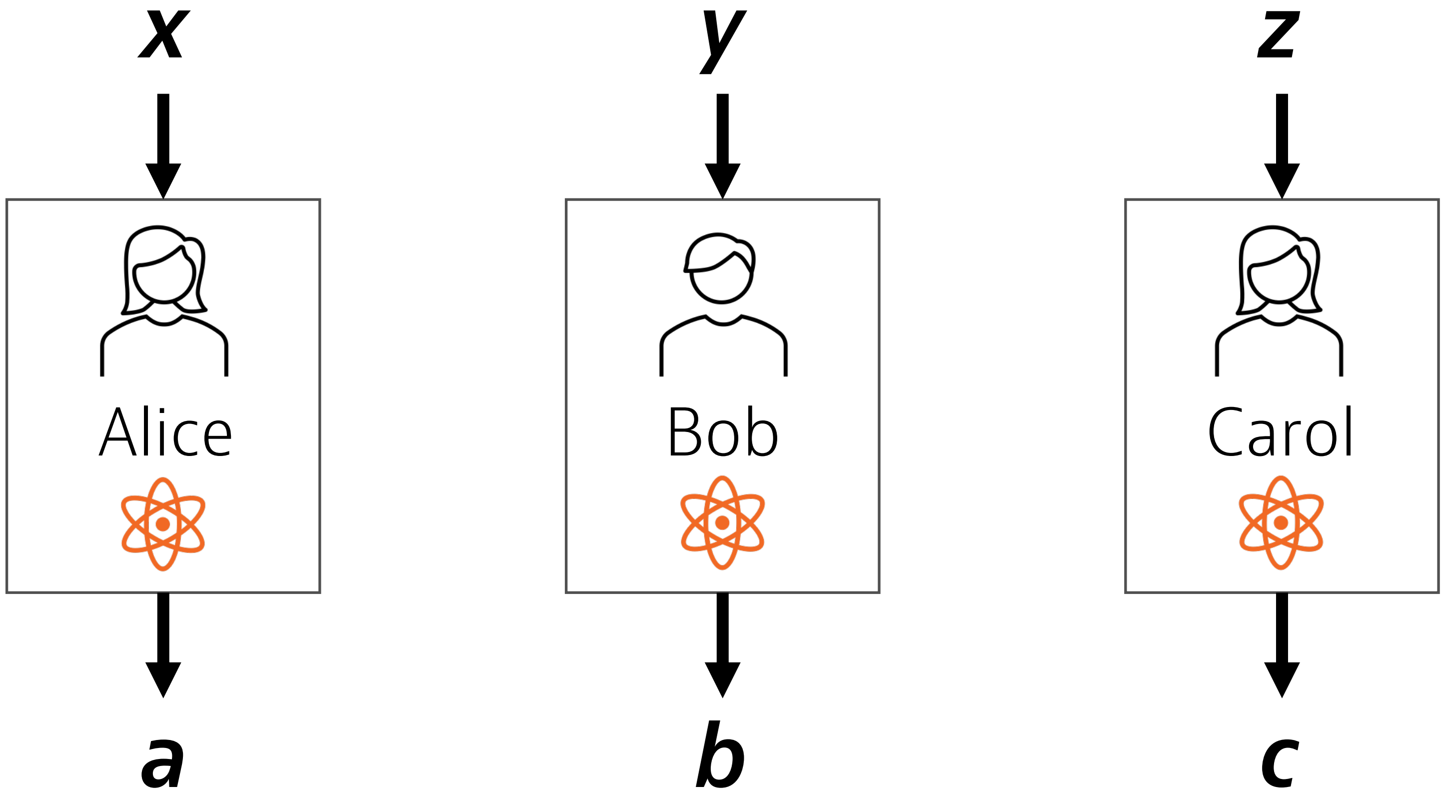


$x + y + z$	$a + b + c$
0	even
1	any
2	odd
3	any

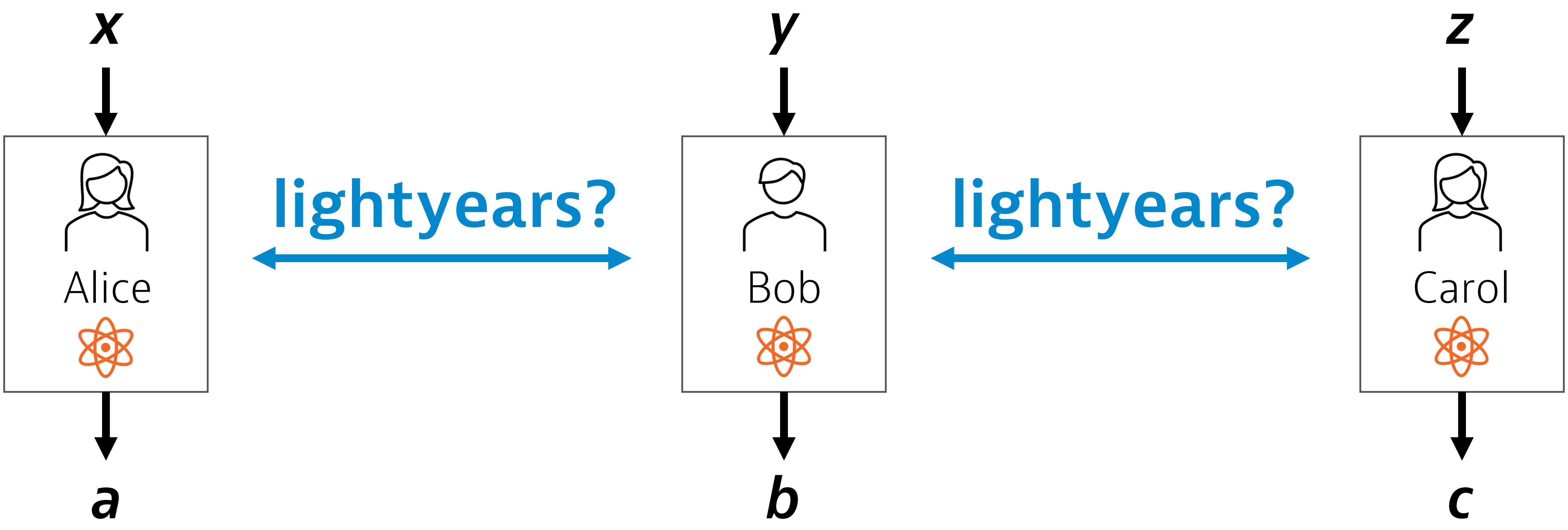


$x + y + z$	$a + b + c$
0	even
1	any
2	odd
3	any

Works with probability 1



No classical explanation without communication



Causality

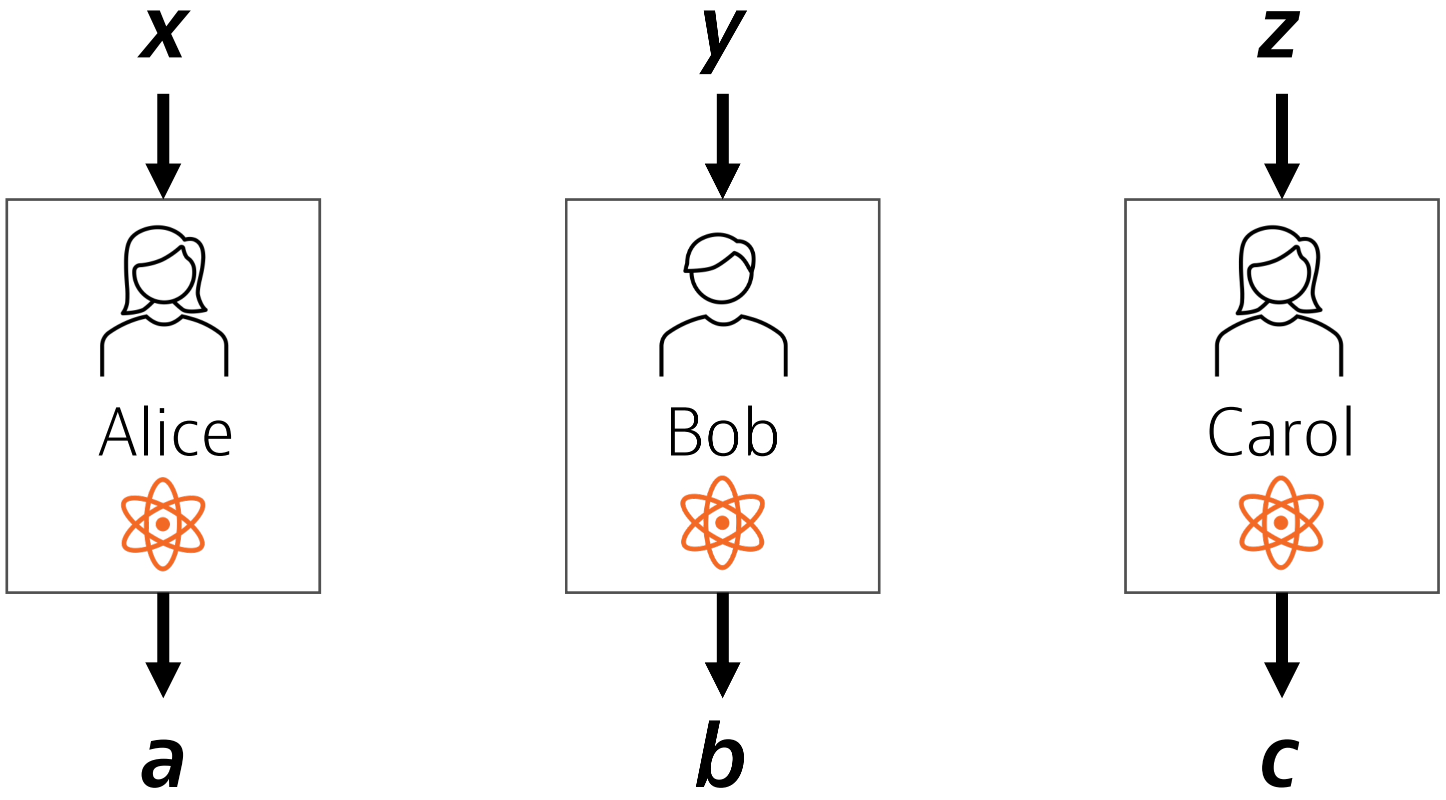
Cornerstones

1. Quantum physics is **nonlocal**
2. But you **cannot violate causality**

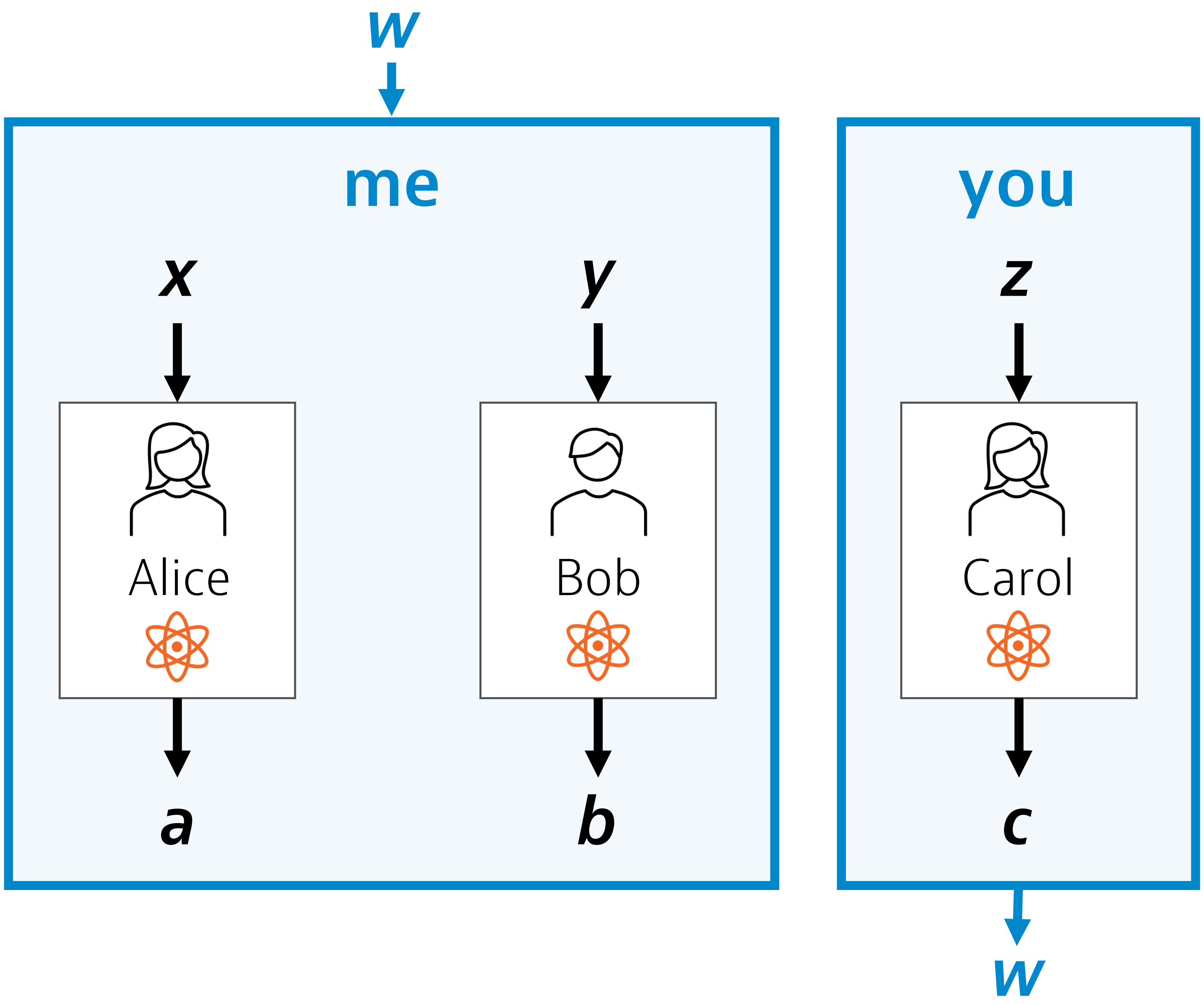
No signaling

- The only classical winning strategy for the GHZ game involves communication
- But you **cannot** use the quantum strategy for communication!
- And this holds in general

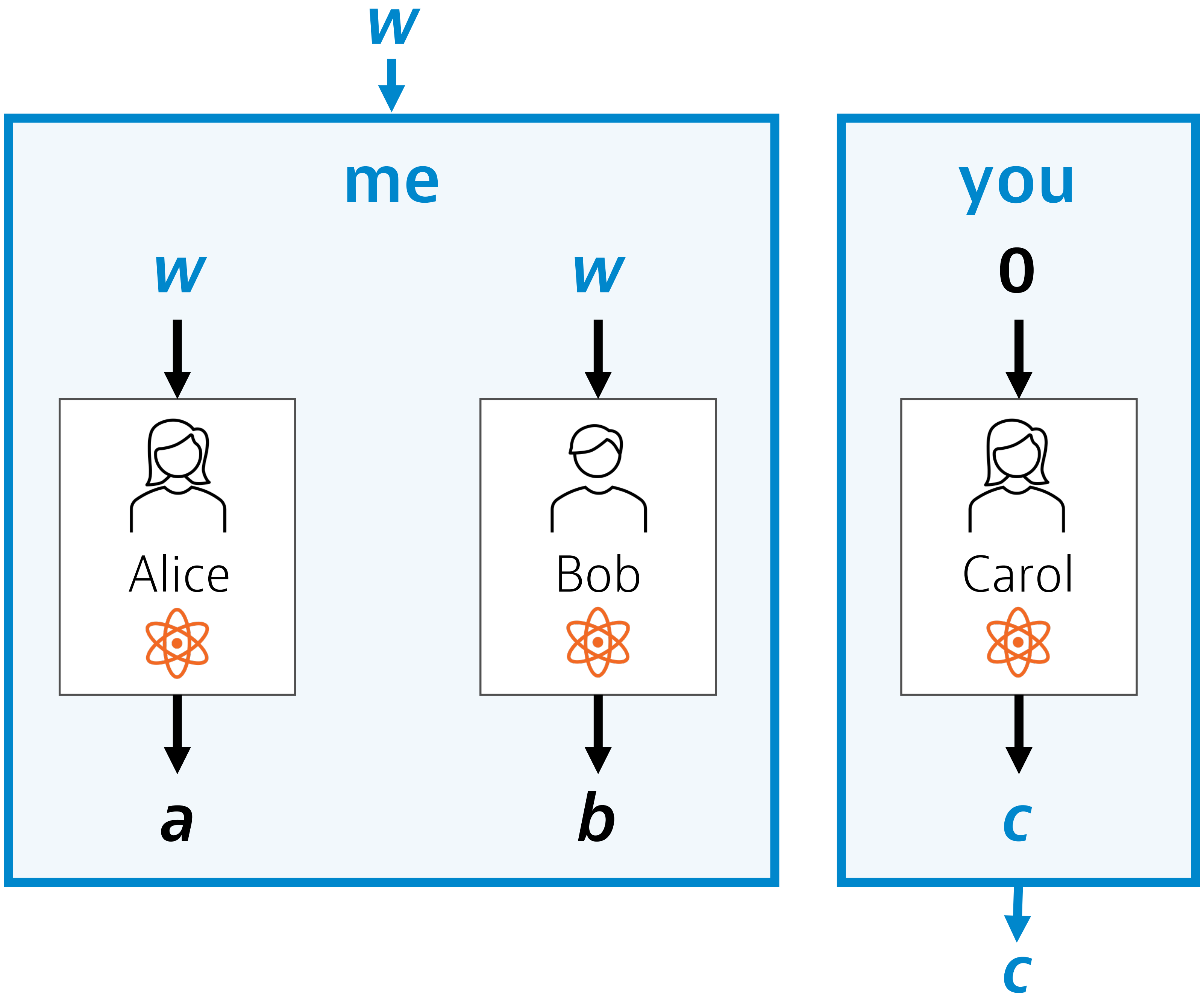
$x + y + z$	$a + b + c$
0	even
1	any
2	odd
3	any



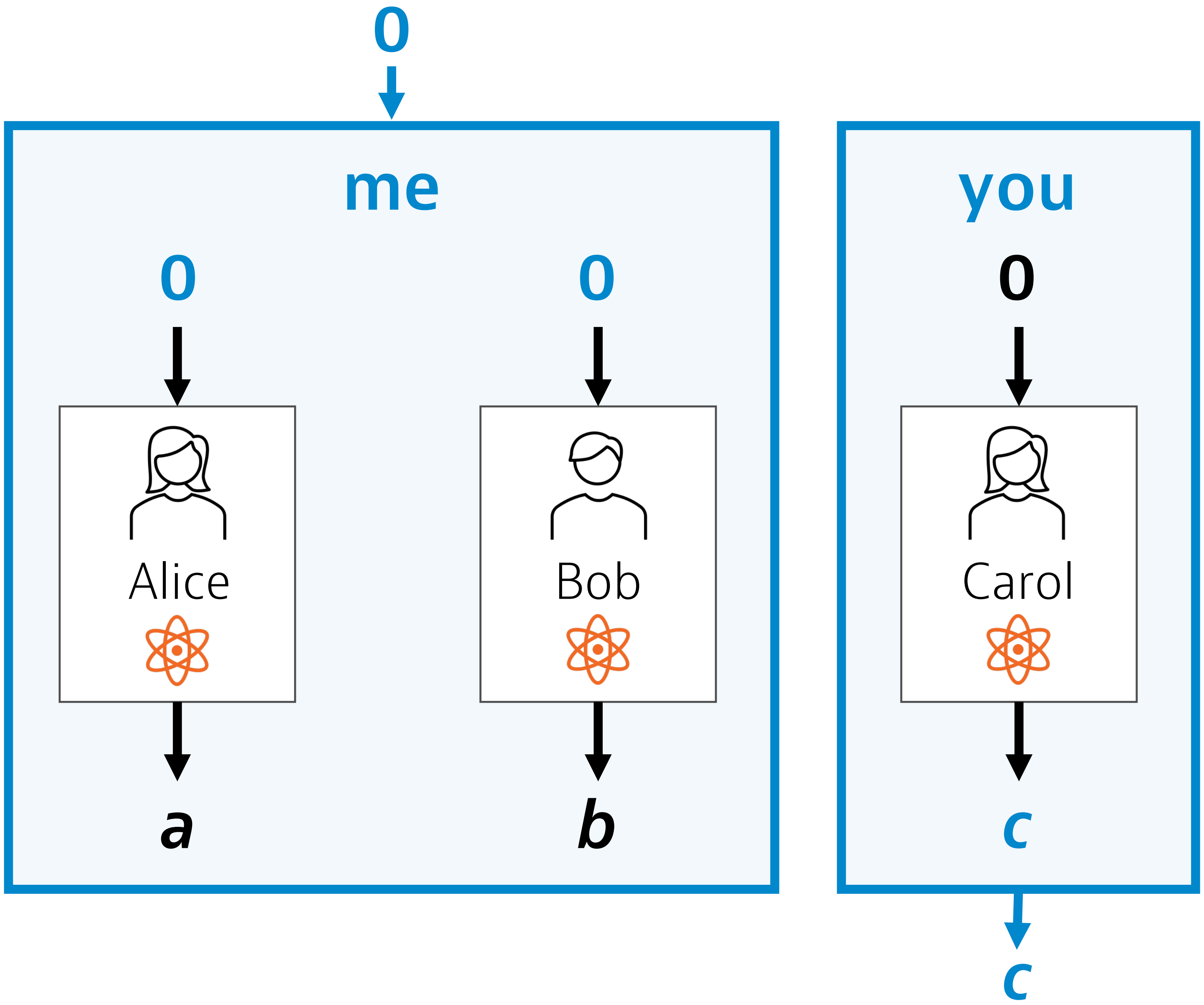
$x + y + z$	$a + b + c$
0	even
1	any
2	odd
3	any



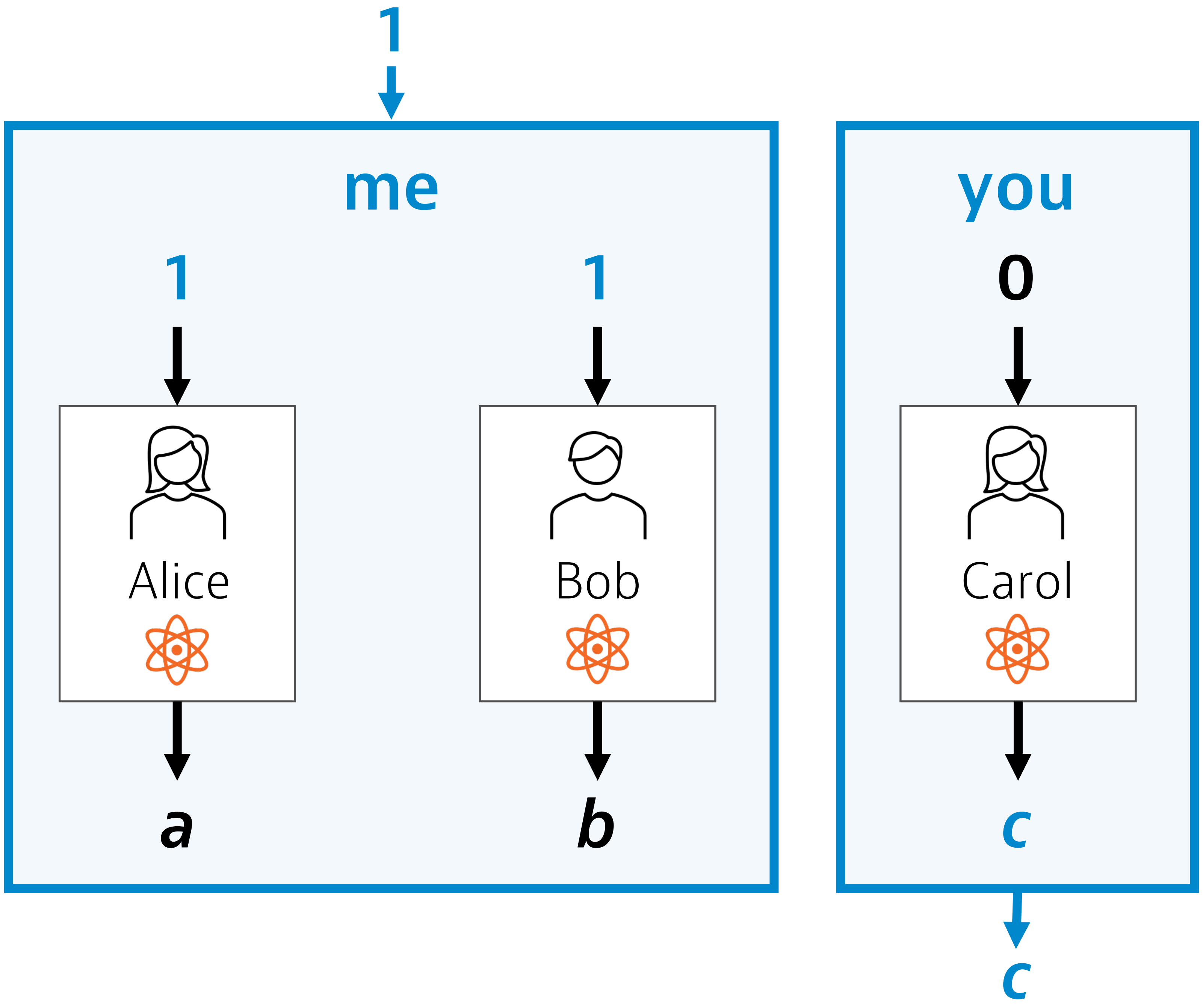
$x + y + z$	$a + b + c$
0	even
1	any
2	odd
3	any



$x + y + z$	$a + b + c$
0	**even**
1	any
2	odd
3	any

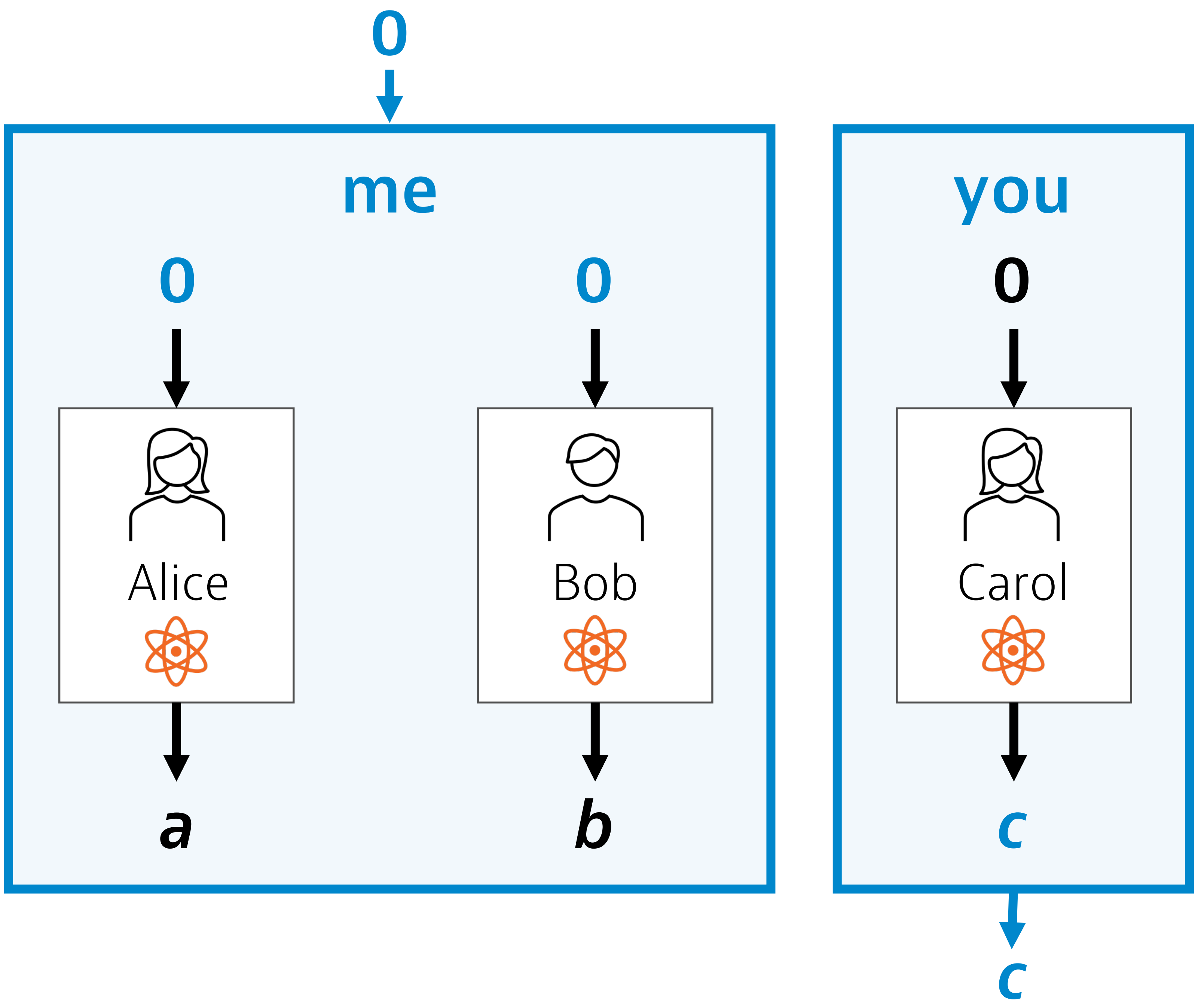


$x + y + z$	$a + b + c$
0	even
1	any
2	**odd**
3	any



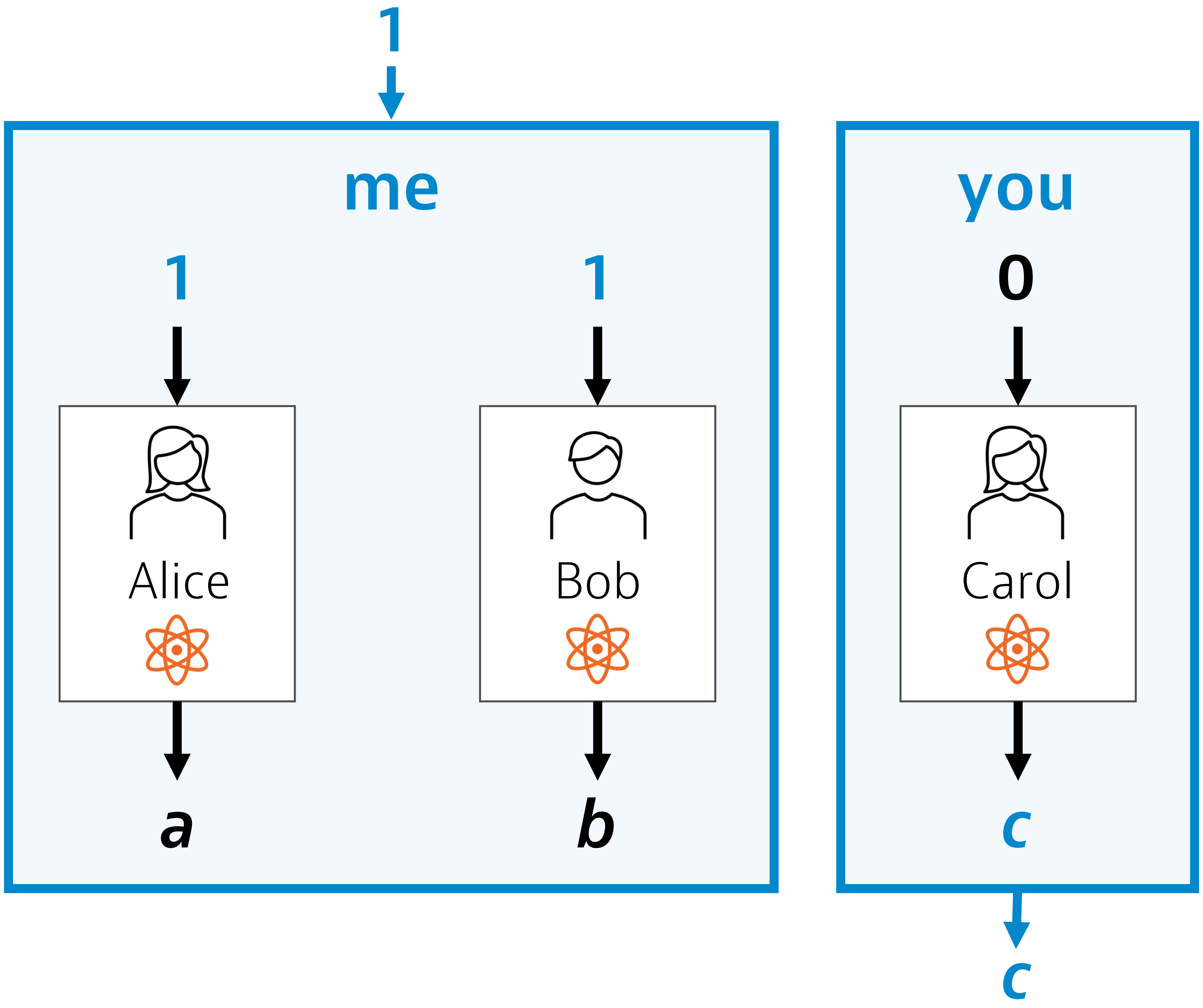
$x + y + z$	$a + b + c$
0	**even**
1	any
2	odd
3	any

a	b	c
0	0	**0**
0	1	**1**
1	0	**1**
1	1	**0**

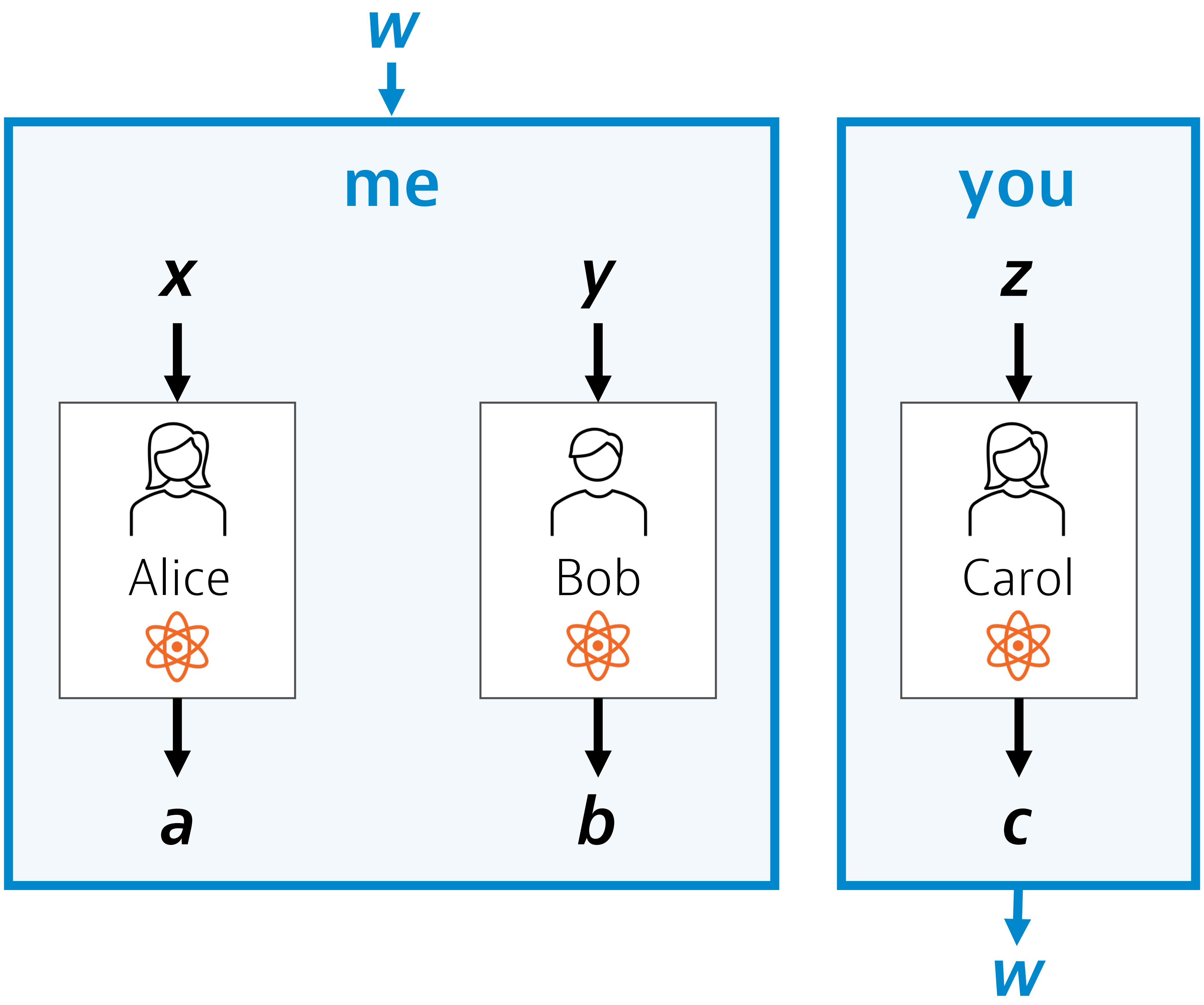


$x + y + z$	$a + b + c$
0	even
1	any
2	odd
3	any

a	b	c
0	0	1
0	1	0
1	0	0
1	1	1

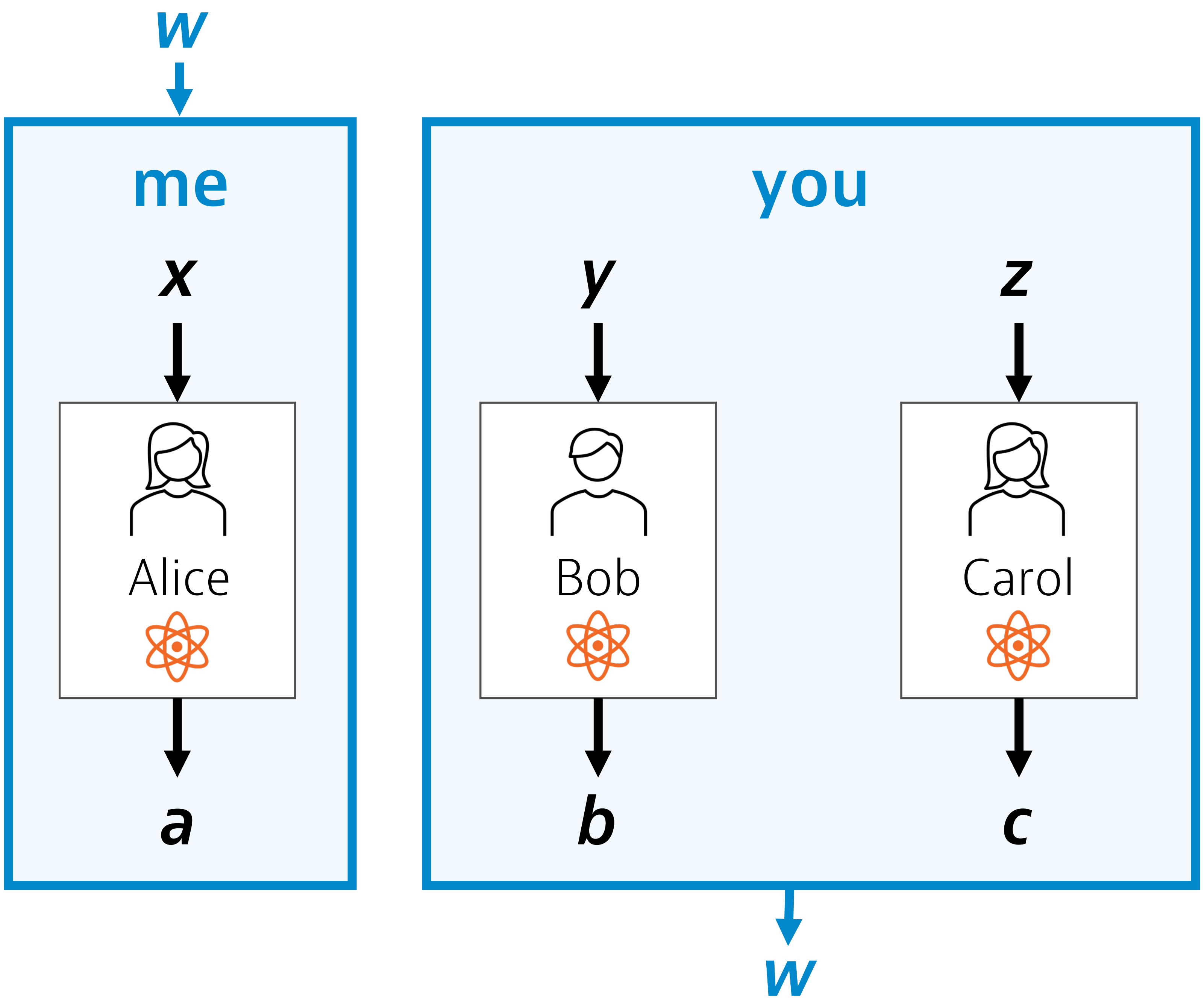


$x + y + z$	$a + b + c$
0	even
1	any
2	odd
3	any



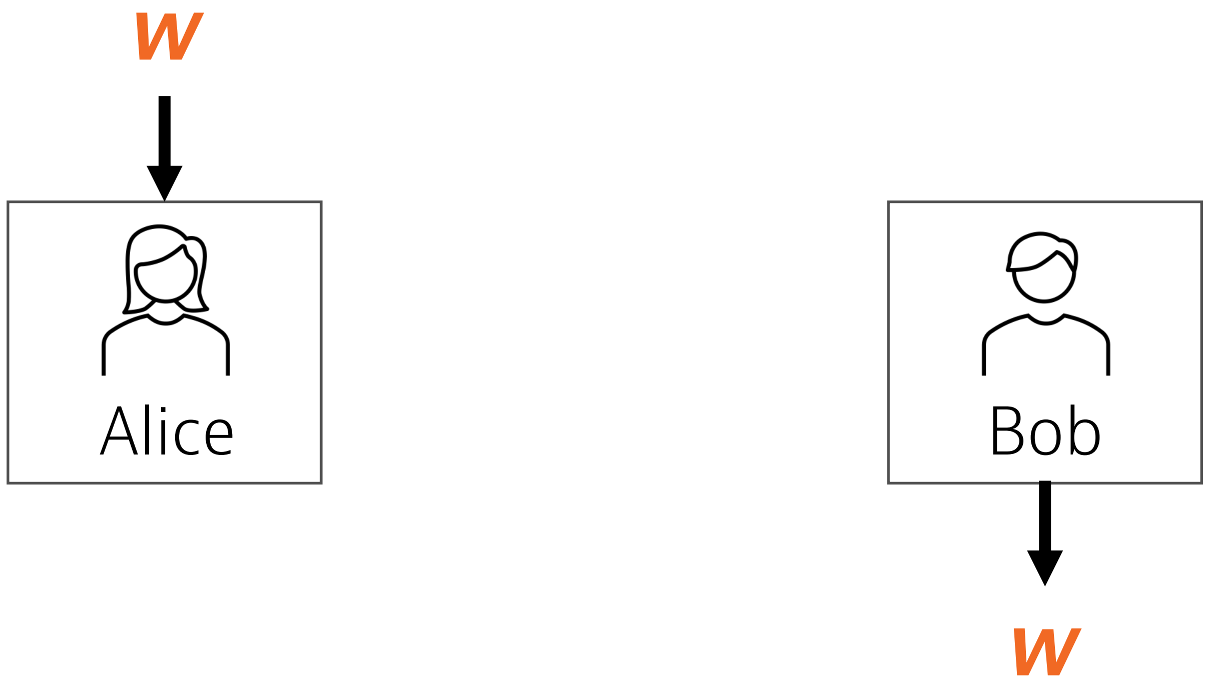
No signaling

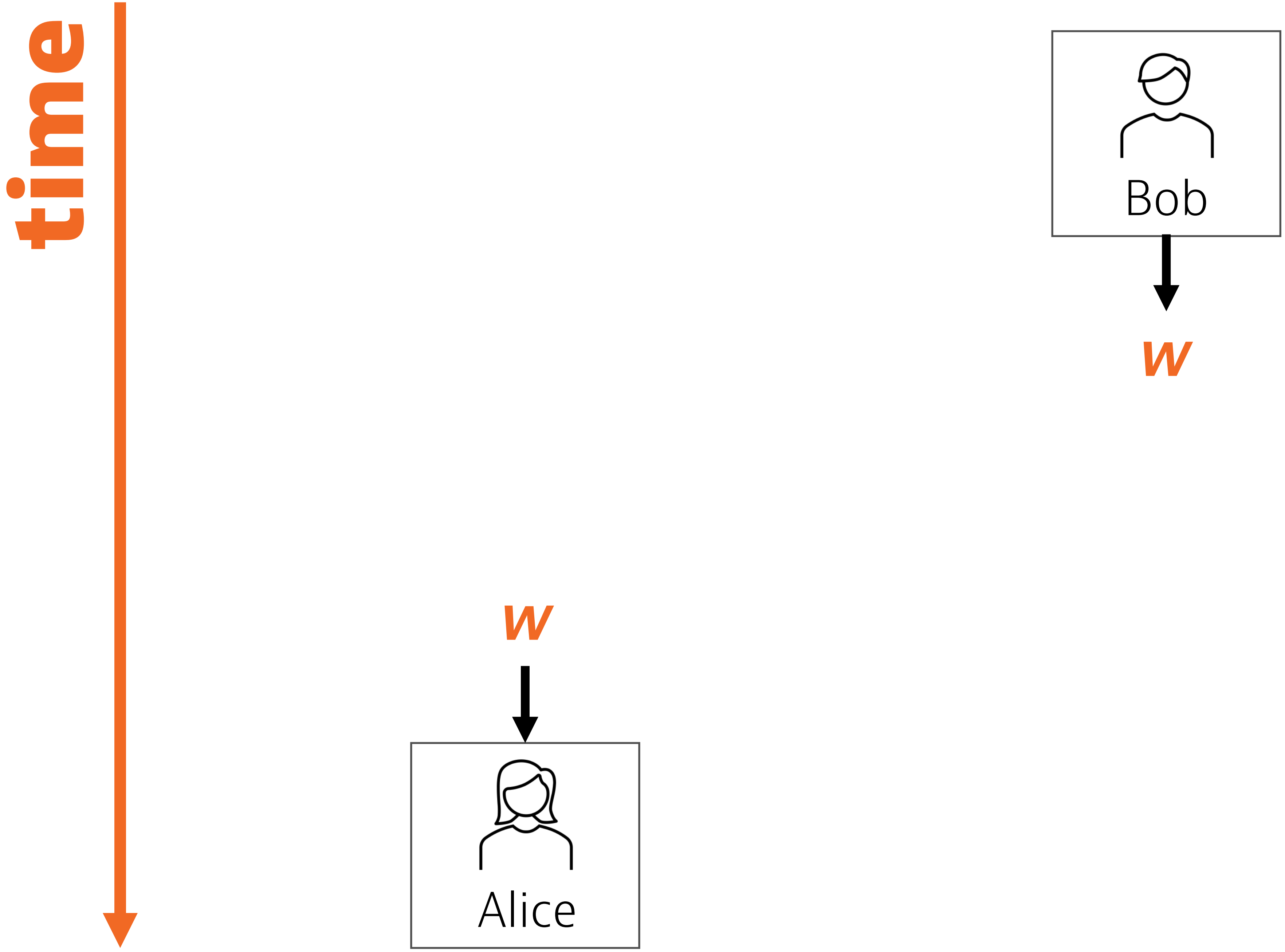
$x + y + z$	$a + b + c$
0	even
1	any
2	odd
3	any



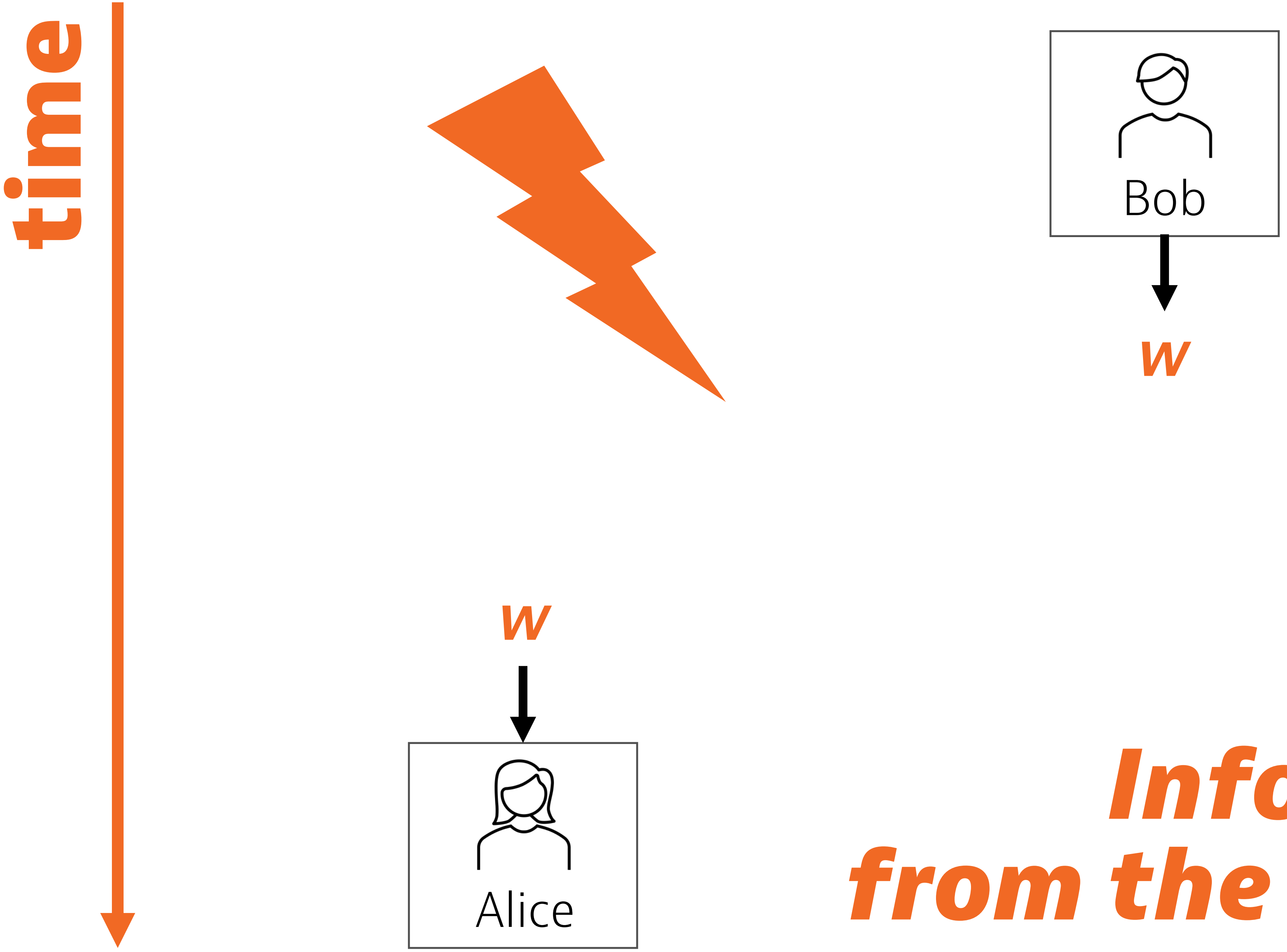
No signaling

Quantum cannot violate causality

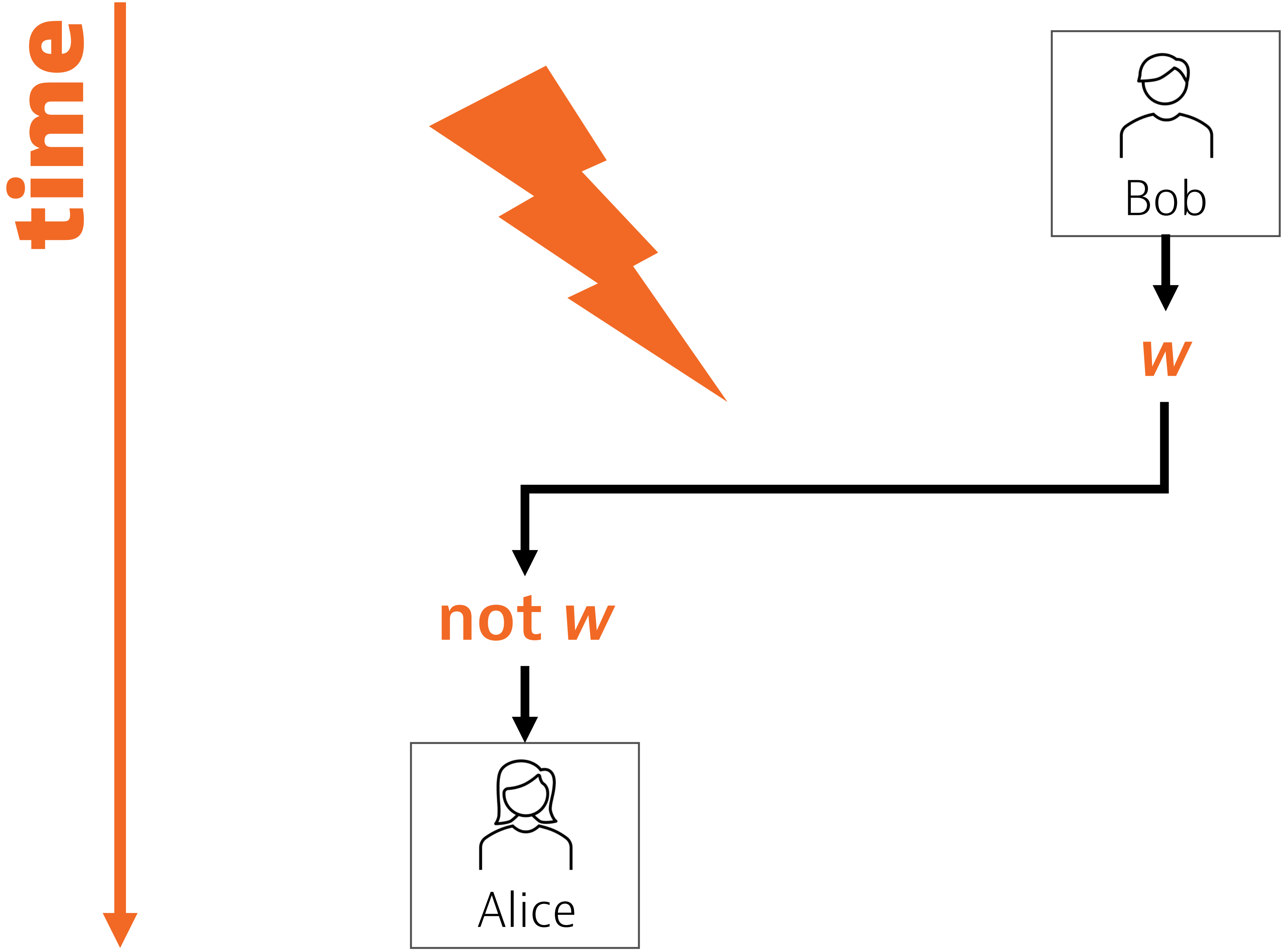

w
Alice
Bob
w

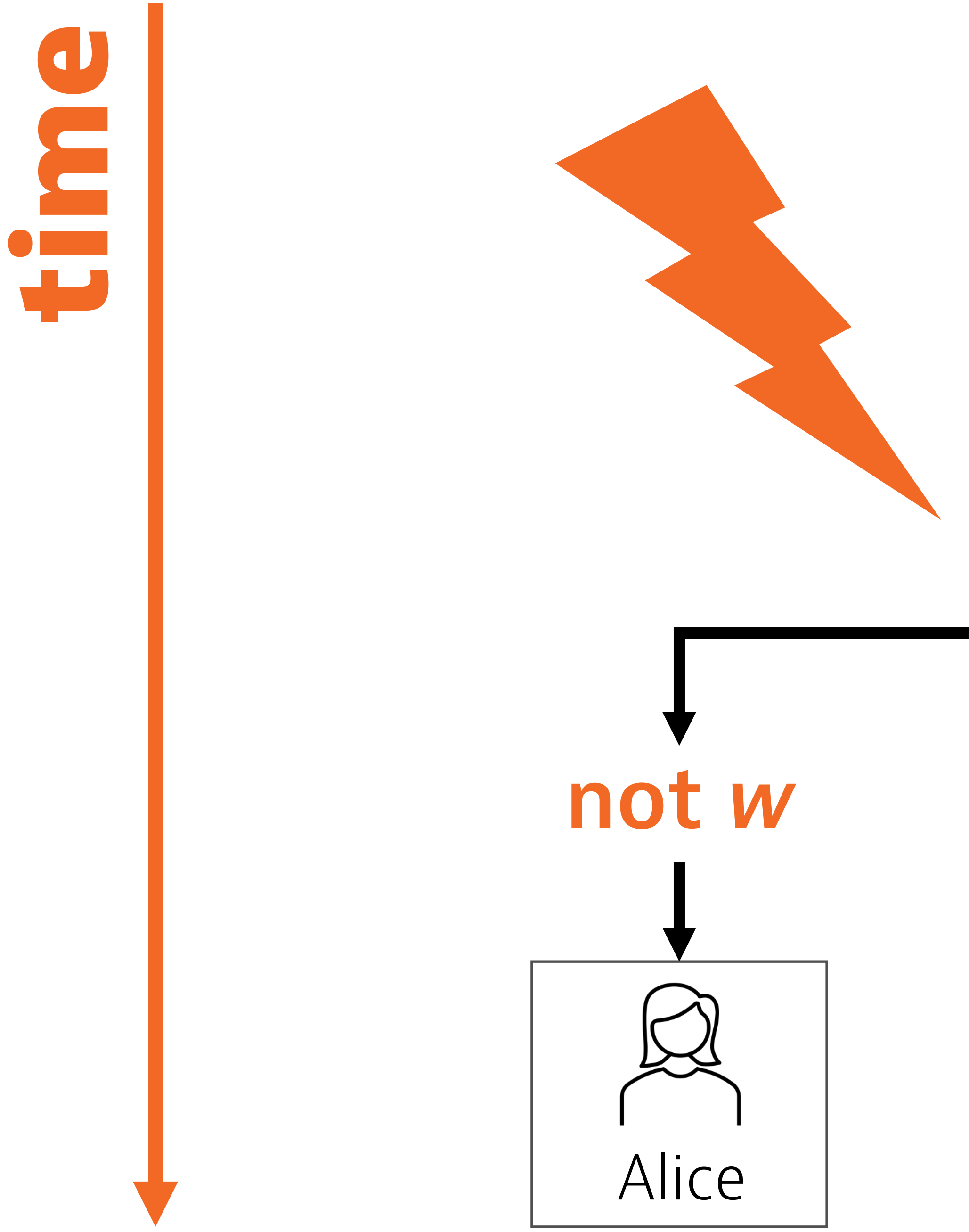


time
Bob
w
w
Alice



Information from the future??


time
Bob
w
not w
Alice



Obviously laws of physics must prevent this...

Recap

1. Quantum physics is **nonlocal:**
   you can do something that ***requires communication in classical settings***
2. But you **cannot violate causality:**
   you ***cannot use quantum magic to communicate***

4 hypothetical models

Classical

Quantum

Does not violate causality

Could violate causality

3 important models

Classical

Quantum

Non-signaling

3 important models

Classical < Quantum < Non-signaling

3 important models

Classical < Quantum < Non-signaling

GHZ game

3 important models

Classical < Quantum < Non-signaling

CHSH game

3 important models

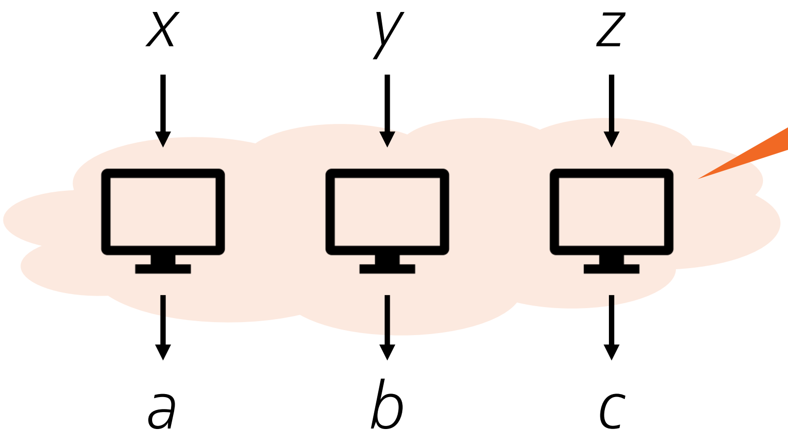
Classical < Quantum < Non-signaling

Recap

- Quantum is magical, but in a very limited way
- We cannot use quantum magic for communication
- Is there anything useful we can do with it?

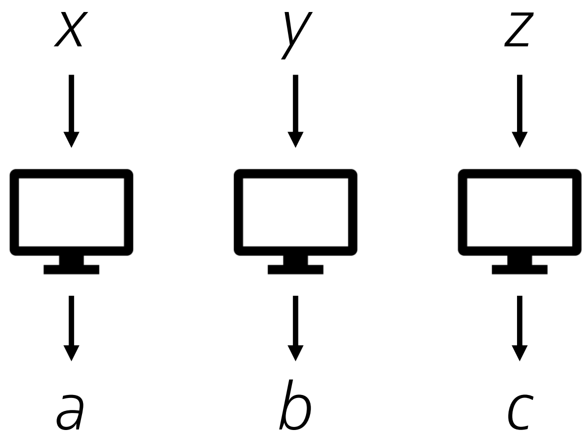
What did we really gain in GHZ?

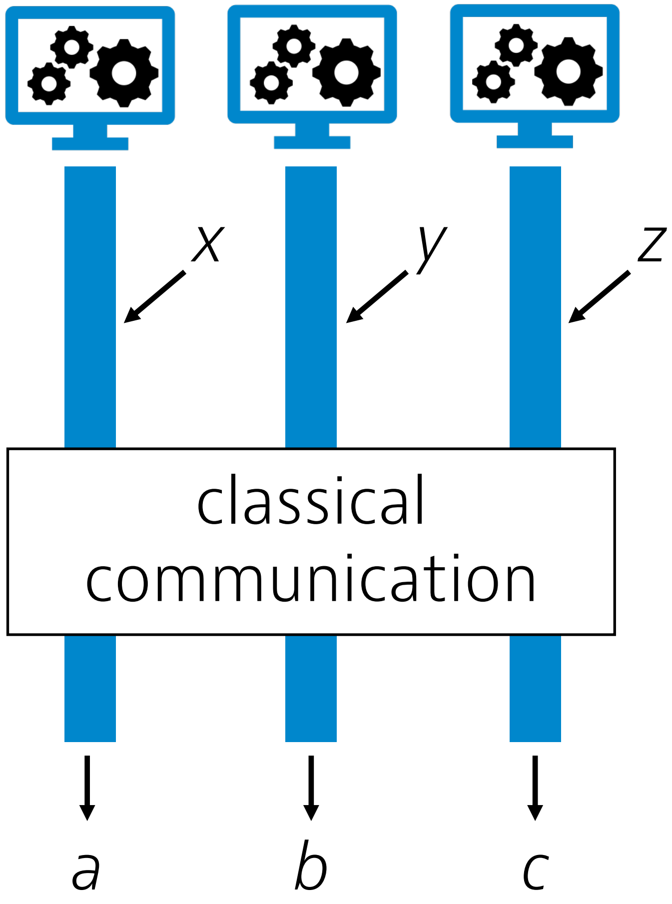
$x + y + z$	$a + b + c$
0	even
1	any
2	odd
3	any



3 computers in
a computer network,
no prior setup

$x + y + z$	$a + b + c$
0	even
1	any
2	odd
3	any





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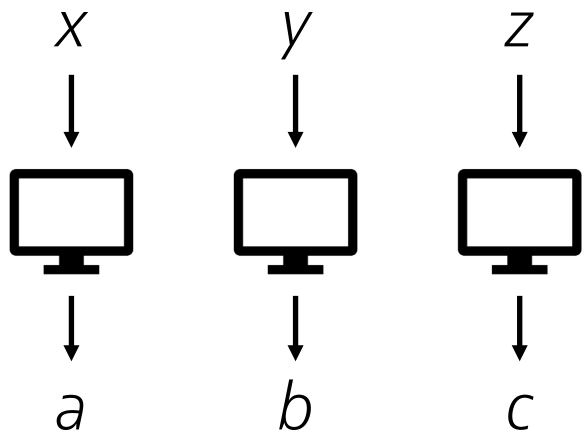
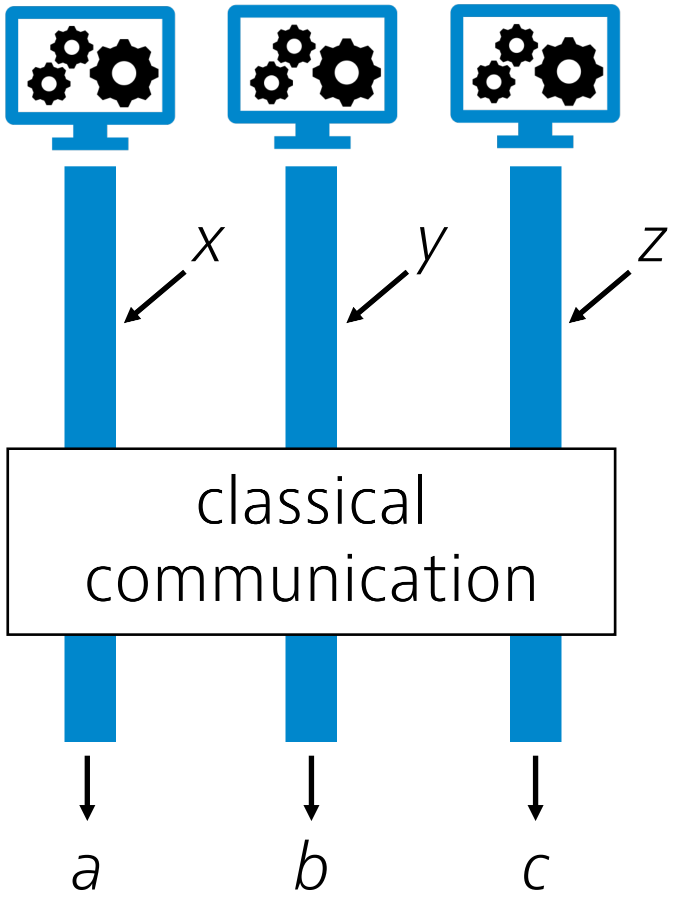
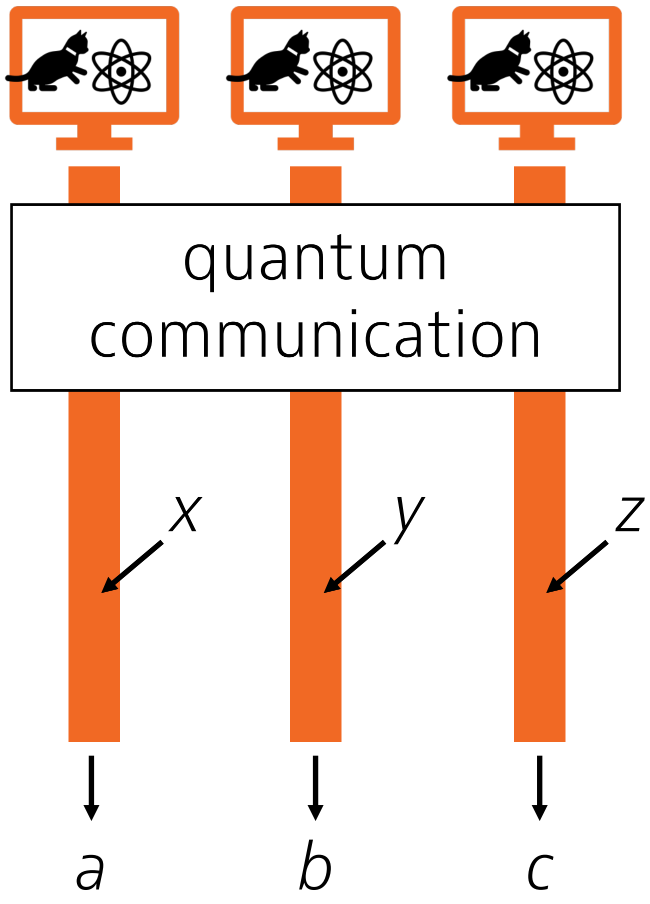




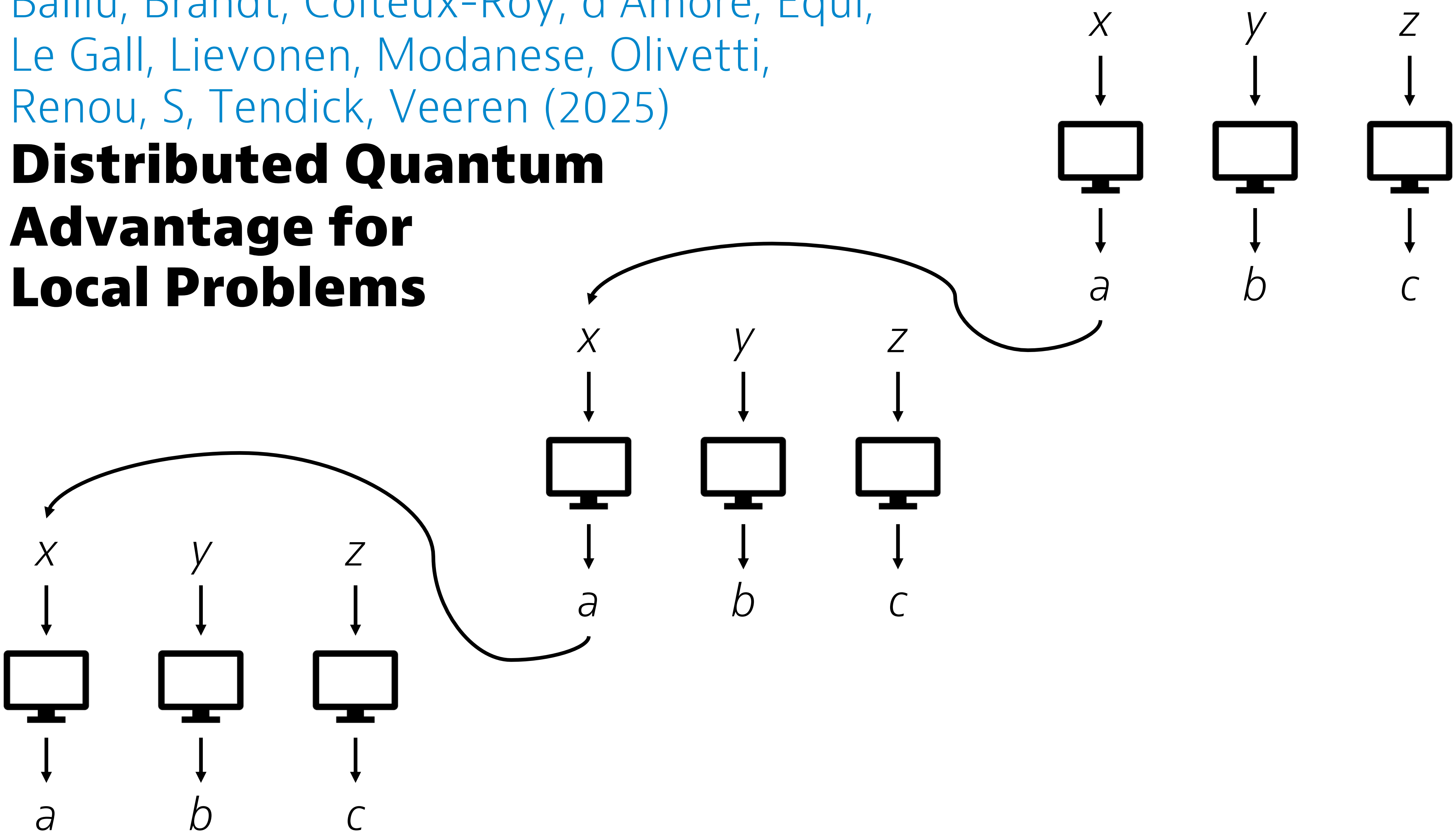


Recap

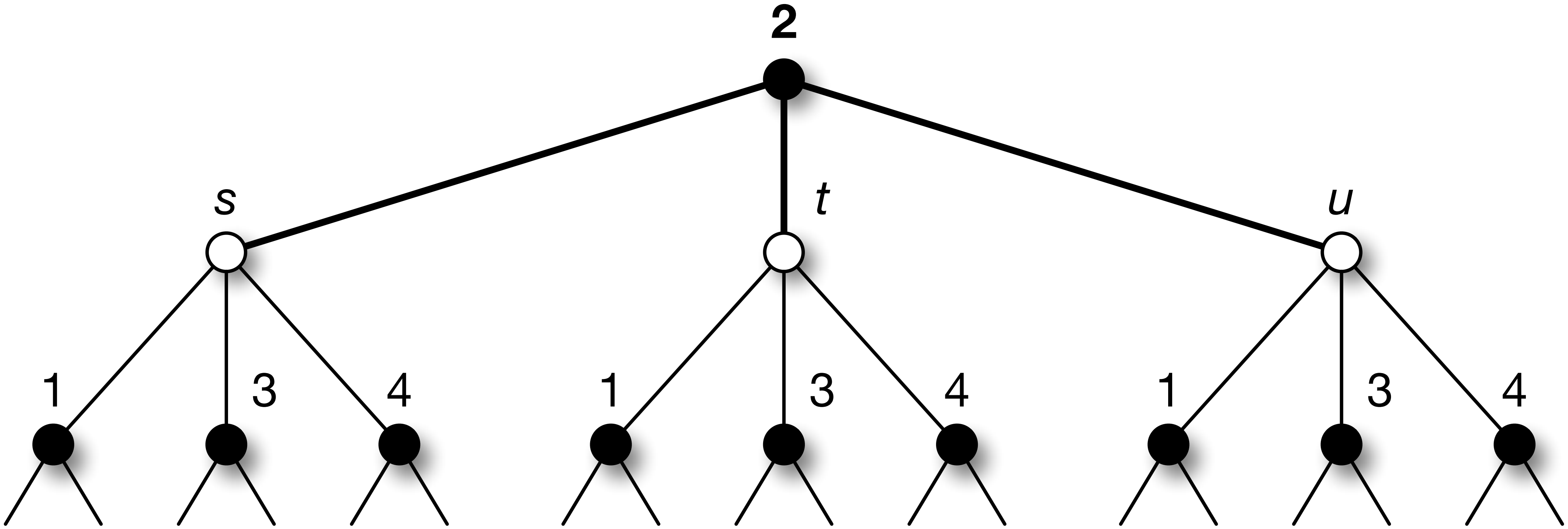
- **Classical:** 1 communication round *after* seeing the inputs
- **Quantum:** 1 communication round *before* seeing the inputs
- It is still 1 round in both cases
- Do we ever gain anything?

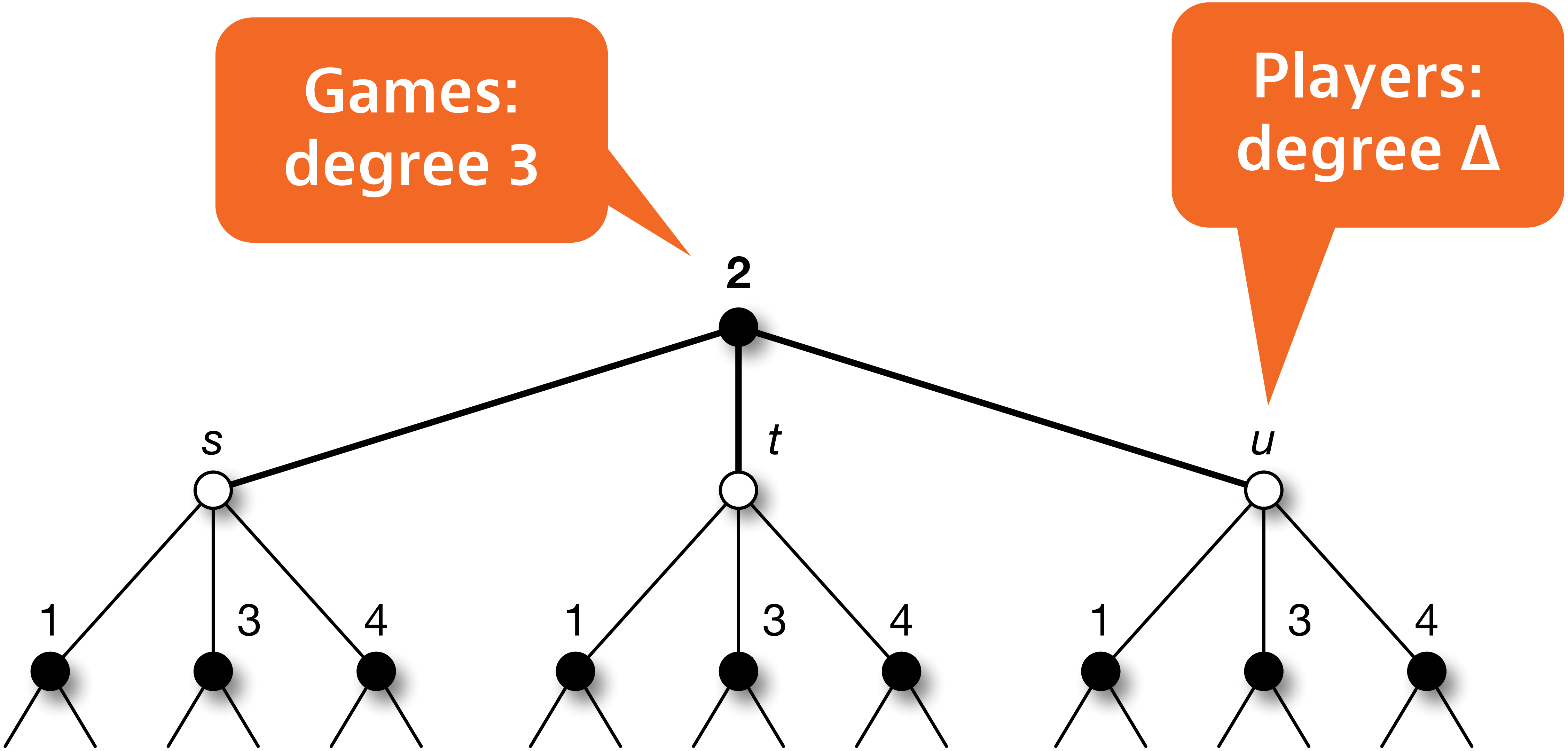
Balliu, Brandt, Coiteux-Roy, d’Amore, Equi, Le Gall, Lievonen, Modanese, Olivetti, Renou, S, Tendick, Veeren (2025)

Distributed Quantum Advantage for Local Problems

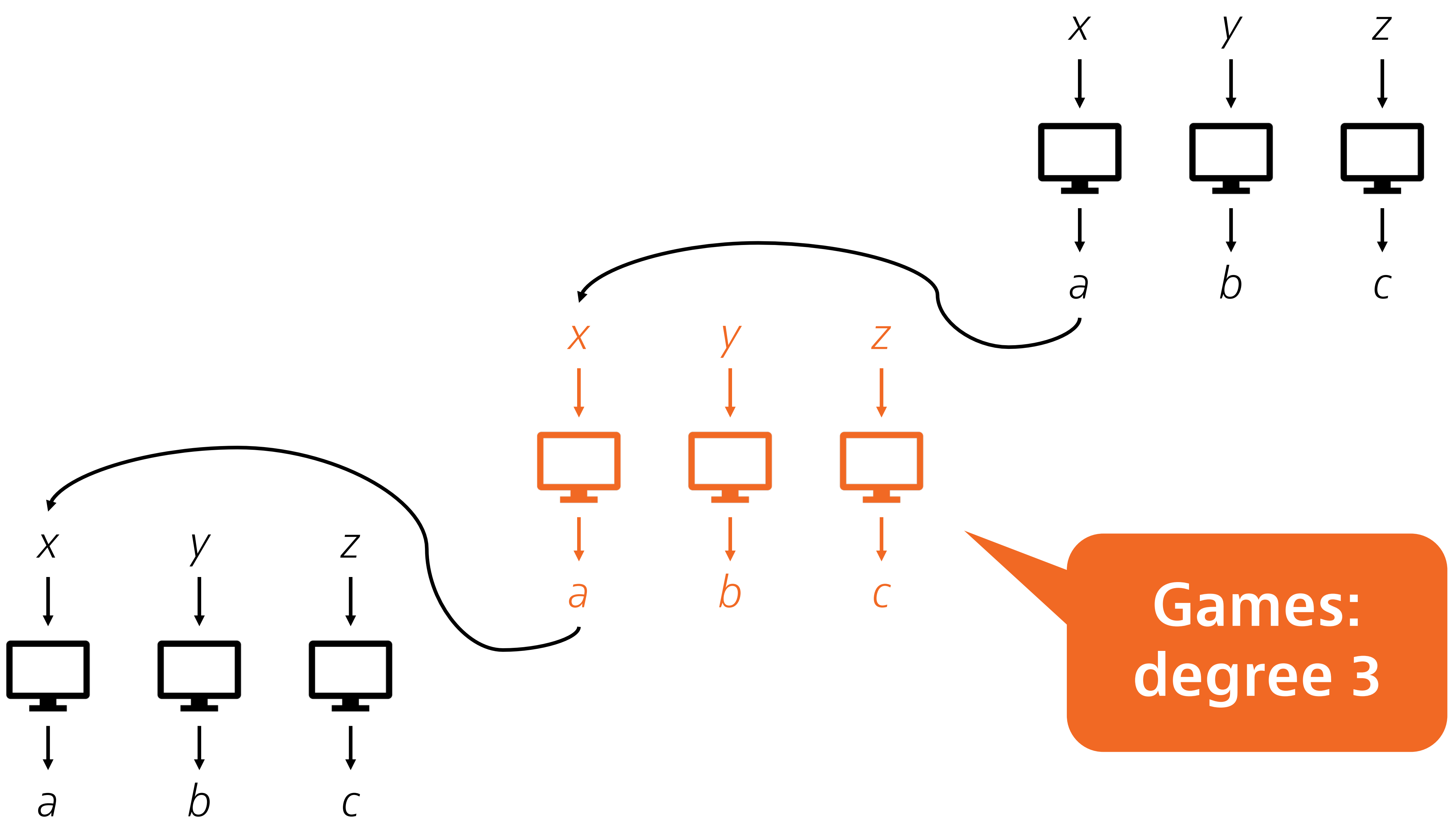


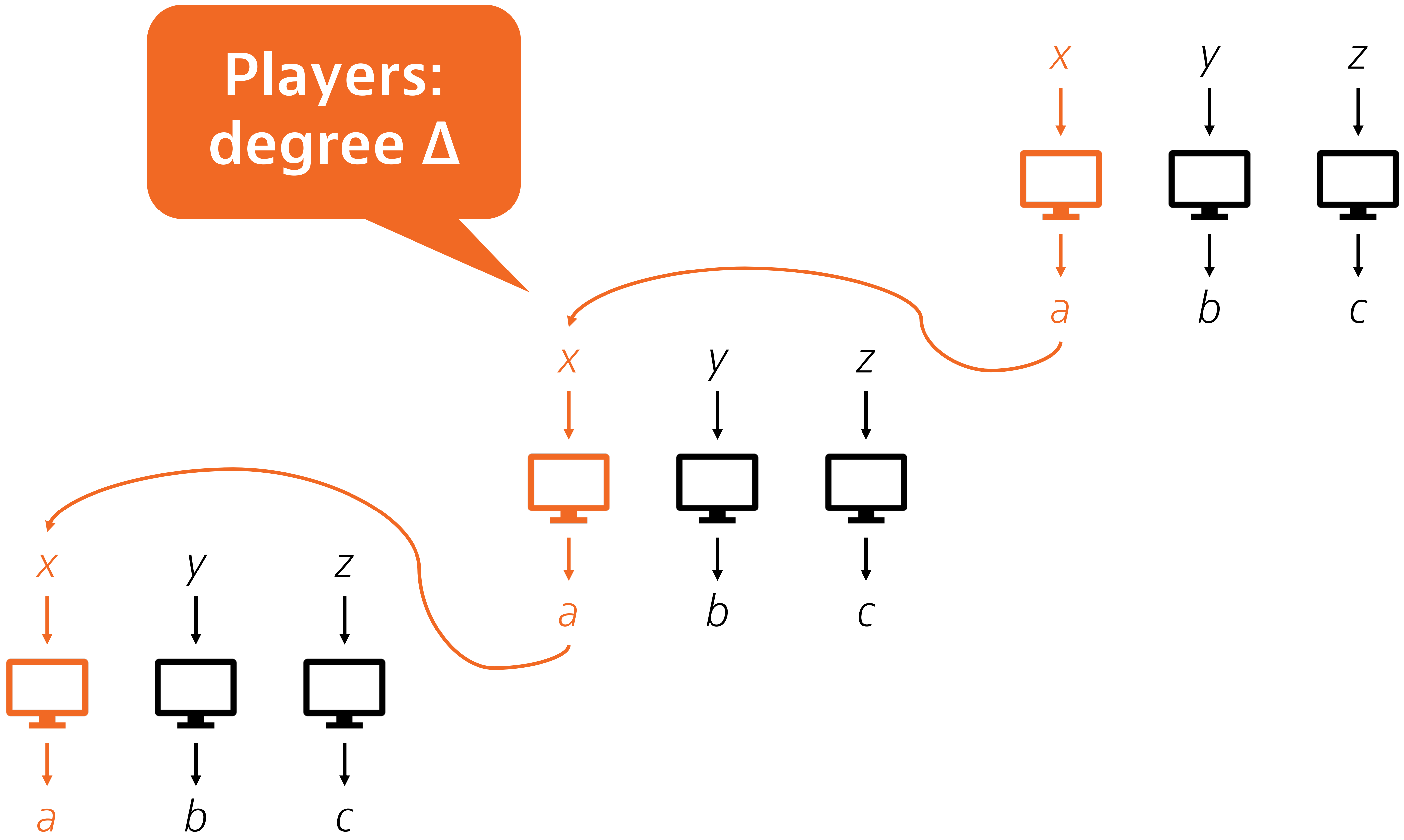
Iterated GHZ problem



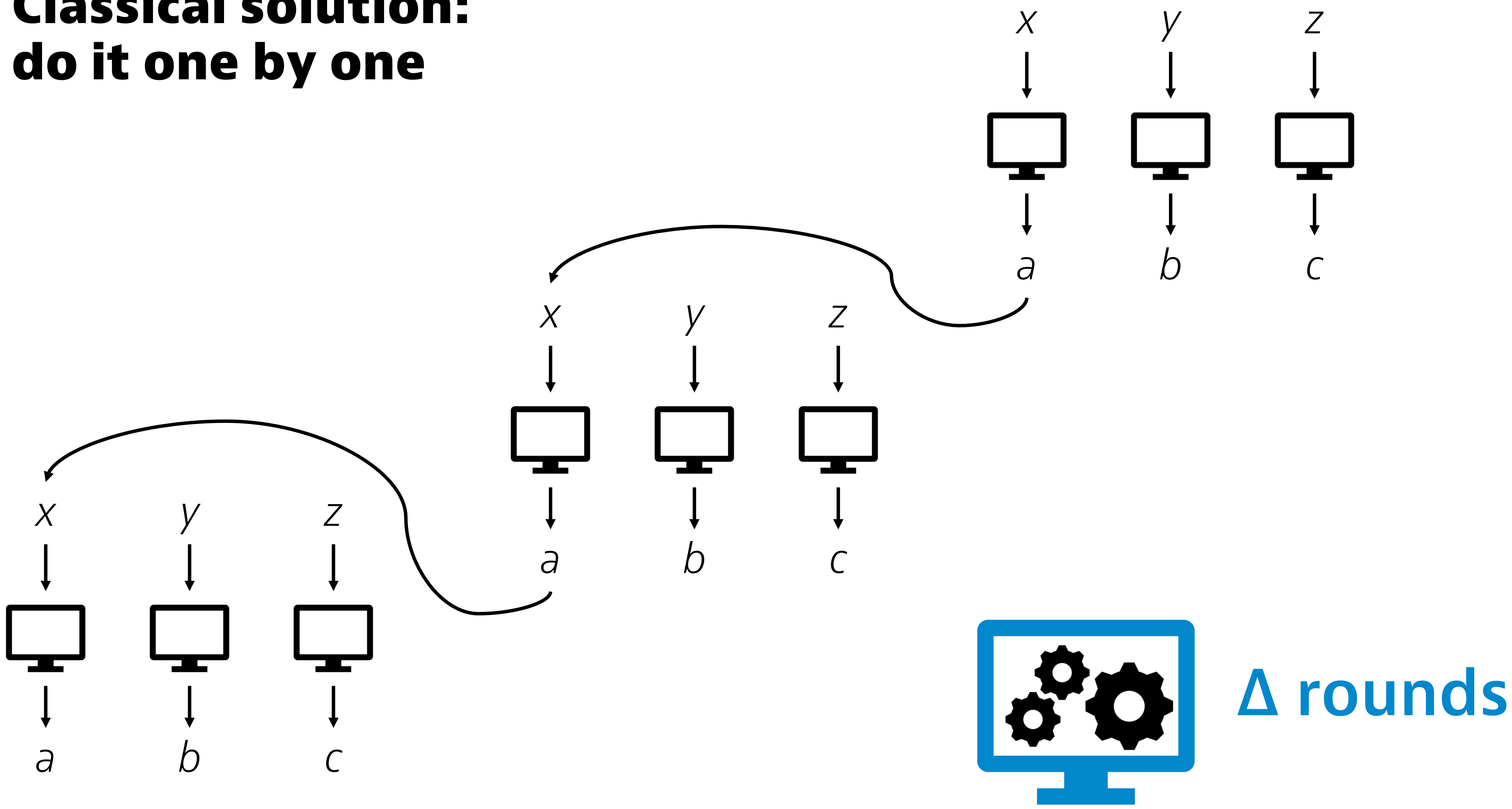


Games:
degree 3
Players:
degree Δ
2
s
t
u
1
3
4
1
3
4
1
3
4

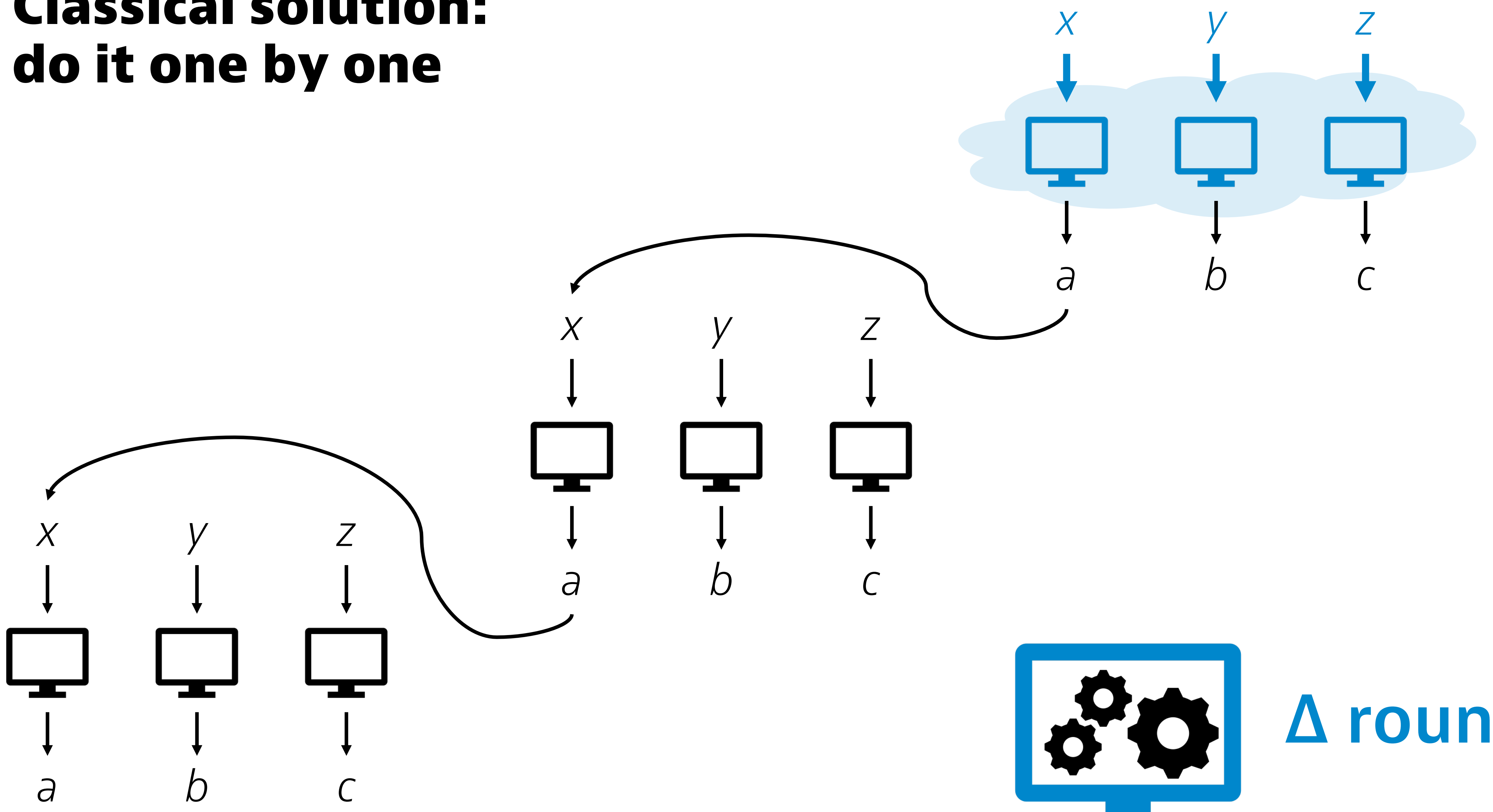

x
y
z
a
b
c
x
y
z
a
b
c
x
y
z
a
b
c
Games:
degree 3


Players:
degree Δ
x
y
z
a
b
c
x
y
z
a
b
c
x
y
z
a
b
c

Classical solution: do it one by one

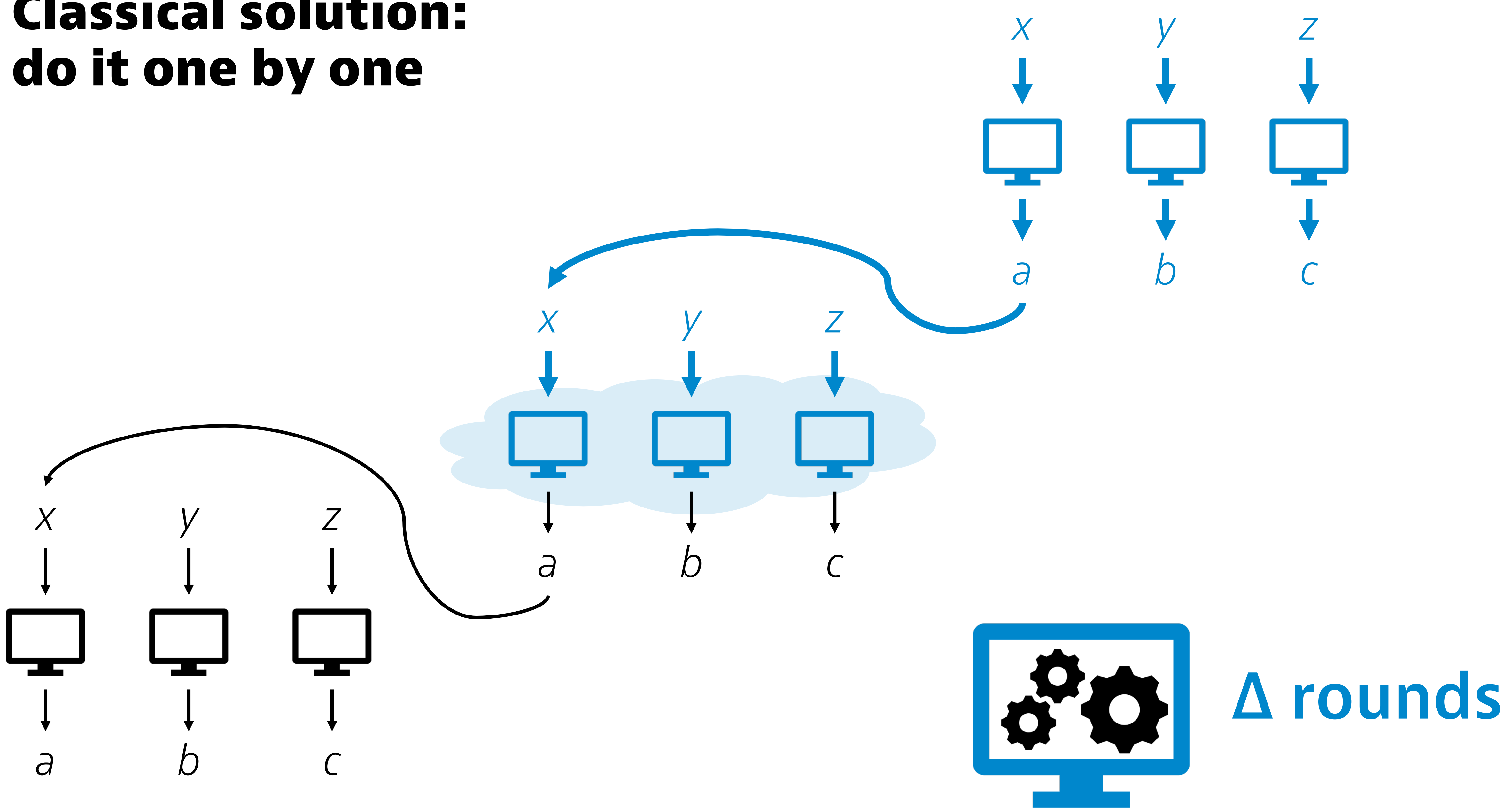


Classical solution: do it one by one

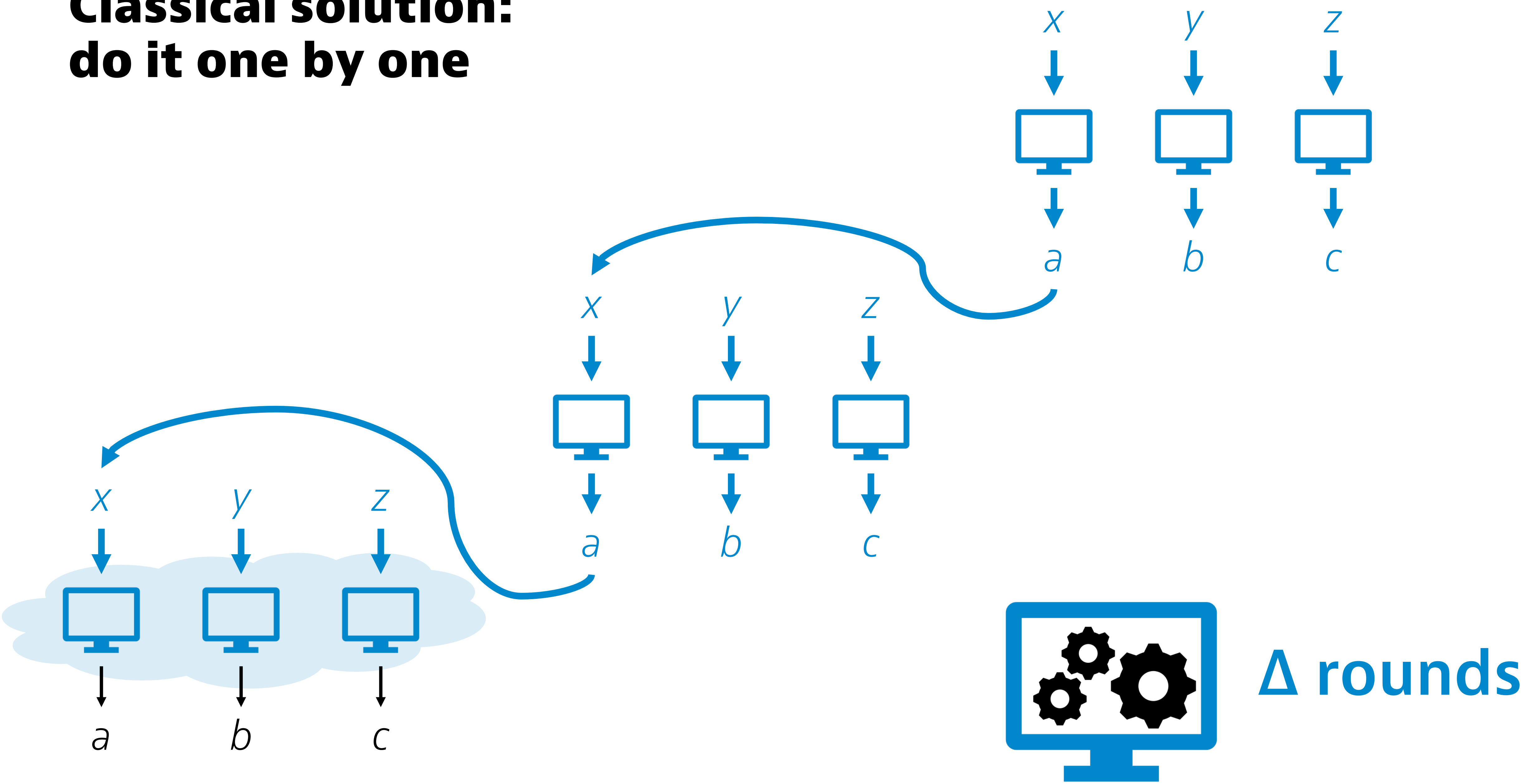


Δ rounds

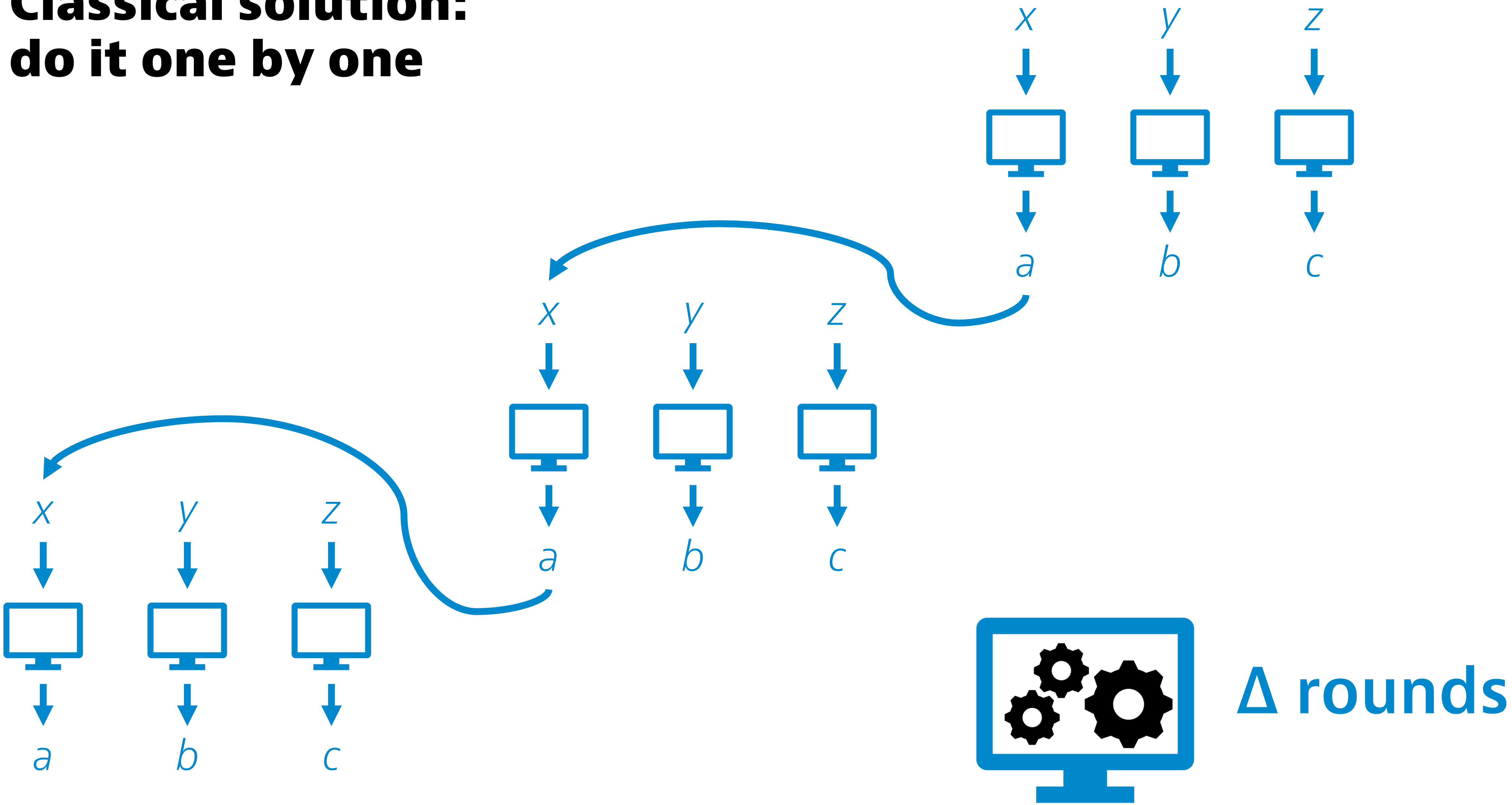
Classical solution: do it one by one



Classical solution: do it one by one

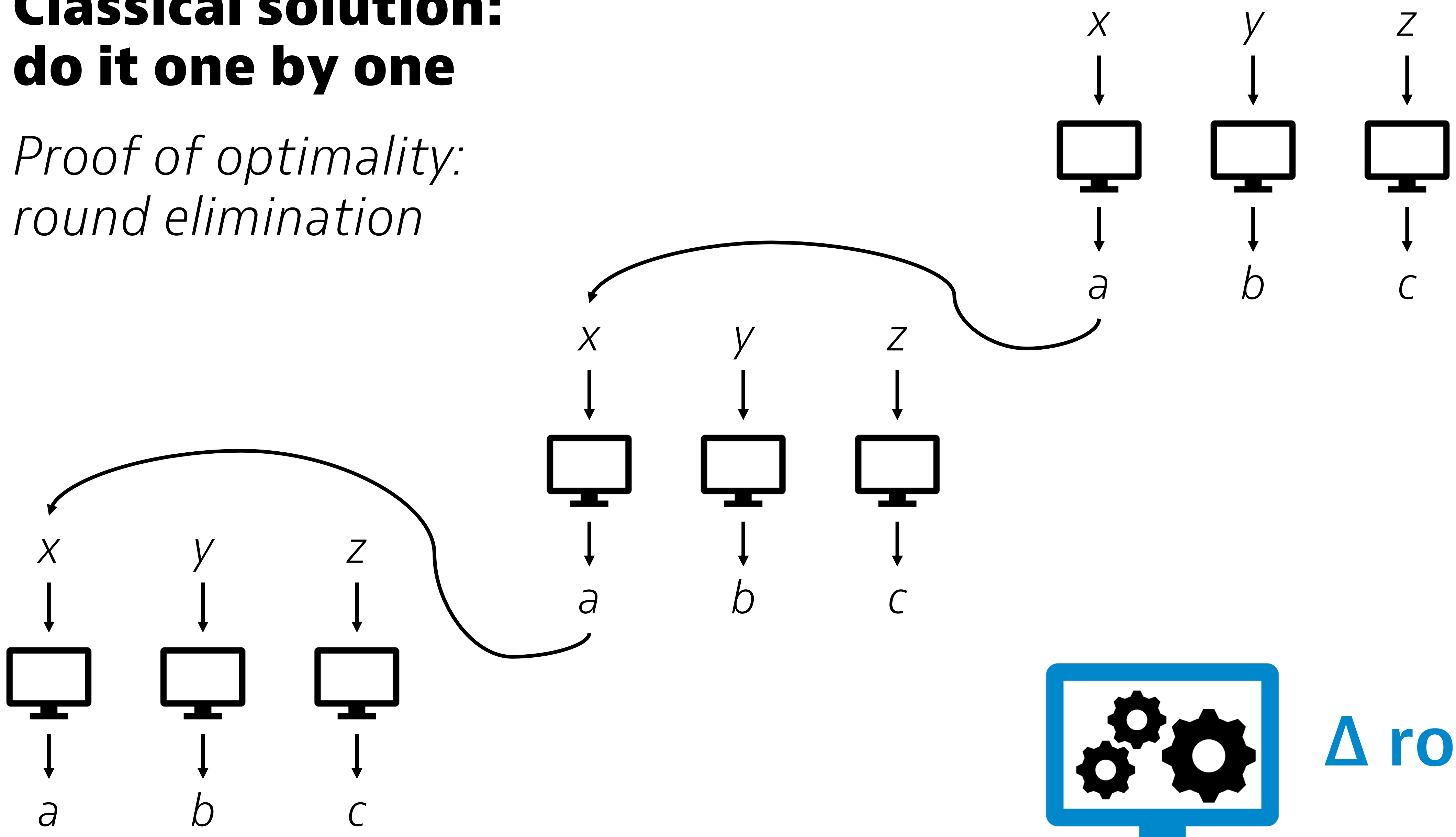


Classical solution: do it one by one



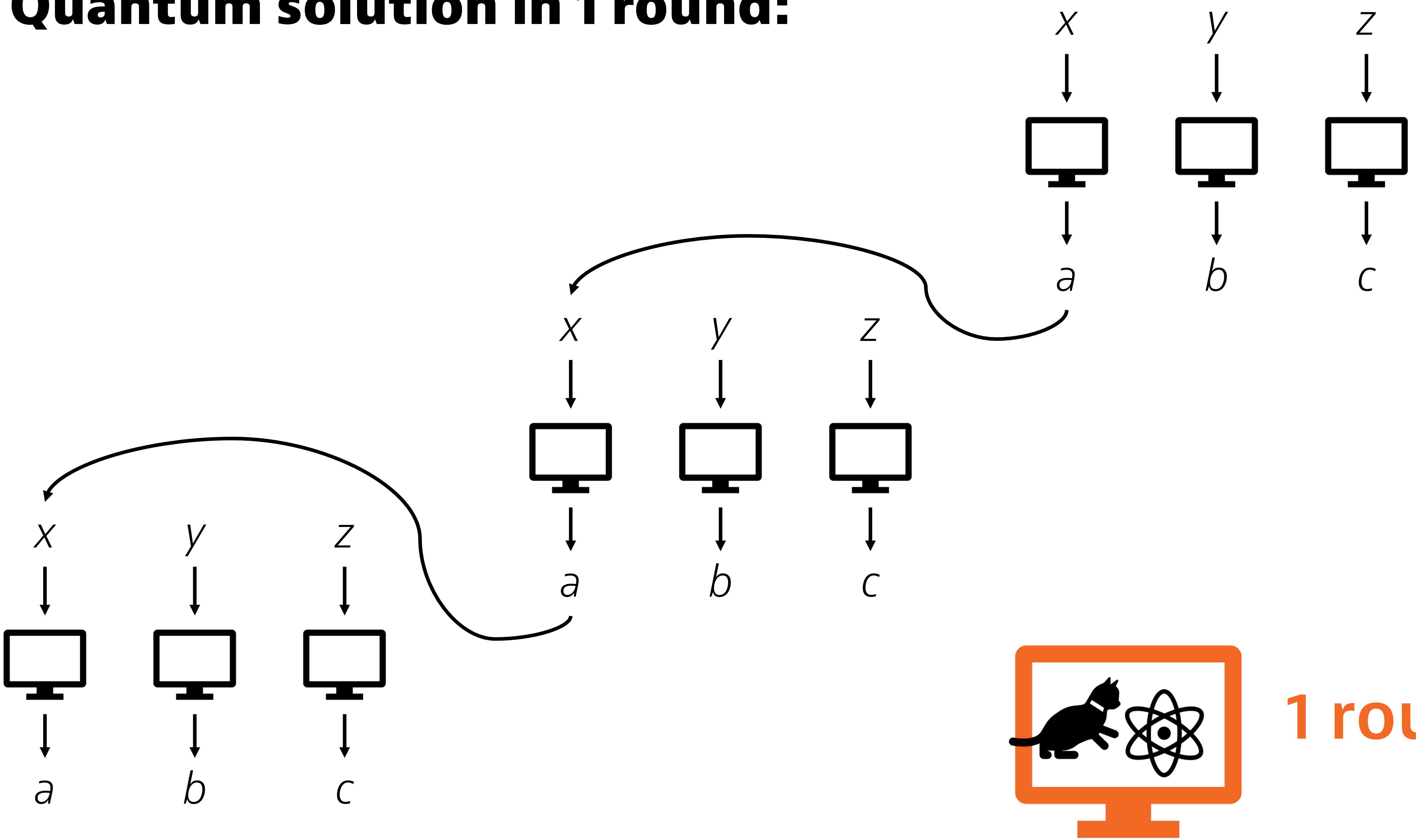
Classical solution: do it one by one

Proof of optimality: round elimination



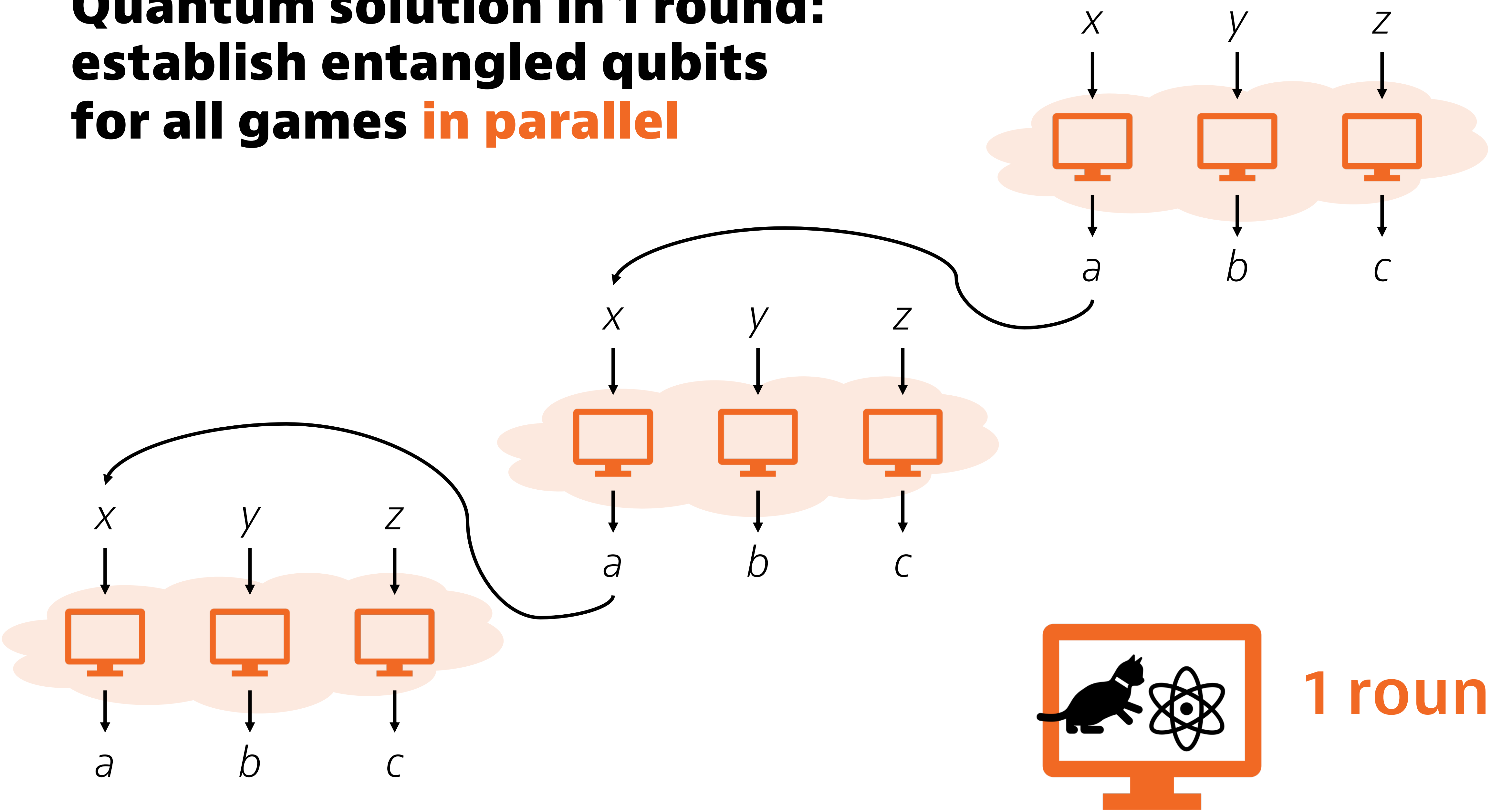
Δ rounds

Quantum solution in 1 round:



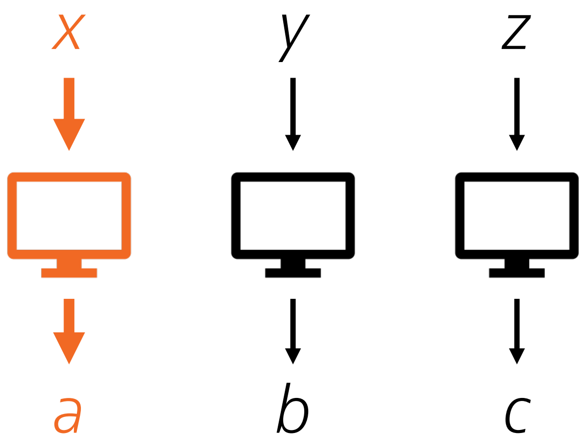
1 round

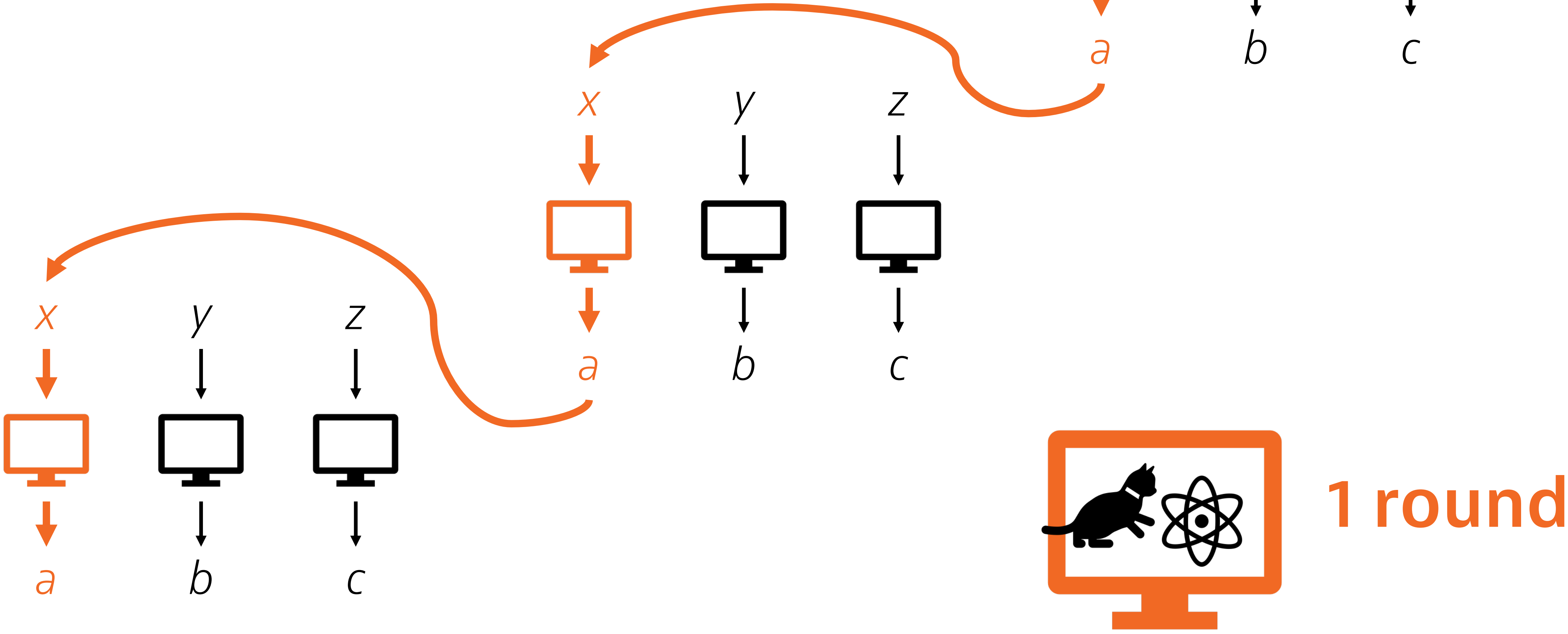
Quantum solution in 1 round: establish entangled qubits for all games in parallel

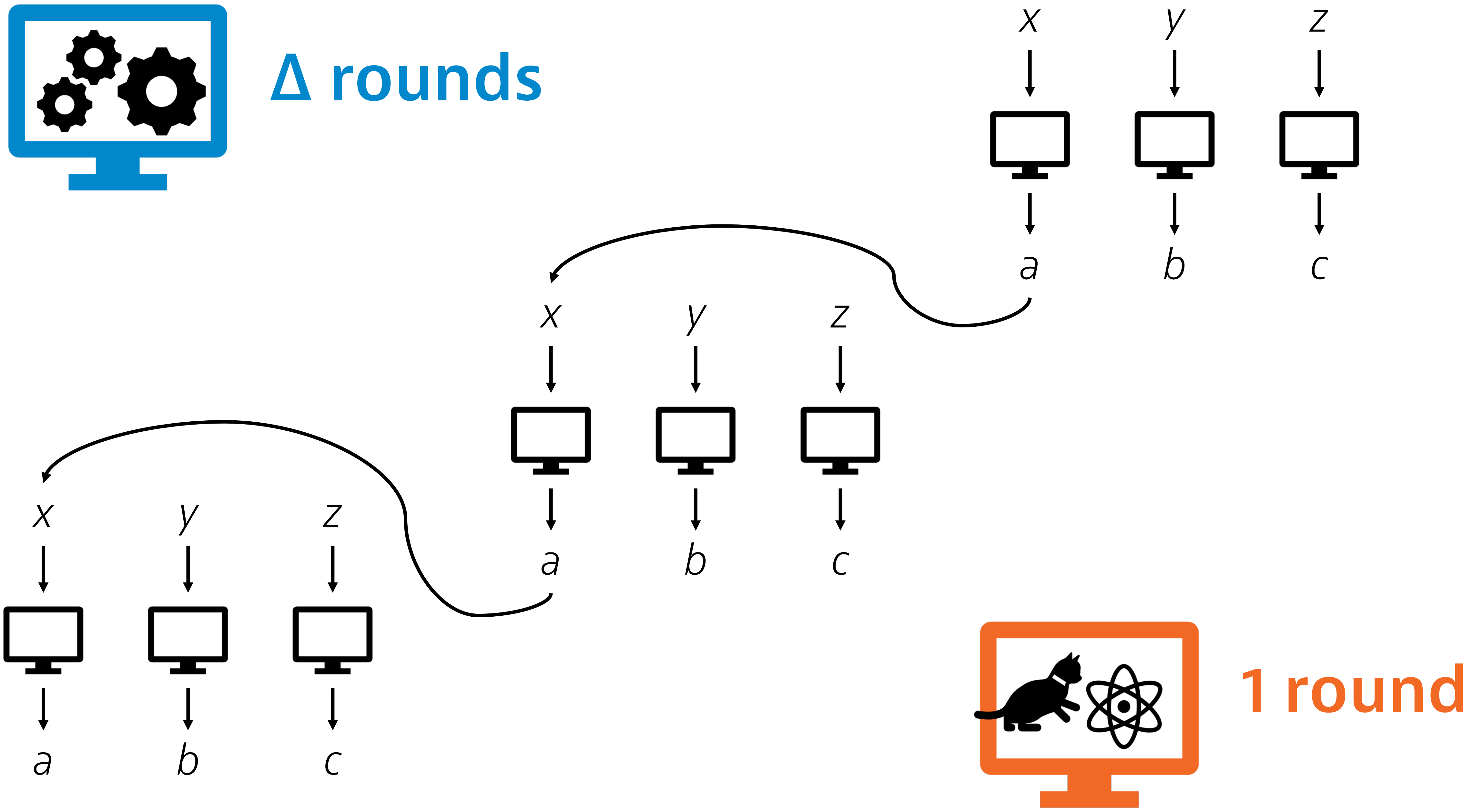


1 round

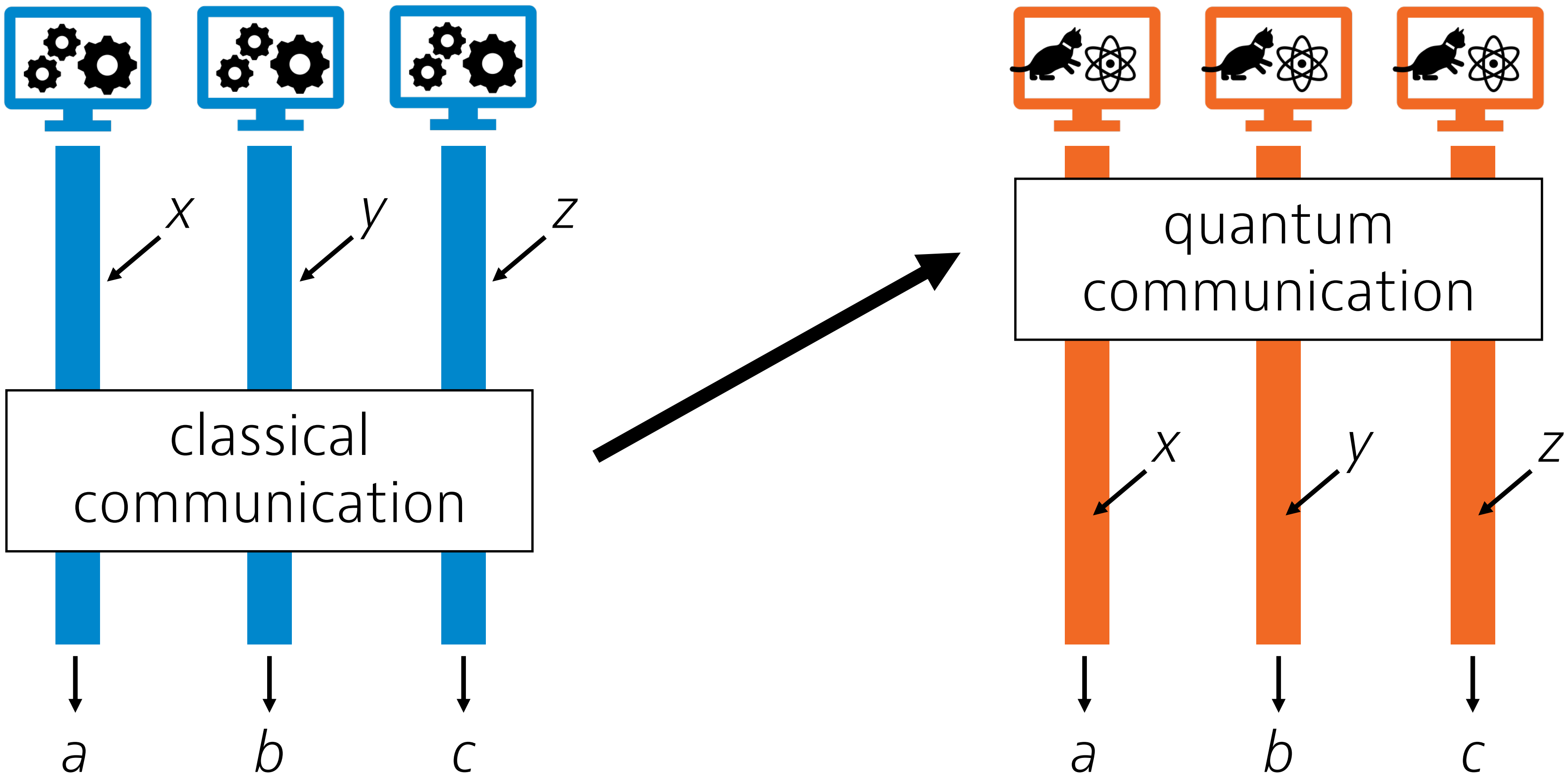
Quantum solution in 1 round: establish entangled qubits for all games in parallel, the rest is local work







Δ rounds
x
y
z
a
b
c
x
y
z
a
b
c
x
y
z
a
b
c
1 round


x
y
z
classical communication
a
b
c
quantum communication
x
y
z
a
b
c

Iterated GHZ problem

Classical

- compute
- communicate
- compute
- communicate
- compute
- communicate …

Quantum

- communicate
- communicate
- communicate …
- compute
- compute
- compute …

Iterated GHZ problem

Classical

- compute
- communicate
- compute
- communicate
- compute
- communicate …

Quantum

- communicate many things in parallel
- lots of local computation

Iterated GHZ problem

Classical

- compute
- communicate
- compute
- communicate
- compute
- communicate …

Quantum

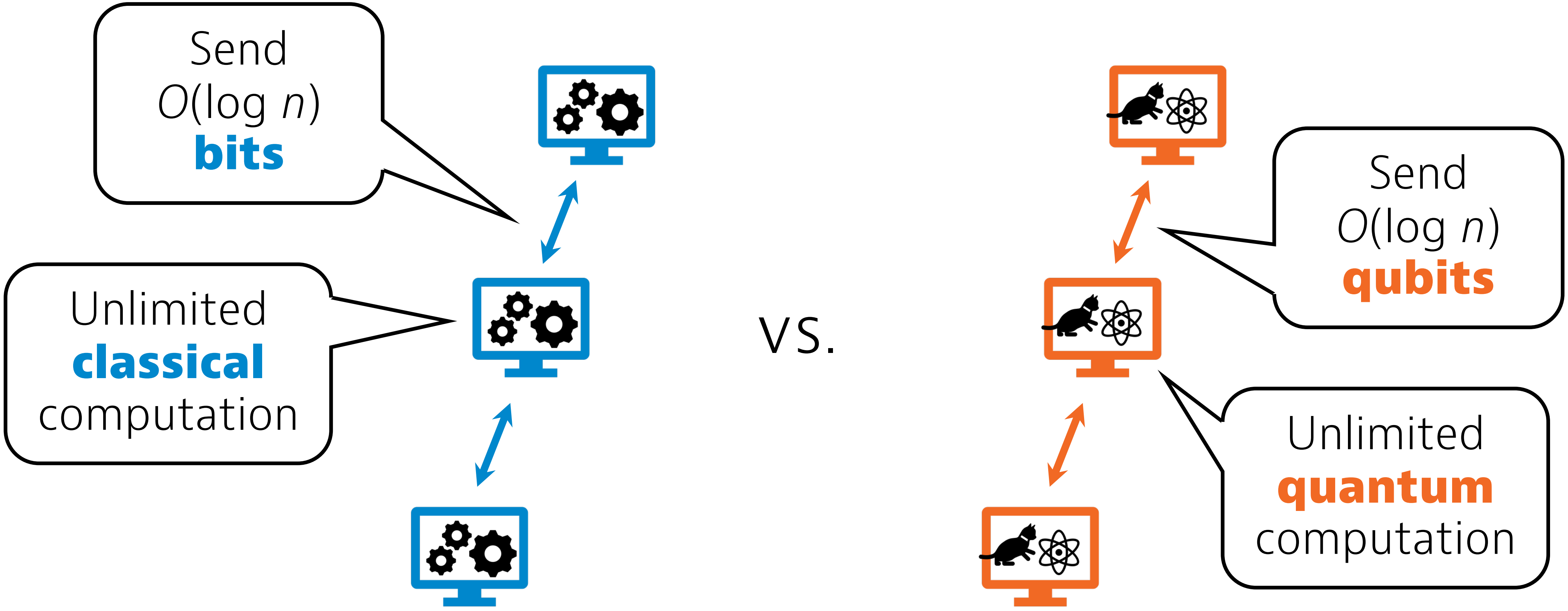
- communicate many things in parallel
- lots of local computation

→ **genuine quantum advantage**

Big picture:

CONGEST &
LOCAL

CONGEST model



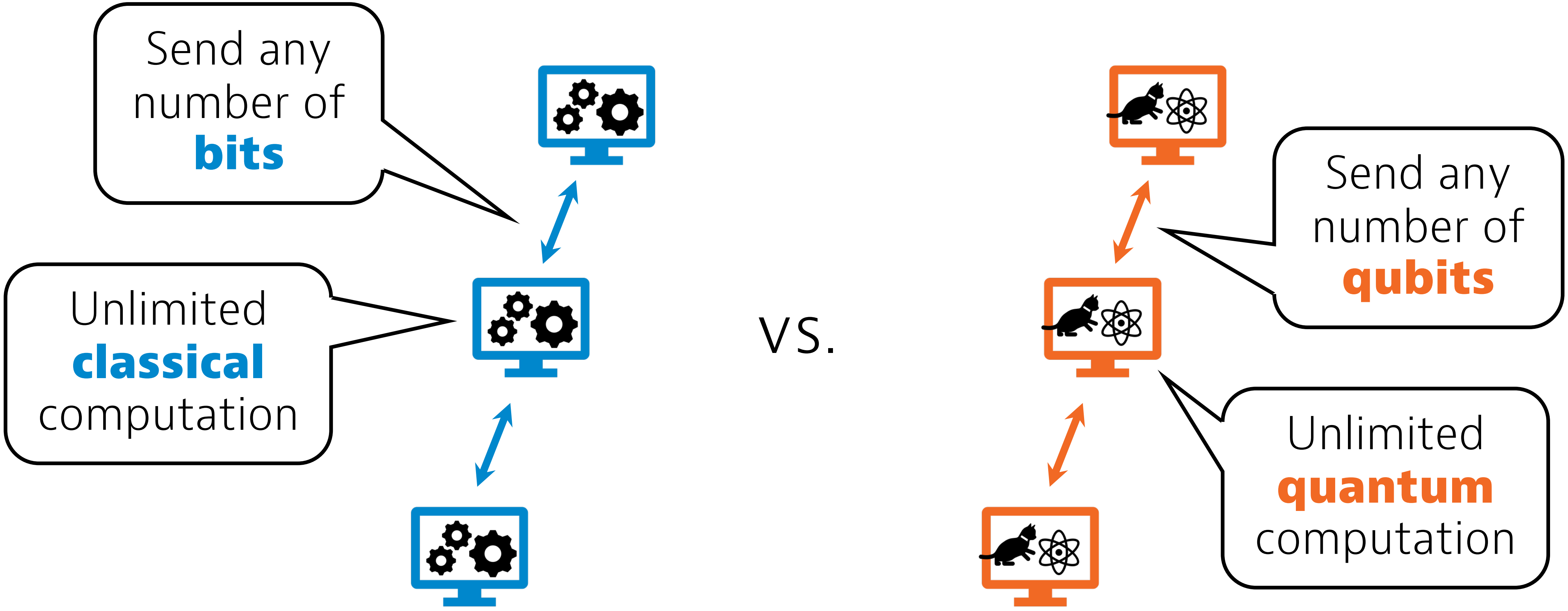
CONGEST vs. **quantum-CONGEST**:
bandwidth-limited setting

Grover helps in CONGEST

- Centralized Grover search:
  - check n candidates in $\approx n^{1/2}$ time
- **Distributed Grover search:**
  - can replace ***n classical search operations*** with ***$n^{1/2}$ quantum search operations***
- Helps with e.g. computing **diameter**

Frischknecht, Holzer, Wattenhofer (2012)
Le Gall, Magniez (2018)

LOCAL
model

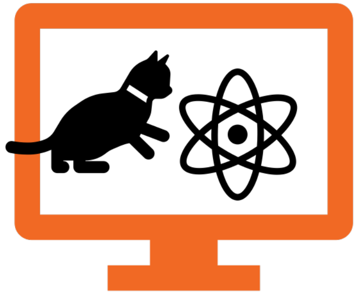


LOCAL vs. **quantum-LOCAL**:
advantage beyond bandwidth savings

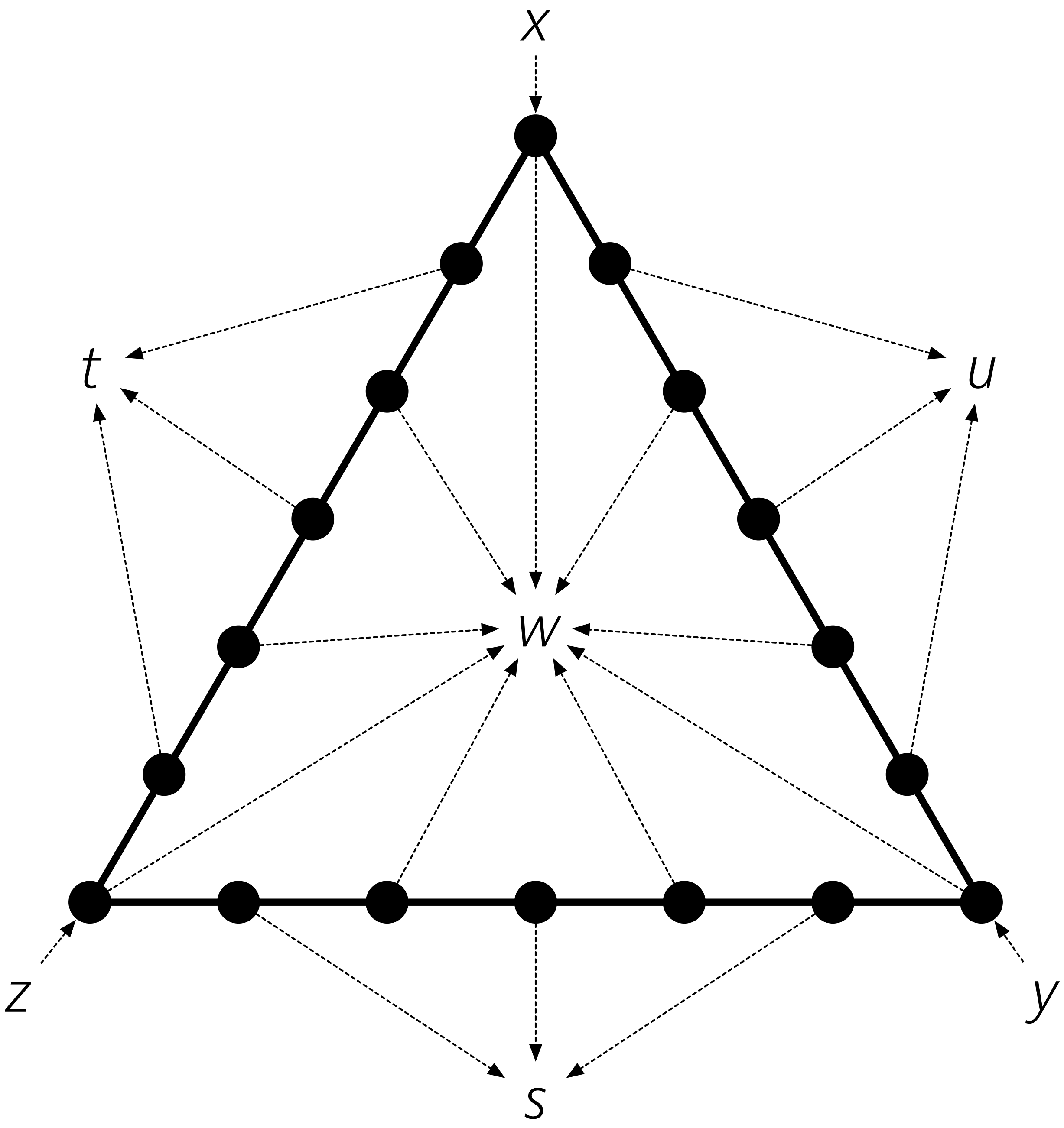
Le Gall, Nishimura, Rosmanis (2019)

Quantum Advantage for the LOCAL Model in Distributed Computing

n/6 rounds

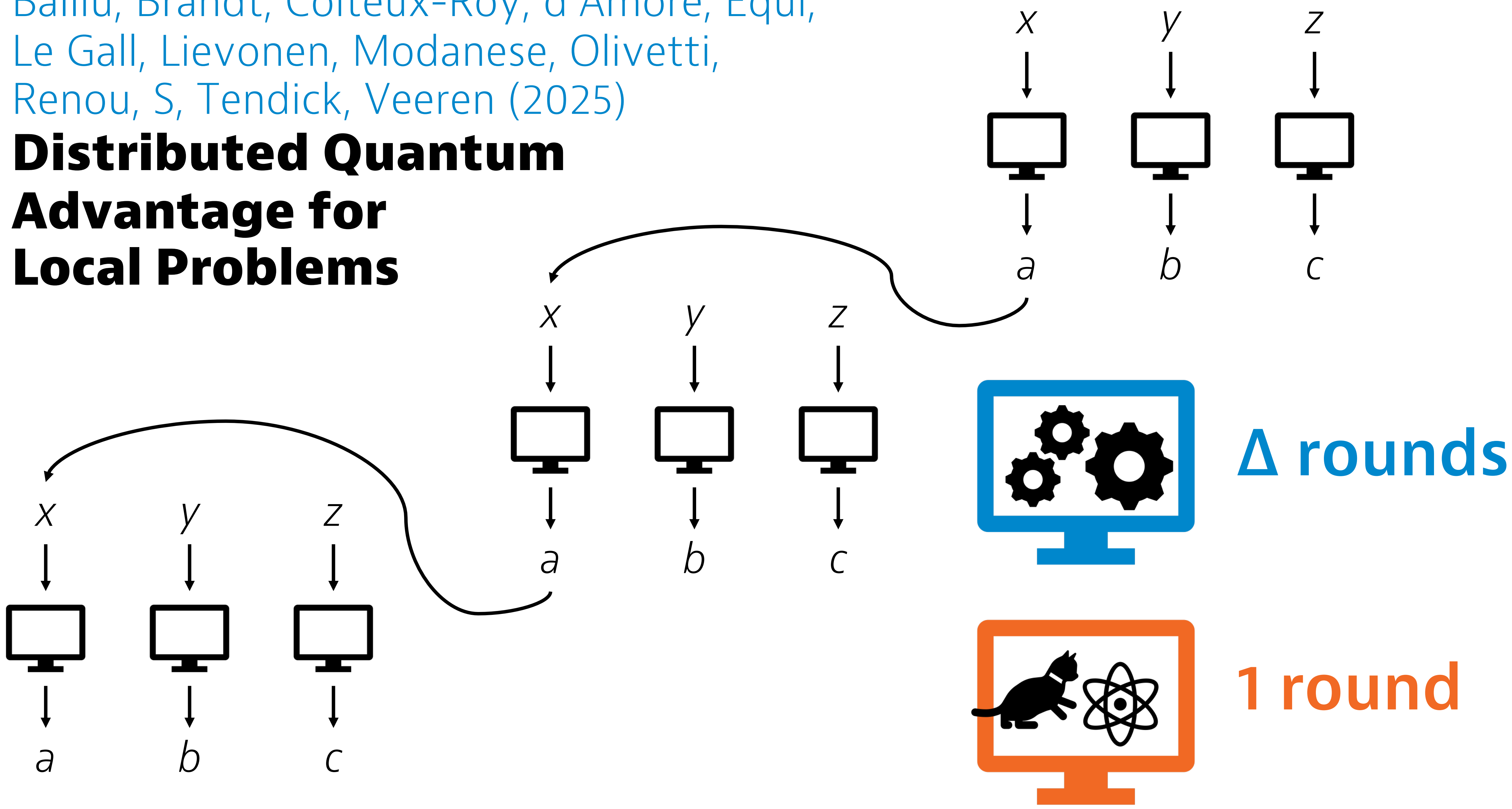


2 rounds



Balliu, Brandt, Coiteux-Roy, d’Amore, Equi, Le Gall, Lievonen, Modanese, Olivetti, Renou, S, Tendick, Veeren (2025)

Distributed Quantum Advantage for Local Problems



… and not much else!

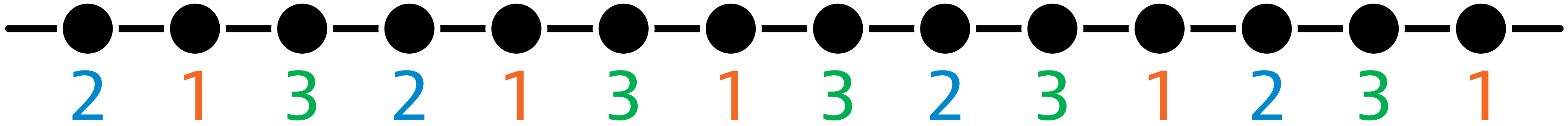
Quantum-LOCAL

- We have **artificial** problems that separate LOCAL and quantum-LOCAL
- Does quantum help with any **practically relevant** problem?

Quantum-LOCAL

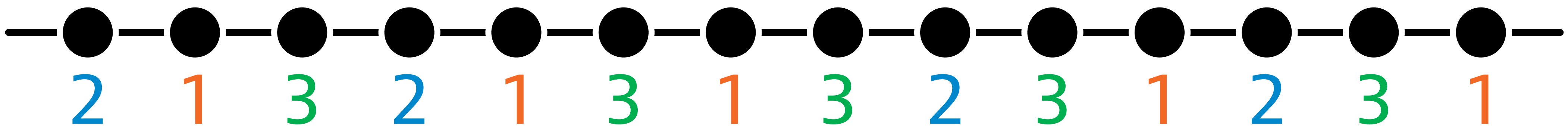
- We have **artificial** problems that separate LOCAL and quantum-LOCAL
- Does quantum help with any **practically relevant** problem?
- ***We do not know!***

3-coloring cycles & paths



Can be solved in $O(\log^* n)$ rounds,and this is tight for ***classical*** *algorithms*

3-coloring cycles & paths



Can be solved in $O(\log^* n)$ rounds,and this is tight for ***classical*** *algorithms*

Does ***quantum*** *help?* ***We do not know!***

Obstacles

Standard technique for showing lack of quantum advantage:

LOCAL ≤ quantum-LOCAL ≤ non-signaling

Obstacles

Standard technique for showing lack of quantum advantage:

LOCAL ≤ quantum-LOCAL ≤ non-signaling

Prove an upper bound here...

... and a matching lower bound here

Obstacles

Standard technique for showing lack of quantum advantage:

LOCAL ≤ quantum-LOCAL ≤ non-signaling

Prove an upper bound here...

No need to touch quantum!

... and a matching lower bound here

Obstacles

Standard technique for showing lack of quantum advantage:

LOCAL ≤ quantum-LOCAL ≤ non-signaling

Obstacles

Standard technique for showing lack of quantum advantage:

LOCAL ≤ quantum-LOCAL ≤ non-signaling

3-coloring is possible in $O(1)$ "rounds"

Obstacles

Standard technique for showing lack of quantum advantage:

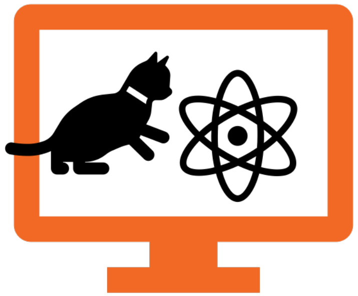
LOCAL ≤ quantum-LOCAL ≤ non-signaling

We must get our hands dirty with quantum

3-coloring is possible in $O(1)$ "rounds"

Summary

Distributed quantum advantage



- **CONGEST:** advantage for *interesting* tasks
- **LOCAL:** advantage for *artificial* tasks so far
- **Key open problems:**
  - 3-coloring cycles in $o(\log^* n)$ rounds?
  - bipartite maximal matching in $o(\Delta)$ rounds?
  - sinkless orientation in $o(\log \log n)$ rounds?